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A SPECIAL PROBLEM OF RECLINING BRIDGES

Tassos P. Avraam

Department of Civil Engineering, National Technical University of Athens
9 Iroon Polytechneiou Str., Zografou Campus, Athens 15780, Greece
e-mail: avraamt@central.ntua.gr

ABSTRACT: This paper deals with the linear dynamic response of a reclining bridge subjected to a load of constant magnitude which moves under the action of its weight, while the bridge goes up. This analysis focuses attention on the effect of the bridge's angular speed on its behavior under the action of a single load (one-axis load), or of a real vehicle model (two-axis load) while the influence of the damping of the beam is taken into account. The so produced oscillations are compared to those caused by a moving load passing the bridge with the maximum speed which get the load during the bridge's turning up. A variety of numerical results and diagrams allows us to draw important conclusions for structural design purpose.

KEYWORDS: Reclining bridges, Variable speeds, Moving loads.

1. INTRODUCTION

The determination of the dynamic effect of moving loads on elastic structures and, particularly, on bridges is a very complicated problem. This multi-parameter problem has been studied by many researchers in order to present reliable solutions.

After the first approximate solutions by Stoke [1], Zimmermann [2], Krylov [3], Timoshenko [4], and Lowan [5], a lot of researchers studied the complete problem including both parameters affecting mainly the dynamic behavior of a beam i.e. the mass of the girder and the mass of the moving load acting on the beam simultaneously [6 to 9].

The problem of the dynamic response of bridges under the action of moving loads is reviewed in detail by Timoshenko [10] and later by Kolousek [11]. One should also mention the excellent monograph on this subject by Fryba [12] and also his studies on the effect of the constant speed and damping on the response of a beam [13, 14].

Many investigators studied a lot of parameters usually neglected but affecting, same times significantly, the dynamic behavior of a bridge.

One can mention, for example, the type of the vehicle by Veletsos and Huang [15], the mass of the moving load by Michaltsos at al. [16], the constants of the springs and dampers by Fertis [17], the bridge's uneven deck by Abdel-Rohmal and Al Duaij [18], Michaltsos [19, 20], the centripetal and Coriolis forces by Michaltsos [21], the critical train's speed by Michaltsos and Raftoyiannis [22].

Although the influence of a variable speed is studied in detail [23], and also the influence of the inclination of a beam [24, 25], the combination of both parameters that appear in a declining bridge is an interesting case.

There is a lot of papers dealing mainly with reconstruction [26], monitoring [27], or design of bascule (reclining) bridges [28], and also the behavior of the deck pavement under different conditions [29, 30], while only a little of these papers study the dynamic behavior under seismic forces (as for example [31]).

The present paper examines the influence of the angular speed of a declining bridge subjected to the action of a load or of a vehicle moving under the action of its weight on a turning up bridge. The so produced deformations are compared to those caused by a moving load on the same bridge at rest.

The current operation codes are very strict. Thus, such a scenario is an unrealistic one. Nevertheless there are some cases of accidents occurring, due to violation of the codes or human negligence.

Having as purpose the study of the problem itself and its effect on the bridge's behavior, we will consider a bridge deck with prismatic cross-section, constant along the bridge-deck length, instead of the usually used one with changed cross-section along the bridge length. The above does not affect the generality of the present study.

Two cases are considered. Firstly the concentrated load and secondly the vehicle (with wheelbase equal to 2d), which both are motionless at $t = 0$ and start to move when the bridge starts to go up. The so-produced oscillations are compared to those caused by a moving load passing the bridge with the maximum speed which gets the load during the bridge's turning up.

The approach is based on the Euler-Bernoulli's beam theory.

At a first view, the problem seems simple, but the resulting equations contain strongly non-linear terms. The so gathered strongly non-linear equations are solved using the Duhamel's and the Euler's gamma integrals, namely using the integrals of the inverse error functions (Gaussian Integrals).

For the determination of the above integrals, it is proposed a easier way, based on the simulation of some terms with simple algebraic and logarithmic functions that one can easily integrate.

A variety of numerical results and diagrams allows us to draw important conclusions for structural design purpose.

