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OPTIMAL LOCATION OF ARCH BRACINGS IN ARCH BRIDGES FOR MAXIMUM BUCKLING LOAD

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ABSTRACT: To overcome the drawbacks of existing design methods for location of arch bracings in arch bridges, a systematic topology optimization based approach for identifying the optimum location of arch bracings is proposed in this paper. The accuracy and efficiency of the proposed method is demonstrated through a numerical example.

KEY WORDS: Arch bridge; Buckling; Location of arch bracings; Optimization.

1 INTRODUCTION

Arch bridges are one of the oldest types of bridges and have great natural strength. However, they can be subjected to a loss of stability under combined loadings. One common and cost-effective approach to increasing the buckling loads of arch bridges is the addition of arch bracing, which supplements the rigidity of base structures by increasing their moment of inertia of cross-sections. The design of arch bracings involves the determination of the location of the added arch bracings. To obtain the maximum advantage from the added arch bracings, the location of the added arch bracings should be placed optimally.

Traditionally an initial feasible location of arch bracings is efficiently designed by following some heuristic design rules. However, there exist some design situations, with special loading conditions or structural singularities for which these simple design rules cannot be applied. Moreover, the design procedure does not seem to be optimal. The location of arch bracings obtained was often improved by the designer in a later stage of the process of obtaining the final design. In these cases it is necessary to use a more direct approach in order to automatically identify the optimal location of arch bracings with minimum designer intervention.

The parametric analysis-based design method has been proposed by Ney et al. [1] as a rational tool for identifying the optimal location of arch bracings. Unfortunately, extensive computational effort may be required to obtain the optimal location of arch bracings if the parametric analysis is repeated for every change in location of arch bracings.

To overcome these difficulties and to increase efficiency in identifying the optimal location of arch bracings, the application of the topology optimization procedure, which has received considerable attention in recent years, has been introduced as an alternative approach. Topology optimization is a tool that can be used to find optimal layout of the structural elements in a given design domain. The application of the topology optimization to shell/plate structures has been studied by a considerable number of investigators including Diaz and Kikuchi [2], Ma and Kikuchi [3], Yang and Chahande [4] and Lam and Santhikumar [5]. However, to the authors' knowledge, its application to an optimal location problem of arch bracings in an arch bridge has not been reported.

The objective of this paper is to propose a systematic topology optimization method to deal with the problem of optimizing location of arch bracings in an arch bridge to maximize buckling load. The optimum location problem is formulated as a nonlinear mathematical programming problem because the objective function in general is a nonlinear function of the design variables representing the location of arch bracing. A numerical example is given to show the efficiency and accuracy of the present method. It is found that the proposed method can be an alternative useful tool for engineering applications.

1 PROBLEM STATEMENT OF OPTIMAL LOCATION OF ARCH BRACINGS

The optimal location problem is to find a design variable vector representing location of arch bracings to maximize the buckling load of arch bridges subjected to the design constraints that are imposed as inequality constraints on design variables. The optimal location problem may be correspondingly stated as follows:

$$\text{Maximize } P_i(V), i = 1,\ldots,k \tag{1}$$

$$\text{Subjected to } L_j \le V_j \le U_j, j = 1,\ldots,n \tag{2}$$

where $P_i(V)$ is the i-th buckling load of arch bridges. Here we assume that P_i is in ascending order, i.e., $P_1 \le P_2 \le P_3$.....General speaking, in application the lowest buckling load P_1 receive the most interest, whereas the higher bucking loads only have a theoretical interest, hence are not important from the designer's point of view; V is the design variable vector representing location of arch bracings; V_j is the nodal coordinates of the finite element model specifying the location of the j-th arch bracing; L_j and U_j are the lower and upper bounds of the location of arch bracing, respectively. k and n are the total number of buckling modes to extract and the total numbers of arch bracings in arch bridges, respectively.

Not that the above optimization problem can be transformed into the following form:

$$\text{Minimize } R_i(V), \quad i = 1,\ldots,k \tag{3}$$

$$\text{Subjected to } L_j \le V_j \le U_j, \quad j = 1,\ldots,n \tag{4}$$

where $R_i(V) = \dfrac{1}{P_i(V)}$ .This is a standard minimization problem with bound constraints, which can be solved by the proposed method discussed next.

2 THE PROPOSED SOLUTION METHOD FOR OPTIMAL LOCATION PROBLEM OF ARCH BRACINGS

2.1 Principle

The proposed method is a hybrid method, consisting of penalty function method (PFM), finite element method (FEM) and the first-order method (FOM). The method is based on three key concepts: (1) transformation of the constrained location problem defined in Eq. (3) to an unconstrained location problem by PFM; (2) actual finite element representation of the constrained location problem defined in Eq. (3) by FEM; and (3) solution of the constrained location problem defined in Eq. (3) by FOM.

