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APPRAISAL OF SIMPLIFIED METHODS FOR THE ANALYSIS OF BOX-GIRDER BRIDGES

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ABSTRACT: This paper has two goals. Firstly to present a simplified method of evaluation of the additional longitudinal stresses that develop under the torsional response both for rectilinear and curved bridges and secondly to perform a comparison with the results deduced from a finite element simulating the real three dimensional structure. It is proved that the simplified method provides a safe estimation of the response and may be used for preliminary design purposes.

KEY WORDS: Box-girder; Torsion; Simplified method; Finite element.

1 INTRODUCTION

Box girders constitute, generally, a good choice for the superstructure of bridges and in most of the cases represent the best solution of all, thanks to their inherent capacity to withstand, in the most efficient way, the torsional stress state that develops. This state in the case of rectilinear girders is due to the eccentric position of traffic loading, whereas in the case of curved bridges in plan is already permanently present due to the girder self-weight alone, apart from any eccentricity of the incoming live loads.

Although under a given torsional moment diagram in a girder no longitudinal stresses develop, according to the classical technical theory of torsion, the existing overall girder deformability as a thin walled section, leads generally to such stresses. These stresses are of rather negligible intensity according to the theory of "warping torsion" if the deformability of the cross section profile is prevented by some inserted transversal diaphragms along the girder, but in their absence –as it is always preferred in the construction– the existing deformability of the cross section leads to substantial longitudinal stresses in addition to the shear stresses developed according to the classical Bredt theory of torsion. It is obvious that the above longitudinal stresses have to be superposed with those due to the longitudinal bending moments of the girder.

To account for this effect, formulas have been suggested on the basis of simulating the cross section profile as a folded plate ([1]-[3]). The present work examines the accuracy of this approximation using a closer to reality simulation with the aid of 3D finite elements. Both rectilinear and curved girders are examined. The comparison shows that the formulas provide conservative results, are on the safe side and may be used for preliminary design purposes.

1. RECTILINEAR GIRDERS

1.1 General loading case

In the typical box section of Fig. 1, an eccentric layout of the traffic load is shown.

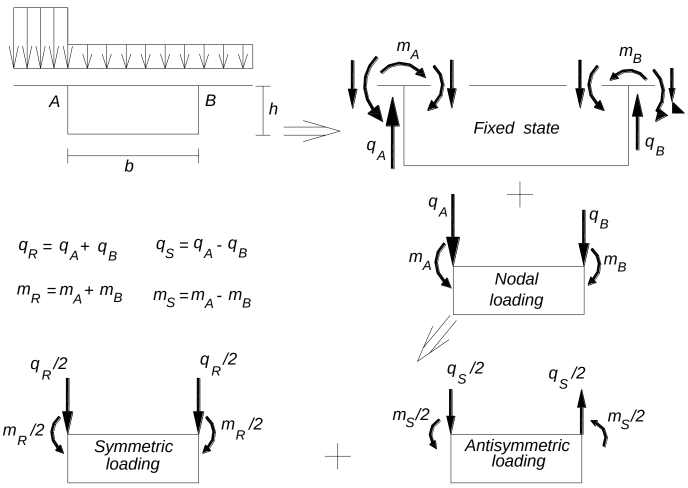

Figure 1. Resolution of eccentric load actions

It may be considered that the state of stress of the girder results from the superposition of the fixed state (I) containing the loads with the appropriate nodal actions and of the state (II) containing only these opposite nodal actions q_A and m_A, together with q_B and m_B.

The fixed state (I) is a trivial one and can be determined directly, so that the state (II) is considered only. This state - provided the section is symmetric- may be always split in a symmetric and an anti-symmetric part.

The symmetric loading causes the normal longitudinal bending and will not be considered further; thus the interest is essentially shifted to the anti-symmetric loading part. This one acts as a distributed torsional load $m_D = (q_S/2)\cdot b + m_s$.

In the present work only an anti-symmetric edge line load q_S will be considered. In this case $m_A = m_B = m_s = m_R = 0$.

1.2 Response due to the deformability of the profile section

The imposed torsional loading causes deformation of the closed section, resulting to longitudinal bending of the section walls, which is coupled with the resulting transverse bending of the section profile itself ([1]). It is pointed out that this response comes additionally to the initially existing Bredt shear flow in the section walls.

In order to investigate the influence of the deformability of the section on the response, the equilibrium of a cut-out girder strip of unit length is at first considered with the antisymmetric loads $q_S/2$ acting at the section edges *A* and *B* (Fig. 2). It is clear that the segment is in equilibrium under the above external forces and the differential shear flow Δv, which is obtained as the resultant of the occurring Bredt shear flows on the two faces of the strip considered.