2. MATHEMATICAL FORMULATION

Let us consider now the declined bridge, shown in figure 1, composed by two cantilever beams of length ℓ, having a prismatic cross-section with constant mass per unit length m, flexural rigidity EI and damping coefficient c, made

from linear, homogeneous and isotropic material. At the instant t=0, there is a load (or a vehicle) F (of mass M) at the edge B of the left bean (fig. 1).
The bridge can go up, turning around A with angular speed φ_o.
At time t, the bridge will rotate by angle

$$\varphi = \varphi_o t \qquad (1.a)$$

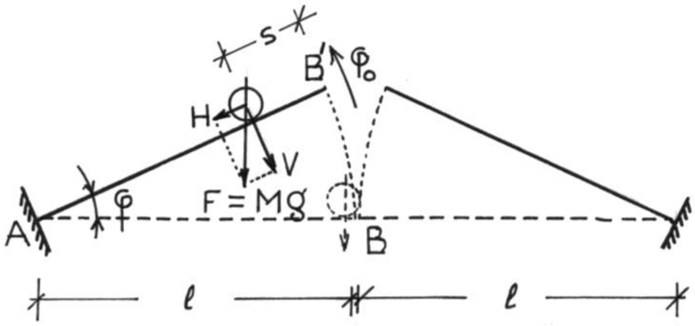

Figure 1: Sketch of a reclined bridge

Therefore the load or the vehicle, according to the Alembert principle will start moving towards the left end of the bridge with acceleration:

$$\gamma = \frac{H}{M} = \frac{F \cdot \sin \varphi}{F/g} = g \cdot \sin \varphi_o t \qquad (1.b)$$

The developed speed of the load or vehicle will be $d\upsilon = \gamma dt$ and therefore:

$$\upsilon = \int_0^t g \cdot \sin \varphi_o t \, dt = \frac{g}{\varphi_o}(1 - \cos \varphi_o t) \qquad (1.c)$$

Finally the travelled distance will be $ds = \upsilon dt$, or:

$$s = \int_0^t \frac{g}{\varphi_o}(1 - \cos \varphi_o t) \, dt = \frac{g}{\varphi_o} t - \frac{g}{\varphi_o^2} \sin \varphi_o t \qquad (1.d)$$

If the load F travels the distance (ℓ-s_o) in time t_o (where s_o is the distance of the initial position of the load F from the end B) will be:

$$\frac{g}{\varphi_o} t_o - \frac{g}{\varphi_o^2} \sin \varphi_o t_o = \ell - s_o \qquad (1.e)$$

Solving the above trigonometric equation (through a graphical or arithmetical way), one can determine the needed time passage t_o.
Finally the needed time for the complete pull up of the bridge is:

$$t_d = \frac{\pi/2}{\varphi_o} = \frac{\pi}{2\varphi_o} \qquad (1.f)$$

2.1 The concentrated load

The simplest and more usual case is the one of a moving load F without consideration of inertia forces (like mass, or centripetal and Coriolis forces, the influence of which has been already studied in [16, 21]). The equation of motion of a reclined bridge under the action of a moving load F is (see fig. 1):

$$EIw''''(x,t) + c\dot{w}(x,t) + m\ddot{w}(x,t) = V \cdot \delta(x - \ell + s) \quad \text{or}$$

$$EIw''''(x,t) + c\dot{w}(x,t) + m\ddot{w}(x,t) = F \cdot \cos\varphi_o t \cdot \delta(x - \ell + s) \qquad (2.a)$$

where $\delta(x-a)$ is the Dirac delta function.

A series solution of equation (2.a) in terms of linear normal modes can be sought in the form:

$$w(x,t) = \sum_n w_n(x,t) = \sum_n X_n(x) T_n(t) \qquad (2.b)$$

where $X_n(x)$ is the shape functions of a cantilever beam, given by many technical books as for example by [32], and $T_n(t)$ are time functions under determination.

Introducing (2.b) into (2.a), we get:

$EI\sum_n X_n'''' T_n + c\sum_n X_n \dot{T}_n + m\sum_n X_n \ddot{T}_n = F \cdot \cos\varphi_o t \cdot \delta(x - \ell + s)$ and because X_n satisfies the equation of the free motion $EIX_n'''' - m\omega_n^2 X_n = 0$, the above becomes:

$$\sum_n X_n \ddot{T}_n + \frac{c}{m}\sum_n X_n \dot{T}_n + \sum_n \omega_n^2 X_n T_n = F \cdot \cos\varphi_o t \cdot \delta(x - \ell + s) \qquad (2.c)$$

Multiplying the latter by X_ρ, integrating over the domain and considering the orthogonality condition, the differential equation of the ρ^{th} mode of the generalized deflection can be written as:

$$\left.\begin{aligned} &\ddot{T}_\rho + \frac{c}{m}\dot{T}_\rho + \omega_\rho^2 T_\rho = \frac{M \cdot g \cdot \cos\varphi_o t}{m\int_0^\ell X_\rho^2 dx} \cdot X_\rho(\ell - s) \\ &\text{with: } s = \frac{g}{\varphi_o} t - \frac{g}{\varphi_o^2}\sin\varphi_o t \end{aligned}\right\} \qquad (2.d)$$

where ω_ρ is the ρ^{th} eigenfrequency of the freely vibrating cantilever.