The determination of location of arch bracing is a problem in constrained optimization. For the constrained optimization, finding points that satisfy all the constraints is often the difficult problem. One approach is to use the PFM for the constrained optimization. In this approach, a constrained problem is transformed into an unconstrained problem by using penalty functions. Two types of penalty functions are commonly used: interior and exterior penalty functions. In this paper, we use the exterior penalty function, which is considered to be the easiest to incorporate into the optimization process. The idea behind this algorithm is to penalize the objective function when it is not satisfying the constraints. By using the exterior penalty function, we transform the constrained problem defined in Eq. (3) into an unconstrained problem:

$$\text{Minimize } \phi(V, r_p) = R(V) + r_p Q(V) \tag{5}$$

where $Q(V)$ is an imposed penalty function; r_p is a multiplier which determines the magnitude of the penalty. A detailed description of the exterior penalty function can be found in [6].

Because the objective function in Eq. (5) is an implicit function of the design variables, the use of the proposed method may involve in evaluation of the implicit objective function. FEM is considered to be the most popular/reliable evaluation method. In this paper, the primary purpose of applying FEM is to compute the value of the implicit objective function. The detailed description of the method can be found in [7]. However, for the sake of completeness, a brief description is given below.

The objective function in Eq. (5) represents the elastic buckling problem of an arch bridge. In FEM the elastic buckling problem is expressed as the following eigenvalue problem:

$$([K_e] - P_{cr}[K_\sigma])\{\phi\} = \{0\} \tag{6}$$

where $[K_e]$ is the linear elastic stiffness matrix; K_σ is the geometric stiffness matrix [7]; P_{cr} is the buckling load; and ϕ is the buckling mode shape. In this paper, the method of subspace iterations is used to compute the values of P_{cr} and ϕ.

Once the above-mentioned constrained optimization problem is successfully transformed into an unconstrained problem, we can easily use any unconstrained optimization methods such as zero-order method and first-order method to solve the constrained optimization problem. In this study, we use the first-order method for solving the unconstrained problem defined in Eq. (5). This first-order method is based on the Fletcher-Reeves variant of the conjugate gradient method, see Ref. [6]. The key aspect of this method is finding a stepping direction, within the design space, and updating the vector V of design variable vector representing location of arch bracings according to

$$V^{n+1} = V^n + \lambda s^n \tag{7}$$

where λ is the parameter obtained by means of a unidirectional minimization. Finally, the vector s represents the stepping direction, see Ref. [6].

2.2 Procedure for the proposed method

The procedure of the proposed method is:

a) Define the optimization problem including the objective function, $P(V)$ and design variables V.
b) Define the initial location of arch bracing in terms of a set of design variables.
c) Transform the constrained problem defined in Eq. (3) into an unconstrained problem using exterior penalty function
d) Calculate the value of objective functions using FEM and evaluate the sensitivity of the objective functions of current design with respect to the design variables.
e) Using the sensitivity information and a suitable unconstrained optimization algorithm, such as FOM, generate a new location of arch bracing (usually of higher buckling load than the previous design) which satisfies the

constraints.

f) If the new location of arch bracing is not optimum, go to step 2; otherwise stop.

3 NUMERICAL EXAMPLE

A numerical example is presented for demonstrating how the proposed method can be used for finding the optimum location of arch bracings in an arch bridge structure. The problem selected for this purpose is of relatively small size consisting of only five I-type arch bracings, but the results and conclusions obtained from this example were the same as obtained from several other small and large problems. A larger problem is not being presented in this paper because of the huge amount of associated data.

3.1 Description of the example bridge

The example bridge studied here is a through arch bridge having a span length of 66m. The bridge consists of twin parallel arches with a 66m span and 13.2m clear width. The shape of the arch bridge is parabolic and the rise to span ratio of arch bridge is 1:4. The arch rib is made from concrete filled steel circular section. The arch bridge deck system consists of two horizontal tied girder, end cross girder, middle cross girder and concrete deck slab. The horizontal tied girder bears not only the horizontal thrust generated by arch ribs but also vertical loads brought over by the transverse beams. In order to increase the lateral stability of the arch bridge, five I-type arch bracings are located symmetrically between two parallel arch ribs. Symmetric requirements for the location of these arch bracings are kept unchanged during location optimization of these arch bracings. The deck is suspended from the two arches by means of 24 suspenders.

3.2 Formulation of optimal location problem of arch bracings in the example bridge

The process of formulating this optimal location problem involves identifying the design variables, the objective function and the corresponding constraints, which are presented below.