Given that the torsional moment along the beam is M_T, the shear flow is $v = \frac{M_T}{2F_k}$. From the strip equilibrium it turns out: $\Delta v = \frac{q_S b_0}{4F_k}$

According to the section dimensions appearing in Fig. 1 it is: $F_k = hb$.

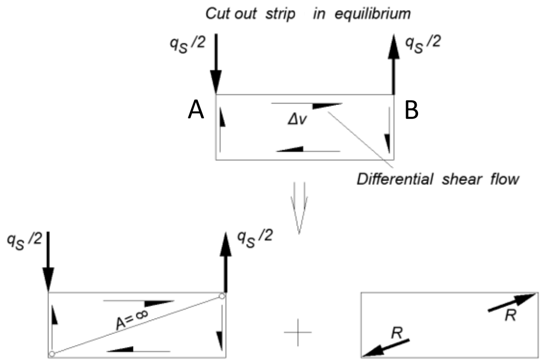

Figure 2. Static analysis of the deformable cross section under load

Obviously, the examined strip tends to deform under the forces $q_S/2$ and Δv. This deformation consists essentially in a change of length of its diagonals and induces an additional state-of-stress for each section wall. So it is considered appropriate to insert a hinged strut of an infinite axial stiffness along the diagonal (Fig. 2). It is clear that this diagonal element develops a tensile axial force *R* (kN/m) distributed along the length of the girder. Applying now the opposite of the acting forces *R* on the respective longitudinal edges of the girder in the absence of the diagonal strut, it is obvious that the superposition of the thus resulting state-of-stress with the one of the blocked strip, will give the final response (Fig. 2).

To examine this last ‘diagonal loading’ it is at first considered that the forces *R* act on a girder having hinged connections at the section edges instead of monolithic ones (Fig. 3). The forces R may be resolved, equivalently, at each edge into the two concurring walls, therefore each one of them can be considered as a longitudinal beam loaded by the corresponding component, developing bending moments M_0 and corresponding normal stresses σ, according to the classical theory of bending. However the resulting strains σ/E at the common edges of the walls with the slabs are not equal -as they should be - and for this reason some distributed longitudinal forces have to be additionally introduced along the edges of each wall, in order to establish the strain compatibility at each edge. It is clear that in this way the initially determined normal stresses σ will be changed.

By following the above analysis, the determination of the longitudinal axial stresses is possible. For example, if we concentrate on the left web of the hinged box section, they can be determined through the classical bending formula on the basis of the moment M_0. It is proved [1] that the moment of inertia I^* used is slightly larger than the normal value I_w for the web by a factor k^w and the new 'neutral axis' lies at a distance y_o from the top fiber, which is somewhat less than its 'normal value', i.e. the half of the web height.

The moment M_0 results from the loading of the left web with the respective component R_w of the 'diagonal force' R. It is found that: $R_w = R{\cdot}h/s$., where s is the length of the diagonals. Thus it is:

$$\sigma_0 = -\frac{M_0}{I^*} y_0, \sigma_u = \frac{M_0}{I^*}(h - y_0) \qquad (1)$$

where $I^* = I_w k^w$

Furthermore it is found: $k^w = \frac{1}{2}\frac{(\alpha + 2)^2 - 1}{3 + \alpha}, y_0 = \frac{1}{2}h$

with $\alpha = \frac{t_0 b}{t_w h}$, where t_0, t_w represent the thickness of both the flanges and of both the webs respectively.

The longitudinal web-beam obeys the following typical differential equation

$$EI^* \frac{d^4 w}{dx^4} = R_w \qquad (2)$$

where R_w and I^* represent the initial distributed web loading and the equivalent moment of inertia respectively, as explained above. It is noted that w represents the in-plane deflection of the web and it is due merely to the assumed deformability of the section.

One may now account for the monolithic connection of the section walls.

The bending deformation of the hinged section profile, under the action of diagonal forces R, has as a consequence the increase of its diagonal length by d (Fig. 3). However, this change cannot be realised without any resistance, given that the transverse stiffness C of the closed monolithic section profile is automatically mobilised. This stiffness is expressed through the relation $r = dC$, as that diagonal pair of forces *r* required to produce the deformation *d* (*C* may be evaluated as the required diagonal force to cause a unit elongation of the closed frame diagonal).