The solution of the above is given by the Duhamel's integral:

$$\left.\begin{aligned}
&T_\rho(t) = e^{-\beta t}(A_\rho \sin\overline{\omega}_\rho t + B_\rho \cos\overline{\omega}_\rho t)\\
&\qquad + \frac{M\cdot g}{\Gamma_\rho}\int_0^t e^{-\beta(t-\tau)}\cos\varphi_o\tau\cdot X_\rho\left(\ell - \frac{g}{\varphi_o}\tau + \frac{g}{\varphi_o^2}\sin\varphi_o\tau\right)\cdot\sin\overline{\omega}_\rho(t-\tau)\,d\tau\\
&\text{where:}\quad \Gamma_\rho = m\overline{\omega}_\rho\int_0^\ell X_\rho^2 dx\,,\quad \beta = \frac{c}{2m}\,,\quad \overline{\omega}_\rho = \sqrt{\omega_\rho^2 - \beta^2}
\end{aligned}\right\}\quad (2.e)$$

In order to determine the integral of the above eq (2e), we express $\sin\varphi_o t$ with a series $\sin\varphi_o t = \sum_n^n (-1)^n \frac{(\varphi_o t)^{2n+1}}{(2n+1)!}$, and neglecting the higher order terms, eq(2e) becomes:

$$\begin{aligned}
&T_\rho(t) = e^{-\beta t}(A_\rho \sin\overline{\omega}_\rho t + B_\rho \cos\overline{\omega}_\rho t)\\
&\qquad + \frac{M\cdot g}{\Gamma_\rho}\int_0^t e^{-\beta(t-\tau)}\cos\varphi_o\tau\cdot X_\rho\left(\ell - \frac{g\varphi_o\tau^3}{6}\right)\cdot\sin\overline{\omega}_\rho(t-\tau)\,d\tau
\end{aligned}$$

The above integral, can be find using the Euler's gamma integrals: $\Gamma(a, z_o, z_1) = \int_{z_o}^{z_1} t^{a-1}e^{-t}dt$, namely using the integrals of the inverse error functions (Gaussian Integrals).

There are also different simplest methods, but with the same accuracy, for the determination of the above integral. The simplest of them is the simulation of the terms that compose the shape function, with simple algebraic and logarithmic functions that one can easily integrate.

Finally the factors A_ρ and B_ρ are determined through the use of the initial conditions:

$$\left.\begin{aligned}
&w(x,0) = \frac{F\ell^3}{3EI}\left(\frac{3x^2}{2\ell^2} - \frac{x^3}{2\ell^3}\right)\\
&\dot{w}(x,0) = 0
\end{aligned}\right\}\quad (2.f)$$

taking into account that, at the starting of the rotation of the bridge, the load F is applied on the point B (see fig. 1) and therefore the bridge gets static deflection only.

2.2 The real vehicle

Let us consider now the biaxial vehicle of figure 2, having wheelbase equal to 2d and distance of its gravity center S from the bridge's surface equal to h. The mass of the vehicle is M, equally distributed on its two axes. Finally we neglect the vehicle's rotatory mass of inertia.

We assume that at t=0, the front axis of the vehicle is located on point B and that for a bridge at rest (φ=0), the load F=Mg is divided equally between the two axles of the vehicle.

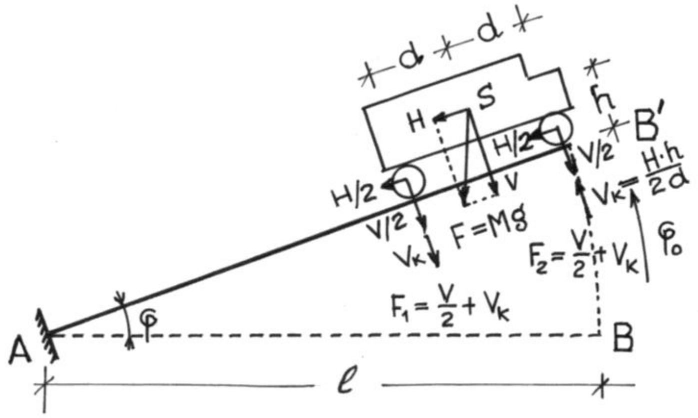

Figure 2: The real vehicle

The forces F_1 and F_2 are:

$$\left.\begin{aligned} F_1 &= \frac{V}{2} + V_k \\ F_2 &= \frac{V}{2} - V_k \end{aligned}\right\}$$

and because:

$$\left.\begin{aligned} V &= M \cdot g \cdot \cos\varphi_o t \\ V_k &= \frac{H \cdot h}{2d} \\ H &= M \cdot g \cdot \sin\varphi_o t \end{aligned}\right\}$$

they can be expressed as follows:

$$\left.\begin{aligned} F_1 &= \frac{M \cdot g}{2} \cdot \cos\varphi_o t + \frac{M \cdot g \cdot h}{2d} \cdot \sin\varphi_o t \\ F_2 &= \frac{M \cdot g}{2} \cdot \cos\varphi_o t - \frac{M \cdot g \cdot h}{2d} \cdot \sin\varphi_o t \end{aligned}\right\} \qquad (3.a)$$

The back wheel will arrive first on A at time t_1 given by the solution of the equation:

$$\frac{g}{\varphi_o} t_1 - \frac{g}{\varphi_o^2} \cdot \sin\varphi_o t_1 = \ell - 2 \cdot d \qquad (3.b)$$

Analogously, the front wheel will arrive on A at time t_2 given by the solution of the equation:

$$\frac{g}{\varphi_o} t_2 - \frac{g}{\varphi_o^2} \cdot \sin\varphi_o t_2 = \ell \qquad (3.c)$$

The analogous of (2.a) equation, for the vehicle of fig. 2 will be:

$$\left.\begin{aligned} EIw''''(x,t) + c\dot{w}(x,t) + m\ddot{w}(x,t) &= \frac{M\cdot g}{2}\cdot\left(\cos\varphi_o t - \frac{h}{d}\sin\varphi_o t\right)\cdot\delta(x-\ell+s) \\ &+ \frac{M\cdot g}{2}\cdot\left(\cos\varphi_o t + \frac{h}{d}\sin\varphi_o t\right)\cdot\delta(x-\ell+s+2d) \end{aligned}\right\} \qquad (4.a)$$

A solution is sought with the form:

$$w(x,t) = \sum_n w_n(x,t) = \sum_n X_n(x) T_n(t) \qquad (4.b)$$

where $X_n(x)$ is the shape functions of a cantilever beam, and $T_n(t)$ are time functions under determination.

Following a similar procedure like the one of §2.1, we conclude to the following equation:

$$\left.\begin{aligned} \ddot{T}_\rho + 2\beta\dot{T}_\rho + \omega_\rho^2 T_\rho &= \frac{Mg}{2m\int_0^\ell X_\rho^2 dx}\left(\cos\varphi_o t - \frac{h}{d}\sin\varphi_o t\right)\cdot X_\rho\left(\ell - \frac{g}{\varphi_o} t + \frac{g}{\varphi_o^2}\sin\varphi_o t\right) \\ &+ \frac{Mg}{2m\int_0^\ell X_\rho^2 dx}\left(\cos\varphi_o t + \frac{h}{d}\sin\varphi_o t\right)\cdot X_\rho\left(\ell - 2d - \frac{g}{\varphi_o} t + \frac{g}{\varphi_o^2}\sin\varphi_o t\right) \end{aligned}\right\} \qquad (4.c)$$

The solution of the above equation is given by the Duhamel's integral:

$$\left.\begin{aligned} T_\rho(t) &= e^{-\beta t}(A_\rho \sin\overline{\omega}_\rho t + B_\rho \cos\overline{\omega}_\rho t) + \\ &\frac{Mg}{2\Gamma_\rho}\int_0^t e^{-\beta(t-\tau)}\left(\cos\varphi_o\tau - \frac{h}{d}\sin\varphi_o\tau\right)\cdot X_\rho\left(\ell - \frac{g}{\varphi_o}\tau + \frac{g}{\varphi_o^2}\sin\varphi_o\tau\right)\sin\overline{\omega}_\rho(t-\tau)\,d\tau + \\ &\frac{Mg}{2\Gamma_\rho}\int_0^t e^{-\beta(t-\tau)}\left(\cos\varphi_o\tau + \frac{h}{d}\sin\varphi_o\tau\right)\cdot X_\rho\left(\ell - 2d - \frac{g}{\varphi_o}\tau + \frac{g}{\varphi_o^2}\sin\varphi_o\tau\right)\sin\overline{\omega}_\rho(t-\tau)\,d\tau \end{aligned}\right\} \qquad (4.d)$$

where Γ_ρ, $\overline{\omega}_\rho$, β are given by eq (2.e).

the above integrals can be determined as in §2.1.