Design variables are independent quantities that are varied in order to achieve the optimum design. For the arch bracing, the design variables must identify the exact location. The location of each arch bracing is defined by a set of control points. Each of these points is defined by three cartesian coordinates (X, Y and Z). To reduce the number of design variables, only five boundary points are taken as control points in this work. Namely, one arch bracing is represented by one boundary point. Thus, the location of the arch bracings is determined by three cartesian coordinates X, Y and Z of these control points. The location of the arch bracings in the example arch bridge with I-type arch bracings is identified by the one cartesian coordinate X of these control points. As shown in Fig.1, the arch bridge model has five I-type arch bracings, and therefore, there are 5 design variables. Due to the symmetry and the simplicity of the problem, only 2 cartesian coordinates of the control points 1 and 2 are defined as independent design variables, the other 2 cartesian coordinates of control points 4 and 5 are symmetric with respect to the independent design variables and the last one cartesian coordinate of control point 3 are kept fixed. If X_i is the one cartesian coordinate of i-th control point, the Eq. (8) represents the vector of design variables.

$$V = (X_1, X_2) \tag{8}$$

Two cases are considered in the following optimization procedures: Case I: only one design variable of X_1 are used; Case II: all two design variables of (X_1, X_2) are used. The initial value of design variables for Cases I and II is shown in Table 1.

Objective function is the dependent variable that must be maximized. One of the most advantageous reasons for using the arch bracings in an arch bridge is their ability to increase buckling stability of the arch bridge. At the present work the elastic buckling load of the arch bridge is selected as an objective function.

Constraints are dependent variables and functions of the design variables that constrain the design. The coordinates of control points representing location of arch bracings are limited in the arch bracing and must be considered as constraints. The location constraint is a bound constraint to ensure that no location violation occurs when design variables are updated. For example, when design variables of coordinates X_1 are considered, their value of coordinates can not exceed the value of coordinates X_2. Table 2 presents the lower and upper bounds imposed on design variables.

Table 1. Initial value of design variables

Design variable	Initial value (m)
X_1	13.000
X_2	23.000

Table 2. Lower and upper bounds imposed on design variables

Design variable	Lower bound (m)	Upper bound (m)
X_1	0	22.9
X_2	0	32.9

3.3 Optimization results and discussion

The initial and optimal designs for Cases I and II are given in Table 3. From Table 3, it can be seen that: (1) arch bracings have moved towards the ends of arch ribs to increase the buckling load of arch bridges. The final buckling loads for Cases I and II are increased from 6.926 to 7.016 and from 6.926 to 7.138, respectively. Even though these increments are not much, the main feature is to observe where to place arch bracings to increase or maintain the largest buckling load; (2) the buckling loads of the optimal designs for Cases I and II obtained by using the proposed method are higher than those of the initial designs. This result shows that the proposed method can be employed for finding optimal arch bracing locations in arch bridge structures; (3) the Case I optimum design is worse than the Case II optimum design in this example. This is due to the fact of having more design variables for Case 2, which means more choices in the optimal design space. The noteworthy difference indicates that the number of design variables can considerably affect the design processes.

Table 3. The initial and optimal designs for Cases I and II

Different design	Case I		Case II		
	X_1	P_1	X_1	X_2	P_1
Initial design	13.000	6.926	13.000	23.000	6.926
Optimal design	11.850	7.016	9.95	19.25	7.138

In order to ensure that the optimization process converged to the global optimum point, the location of arch bracings was optimized using several start points. Table 4 shows the first selected start point and three different start points with their corresponding coordinates and buckling load after optimization. It can be seen that the maximum difference between the obtained coordinates is about 0.4 m or 3.12% and between the buckling loads is about 0.008 or 0.11%. Therefore, there is confidence that the convergence to the global optimum point is achieved.

Table 4. Location of arch bracing and buckling load obtained from different start points

Start point	Case I		Case II		
	X_1	P_1	X_1	X_2	P_1
I	11.850	7.016	9.95	19.25	7.138
II	11.850	7.016	10.0	19.45	7.136
III	11.850	7.016	10.15	20.20	7.131
IV	11.850	7.016	9.95	19.15	7.139

4 CONCLUSIONS

The location design of arch bracings in an arch bridge structure has been treated as a constrained optimization problem in which the elastic buckling load has been used to construct the objective function of the optimization problem. An efficient, accurate, and robust algorithm is developed to solve the optimization problem and identify the location of arch bracings by maximizing the objective function. The proposed algorithm integrates the concepts of the penalty function method, finite element method and the first-order method. Penalty function method is used to transform the constrained optimization problem to an unconstrained optimization problem. Finite element method is adopted to compute values of implicit objective functions. First-order method is used to solve the unconstrained optimization problem. An example is presented to demonstrate the practical application of the proposed method. The optimal location results obtained by using several different start points demonstrate the robustness of the proposed method.

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