Then the tendency of the hinged section profile to be deformed by *d*, is counteracted by the resistance *r* of the monolithic closed frame and in this way the web is subjected, apart to its ‘initial’ loading R_w also to the loading of the component r_w of the force *r*, obviously with the opposite sense. In the same way as before: $r_w = r \cdot (h/s)$.

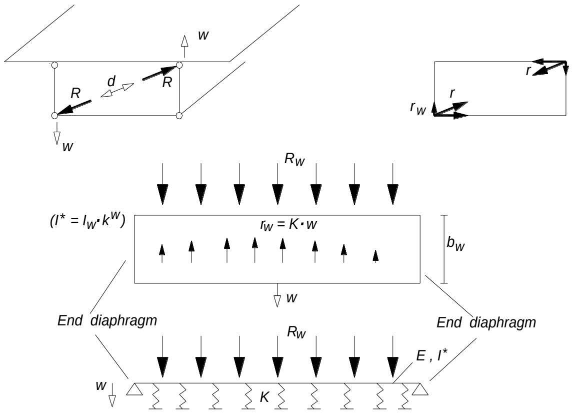

Figure 3. Equivalent beam on elastic foundation

Taking into account that the bending deformation *w* of the web may be expressed through the diagonal deformation *d* by the relation $d = wD$, where [2,3]:

$$D = \frac{4h}{s} = \frac{4h}{\sqrt{h^2 + b^2}} \tag{3}$$

it is found that the component $r_w = wDC(h/s)$. Using $K = DC(h/s)$ one may write down the differential equation for the web beam restored to the rigid section profile, and

$$EI^* \frac{d^4w}{dx^4} = (R_w - r_w) = R_w - wK \tag{4}$$

taking finally the form:

$$EI^* \frac{d^4w}{dx^4} + Kw = R_w \tag{5}$$

This equation is recognized as the typical equation of a beam on elastic foundation with modulus of subgrade reaction *K*. Indeed, the web is carried by the elastic support offered by the profile resistance when undergoing diagonal deformation.

2 CURVED GIRDERS

The box girder is particularly suitable for bridges curved in plan. These bridges have a permanent torsional response caused even by non-eccentric loads, as e.g. the self-weight of the girder.

2.1 Determination of bending and torsional response

The girder is assumed to have a constant radius of curvature equal to *R*. The loading consists of a vertical distributed load *q* passing through the shear centre of the cross-section, as well as a distributed torsional moment m_D. It is reminded here that exactly as in the case of grids, any part of the beam must satisfy three conditions of equilibrium, namely with respect to vertical forces, as well as with respect to the projections of moment vectors on two arbitrary horizontal axes.

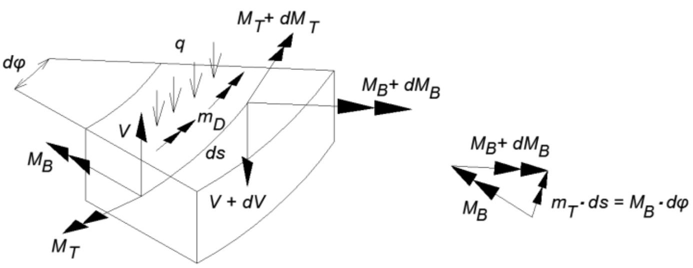

Figure 4. Equilibrium of a curved box girder segment

The equilibrium equations of an elementary segment of length *ds* forming an angle $d\varphi$ ($1/R = d\varphi/ds$), may be written in the form:

$$\frac{d^2M_B}{ds^2} = -\left(q - \frac{1}{R}\frac{dM_T}{ds}\right) \tag{6}$$

$$\frac{dM_T}{ds} = -\left(\frac{M_B}{R} + m_D\right) \tag{7}$$

If the arc span length *L* is much smaller than the radius of curvature *R*, (i.e. $L/R < 0.3$) it may be concluded that the term - $(1/R)\cdot(dM_T/ds)$ on the right side of equation (6) is much smaller than *q*.

Thus, the first equation results to: $\frac{d^2M_B}{ds^2} = -q$

i.e. resembling the equilibrium relation of a rectilinear beam between the bending moment and the load. This means that, under the above conditions, the bending moments of the curved girder may be approximated by the bending moments of a straight beam of span *L* equal to the arch length of the girder. This equivalence, for a simply supported girder, is illustrated in Fig.5.