Finally the coefficients A_ρ and B_ρ are determined by the initial conditions:

$$\left.\begin{aligned}
&w_{o1}(x,0)=\frac{F_1}{2}\cdot\frac{(\ell-2d)^3}{3EI}\left[\frac{3x^2}{2(\ell-2d)^2}+\frac{x^3}{2(\ell-2d)^3}\right]+\frac{F_2}{2}\cdot\frac{\ell^3}{3EI}\left[\frac{3x^2}{2\ell^2}+\frac{x^3}{2\ell^3}\right]\\
&\text{for}\quad 0\le x\le \ell-2d\\
&w_{12}(x,0)=w_{o1}(\ell-2d,0)+2d\cdot w'_{o1}(\ell-2d,0)+\frac{F_2}{2}\cdot\frac{\ell^3}{3EI}\left[\frac{3x^2}{2\ell^2}+\frac{x^3}{2\ell^3}\right]\\
&\text{for}\quad \ell-2d\le x\le \ell\\
&\dot{w}(x,0)=0
\end{aligned}\right\}\qquad (4.e)$$

2.3 Moving load on a bridge at rest

2.3.1 The concentrated load

We consider the load F, which enters the bridge from point A and moves with constant speed υ.

Then, if F is the only dynamic load applied on the bridge, the equation of motion will be:

$$EIw''''(x,t)+c\dot{w}(x,t)+m\ddot{w}(x,t)=F\cdot\delta(x-\alpha)\qquad (5.a)$$

where α is the position of the load F on the beam at time t.

We shall seek a solution in the form of separate variables such as:

$$w(x,t)=\sum_n X_n(x)T_n(t)\qquad (5.b)$$

where X_n is the n^{th} shape function of the cantilever beam and T_n the corresponding time function, to be determined. Following the same procedure like the one of §2, we arrive to the following equation for the ρ^{th} time function:

$$\ddot{T}_\rho+\frac{c}{m}\dot{T}_\rho+\omega_\rho^2T_\rho=\frac{F}{m\int_0^\ell X_\rho^2dx}\cdot X_\rho(\upsilon t)\qquad (5.c)$$

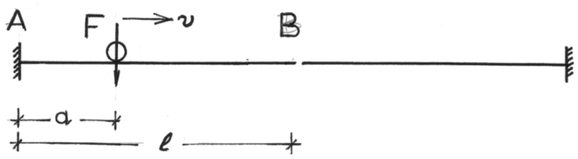

Figure 3: Load on the non-moving bridge

The solution of the above is given by the Duhamel's integral:

$$\left.\begin{aligned} &T_\rho(t) = \frac{M \cdot g}{\Gamma_\rho} \int_0^t e^{-\beta(t-\tau)} X_\rho(\upsilon\tau) \cdot \sin\overline{\omega}_\rho(t-\tau)\,d\tau \\ &\text{where:}\quad \Gamma_\rho = m\overline{\omega}_\rho \int_0^\ell X_\rho^2 dx\,, \quad \beta = \frac{c}{2m}\,, \quad \overline{\omega}_\rho = \sqrt{\omega_\rho^2 - \beta^2} \end{aligned}\right\} \qquad (5.d)$$

2.3.2 The real vehicle

We consider now the vehicle of figure 4, which enters the bridge from point A and moves with constant speed υ. Then, the equation of motion will be:

$$EIw''''(x,t) + c\dot{w}(x,t) + m\ddot{w}(x,t) = F_1 \cdot \delta(x-\alpha) + F_2 \cdot \delta(x-\alpha+2d) \qquad (6.a)$$

Seeking again for a solution in the form of separate variables such as:

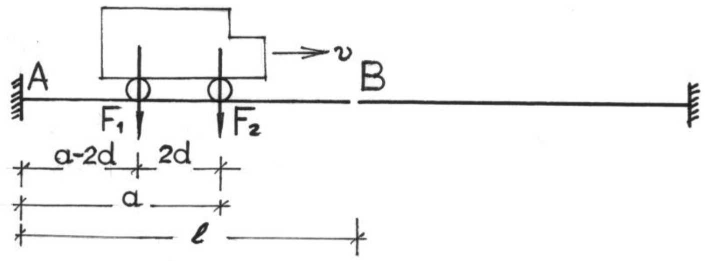

Figure 4: Vehicle on the non-moving bridge

$$6\,w(x,t) = \sum_n X_n(x) T_n(t) \qquad (6.b)$$

and applying the same procedure like the one of §2, we arrive to the following equation for the ρ^{th} time function:

$$\left.\begin{aligned} T_\rho(t) = {} & \frac{F_1}{\Gamma_\rho} \int_0^t e^{-\beta(t-\tau)} X_\rho(\upsilon\tau) \cdot \sin\overline{\omega}_\rho(t-\tau)\,d\tau + \\ & + \frac{F_2}{\Gamma_\rho} \int_{t_1}^t e^{-\beta(t-\tau)} X_\rho(\upsilon\tau - 2d) \cdot \sin\overline{\omega}_\rho(t-\tau)\,d\tau + \\ \text{where:}\quad & \Gamma_\rho = m\overline{\omega}_\rho \int_0^\ell X_\rho^2 dx\,, \quad \beta = \frac{c}{2m}\,, \quad \overline{\omega}_\rho = \sqrt{\omega_\rho^2 - \beta^2}\,, \quad t_1 = \frac{2d}{\upsilon} \end{aligned}\right\} \qquad (6.c)$$

3. NUMERICAL RESULTS AND DISCUSSION

We consider the reclining bridge of figure 1, with the following data: ℓ=30m, I=0.04m^4, m=400 kg/m, c=1500 Nsec/m.

We will study the behavior of the bridge for the following three angular speeds: $\varphi_o = 0.06,\ 0.08,\ \text{and}\ 0.10\,\text{rad/sec}$.

As for the loads, we will study firstly the case of a concentrated load F=200 kN and after the case of a beaxial vehicle having wheelbase 2d=6m, h=1.5m and F=200 kN, and of a beaxial vehicle with wheelbase 2d=3m, h=1.5m and F=40kN.

3.1 The concentrated load

The time passages of the load F for each one angular speed are determined from equation (1.e) with s_o=0, as follows:

angular speed: φ_o= 0.06 rad/sec, time passage: t_o=6.712 sec
angular speed: φ_o= 0.08 rad/sec, time passage: t_o=6.106 sec
angular speed: φ_o= 0.10 rad/sec, time passage: t_o=5.675 sec

Applying the formulae of §2.1, for a load F=200 kN, we obtain the diagrams of fig.3, which show the vibrations of the bridge from $t = 0$ to $t = \pi / 2\varphi_o$, when the bridge becomes vertical. For t > t_o the bridge vibrates freely.

From the diagrams of figure 5 we see that the dynamic deflections are ~19% greater than the static ones. We observe also that at the beginning of the motion, the deflections are almost the same for any speed, while, after some time, the lower angular speeds produce greater deflections.

Finally the bridge stops to vibrate free, before to come to its final vertical position. The time t_p needed for the complete pull up of the bridge is $t_p = 15.701,\ \text{for}\ \varphi_o = 0.10$, $t_p = 19.635,\ \text{for}\ \varphi_o = 0.08$ and $t_p = 26.180,\ \text{for}\ \varphi_o = 0.06$.

3.2 The real vehicle

We will study the behavior of a bridge, under the action of a real vehicle, comparing simultaneously the bridge's behavior with the one under the action of a concentrated load of equal magnitude.

We note that from now on we will continue keeping only the angular speed φ_o= 0.06 rad/sec, because, according to the previous paragraphs, this speed produces the worse dynamic behavior.

For a vehicle of weight F=200 kN, 2d=6.0m and h=1.5m we obtain the following diagrams of figure 6.

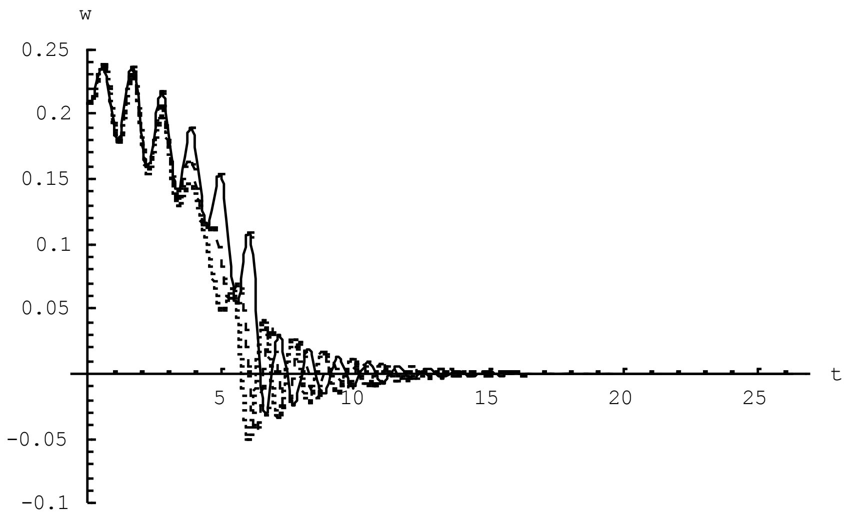

Figure 5: Oscillations of the end B of the bridge under the action of a moving load for different angular speeds ___ φ_o=0.06, _ _ _ φ_o=0.08, - - - φ_o=0.10 rad/sec