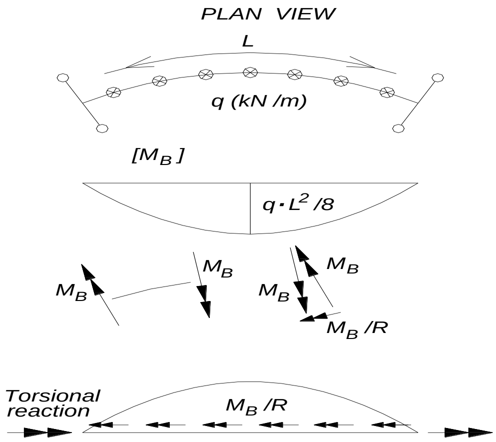

Figure 5. Static analysis of a curved beam in plan

2.2 The response of the cross sectional walls

The torsional response of the girder implies, except of the Bredt peripheral flow, an additional straining of the box section walls. This comes as a result from the way the gravity loads are introduced to the girder.

First it should be noted that the acting compressive forces *D* and tensile forces *Z* in the curved top and bottom flanges respectively, cause distributed deviation forces *q*.

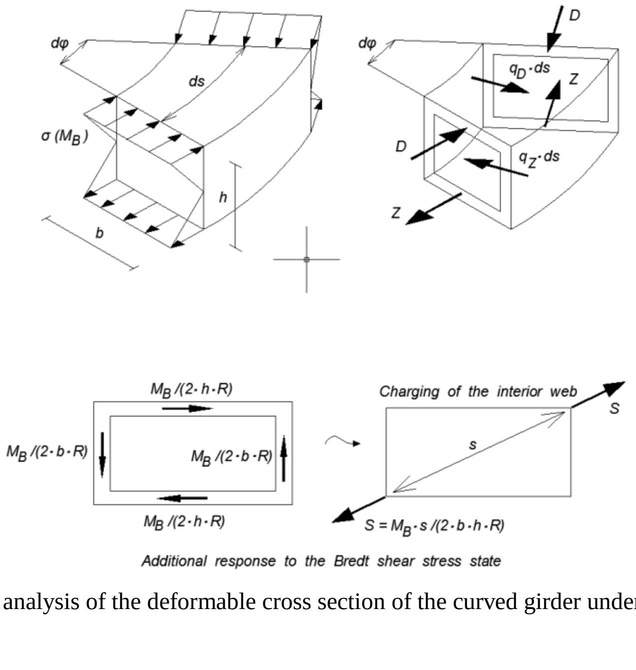

Figure 6. Static analysis of the deformable cross section of the curved girder under load

It is clear from Fig.5, that the transversely distributed equal and opposite forces q_D and q_Z, which the top and bottom slab is obliged to take up respectively, create a torsional load per unit of curved length, being nothing else than the one resulted from the vectorial variation of bending moments, as it was examined in the previous section. It is [2]:

$$q_D = \frac{D}{R} = \frac{M_B}{hR}, \quad q_Z = \frac{Z}{R} = \frac{M_B}{hR} \text{ and given that D=Z we have } q_D h = \frac{M_B}{R}$$

Thus, it can be seen that, even without the action of an externally applied torsional moment m_D, merely the existence of bending along a curved axis implies the imposition of a distributed torsional load (M_B/R), according to equation (7): The cut out strip of unit length, receiving the forces q_D and q_Z at its top and bottom side respectively, is in equilibrium with the developed Bredt shear flow at both its faces (Fig. 5). The resultant of these two flows is the so-called *differential shear*. According to Bredt's formula it is:

$$\frac{dv}{ds} = \frac{dM_T}{ds}\frac{1}{2bh} = -\frac{M_B}{2bhR} \tag{8}$$

Thus the strip being in equilibrium as a plane structure, under the loads q_D, q_Z and dv, gives rise to the self-equilibrated diagonal loading of the profile

$$S = \frac{M_B}{2R}\sqrt{\frac{1}{b^2} + \frac{1}{h^2}} \tag{9}$$

as shown in the figure, which causes longitudinal bending of the walls as well as transverse bending of the section profile, as it has been examined in detail for the rectilinear girder.

Although the analogy is not quite accurate, for the preliminary design needs and for ratios L/R<0.3, it may be considered that the left web wall takes the downward parabolically distributed along the length of the girder according to the Fig. 5 load $S_w = M_B/(2 \cdot R \cdot b)$, acting, as in the case of rectilinear beam, like a beam resting on an elastic foundation with a 'subgrade modulus' K, as examined previously.

3 NUMERICAL EXAMPLES

The above theory was tested on two simply supported box girder bridges, a straight one and a curved one. Both the bridges of a 40m span were designed to have a cross section with b=6.2m and h=2.5m, top and bottom slab of thickness of 0.25m and a web thickness of 0.5m.