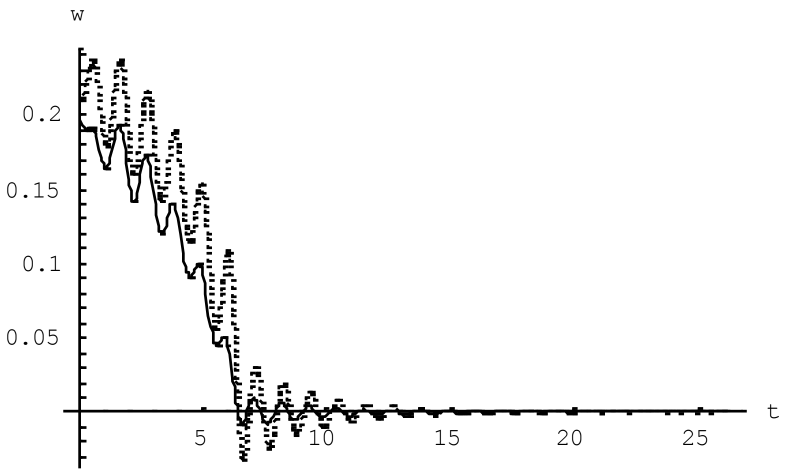

Figure 6: Oscillations of the end B of the bridge turning up with angular speeds φ_o=0.06, under the action of a the real vehicle with great wheelbase (__), or of a the concentrated load (- - -).

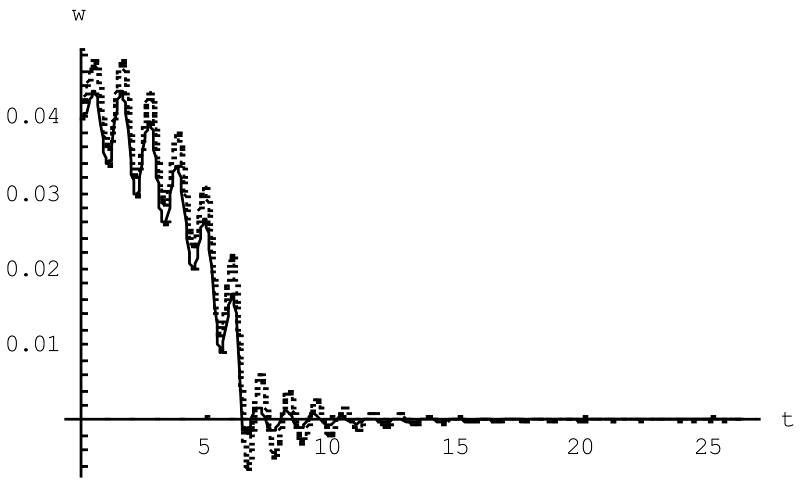

Figure 7: Oscillations of the end B of the bridge turning up with angular speeds φ_o=0.06, under the action of a the real vehicle with small wheelbase (__), or of a the concentrated load (- - -).

For a vehicle of weight F=40 kN, 2d=3.0m and h=1.5m we obtain the diagrams of figure 7.

3.3 Loads on a bridge at rest

3.3.1 The concentrated load

Let us consider now the studied bridge at rest.

At t=0, the load of §3.1 (F=200 kN), enters the bridge with constant speed equal to $\upsilon = \frac{g}{\varphi_o}(1-\cos\varphi_o t_o)$, with $\varphi_o = 0.06\,\text{rad/sec}$ and $t_o = 6.712\,\text{sec}$. The above considered speed is the one that has the load at t_o=6.712 sec, when exits the bridge (see §3.1).

Applying equation (5.d) we obtain the plot of figure 8, where are also drawn the oscillations of the reclined bridge, gathered in §3.1 for $\varphi_o = 0.06\,\text{rad/sec}$.

One can easily ascertain that the produced oscillations in the case of the reclined bridge, compared to those produced by a load moving on the bridge at rest are ~20% greater $\frac{0.24-0.20}{0.20}\cdot 100 = 20\,\%$.

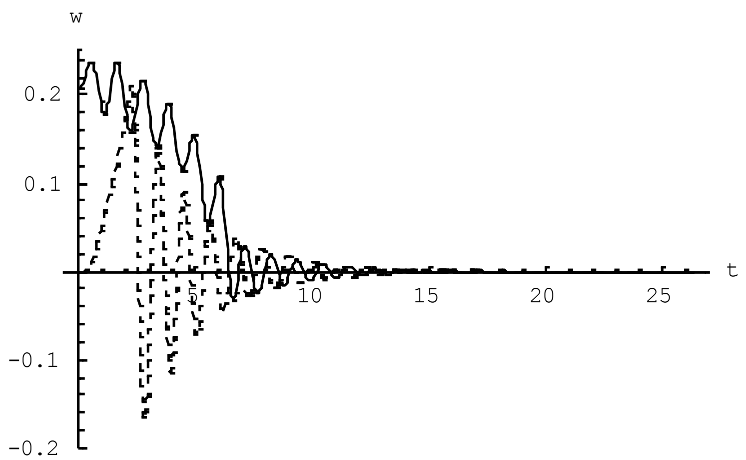

Figure 8: Oscillations of the end B of the bridge turning up with angular speeds φ_o=0.06 (__), compared to those produced by a moving load (- - -).

3.3.2 The real vehicle

Considering again the bridge at rest, the vehicle of §3.2 (with 2d=6m), enters the bridge with constant speed equal to $\upsilon = \frac{g}{\varphi_o}(1-\cos\varphi_o t_o)$, with $\varphi_o = 0.06\,\text{rad/sec}$ and $t_o = 6.712\,\text{sec}$. The above considered speed is the one that has the vehicle at t_o=6.712 sec, when exits the bridge (see §3.2).

Applying equation (6.c) we obtain the plot of figure 9, where are also drawn the oscillations of the reclined bridge, gathered in §3.2 for $\varphi_o = 0.06\,\text{rad/sec}$.

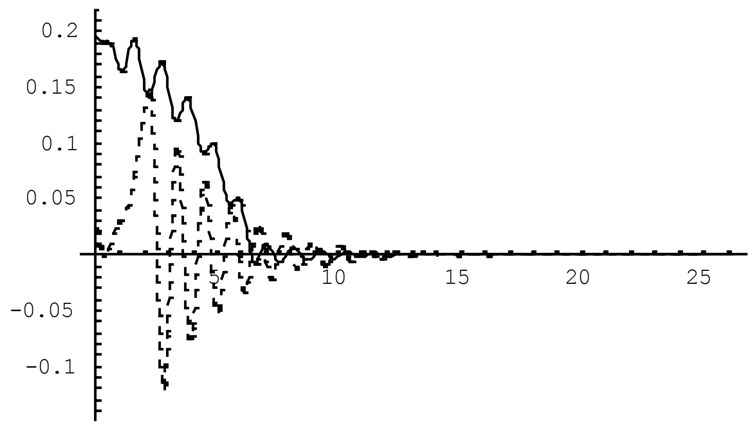

Figure 9: Oscillations of the end B of the bridge turning up with angular speeds φ_o=0.06 (__), compared to those produced by a real vehicle (- - -).

One can easily ascertain that the produced oscillations in the case of the reclined bridge, compared to those produced by a real vehicle moving on the bridge at rest are ~20% greater $\frac{0.21-0.15}{0.15}\cdot 100 = 40\,\%$.

4. CONCLUSIONS

From the results of the model considered, one can draw the following conclusions:

1. The real vehicle model is, of course, more accurate than the one of the concentrated load.
2. At the beginning of the bridge pull up, the dynamic deflections are, for any angular speed, practically equal, while after some instants, the lower angular speeds produce greater deflections.
3. For small wheelbases (with 2d< ℓ/10), the results are similar to the ones produced by a concentrated load with equal magnitude. This fact is also pointed out in [16, 21].
4. The dynamic deflections for concentrated loads or vehicles with small wheelbase are ~19% greater than the static ones, but this difference becomes small or negligible for great wheelbases (with 2d> ℓ/5).
5. After the load's exit, the bridge vibrates freely. This last motion ends before the complete pull up of the bridge.
6. The each time codes in force have to take into account the fact that the produced oscillations by a load or vehicle rolling freely on a turning up bridge, are significantly greater than the ones produced by the same load or vehicle moving on the bridge at rest.

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