3.1 Straight bridge

The straight box-girder bridge was subjected to a couple of anti-symmetric line loads only extending over 8m symmetrically about the mid-point of the bridge.

4466 shell elements were used to discretise the girder. The results of the FE program [4] (Fig.7) showed a value of approximately 343 KN/m^2 at the top and bottom of the element located at the middle of the bridge. The beam on elastic foundation having inertia and spring data according to section 1.2, was loaded by a uniform load spanning 8 m, over the centre, and provided a respective value of 436 KN/m^2, meaning a difference of roughly 21,3 %, showing that the theory is quite conservative and on the safe side.

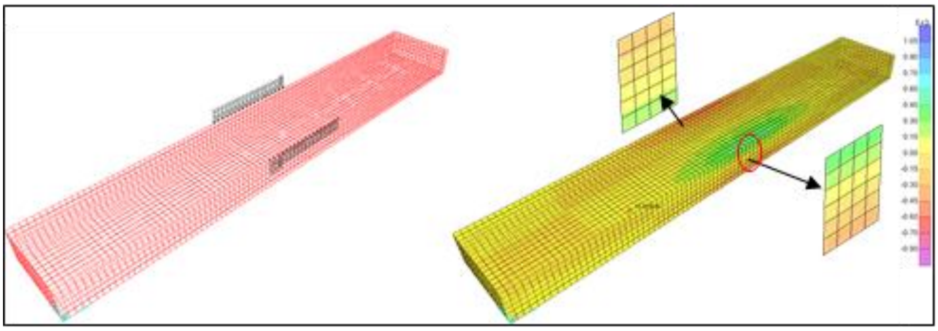

Figure 7. 3D FE results and distribution of stresses along the middle section

3.2 Curved Bridge

The discretization of the bridge with 4466 shell elements may be seen in Fig. 8.

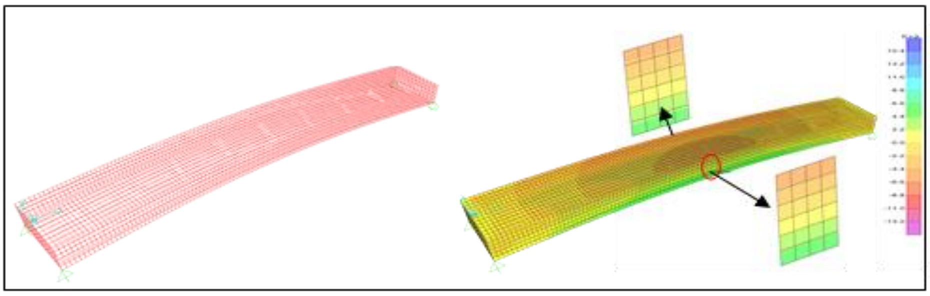

Figure 8. 3D FE results and distribution of stresses along the middle section

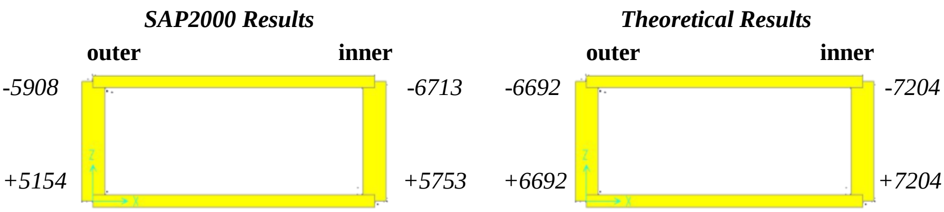

Figure 9. Comparison of analytical and numerical results

The bridge is subjected to its own weight. According to the analysis exposed in section 2.2 a curved beam on elastic foundation having inertia and spring data the same as for the straight bridge was considered, which was loaded by a parabolic continuous loading over the whole length of the beam. This beam provided results which were superposed with those due to pure bending. The comparison with the results of the FE program [4] may be followed in Fig. 9. One may see that the theory results are on the safe side with a maximum discrepancy of approximately 15%.

4 CONCLUSIONS

The deformability of the box section of a bridge girder under a torsional response leads to non-negligible longitudinal stresses which can't be predicted according to the classical theory of torsion (Bredt). However, following the deformability of the folded plate system it is possible to derive the above stresses using a methodology reducing the whole problem to the response of a beam on elastic foundation both for the rectangular and the curved girder. Comparison with FE results shows that this methodology leads to results lying on the safe side making it suitable for preliminary design purposes.

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