International Journal of Bridge Engineering (IJBE), Vol. 3, No. 1, (2015), pp. 33-48

NONLINEAR STATIC RESPONSE OF SELF-ANCHORED CABLE SUPPORTED BRIDGES

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ABSTRACT: A parametric analysis of the nonlinear static behaviour of self-anchored long-span bridges is here carried out by using a 3D nonlinear finite element model of the bridge. Both cable-stayed bridges with a fan-shaped arrangement of stays and combined cable-stayed-suspension bridges are considered in the numerical investigations. The importance of an accurate description of geometrically nonlinear effects, arising from the cables nonlinear behavior in coupling with the instability effect of axial compression in girder and pylons, is pointed out by means of comparisons with results obtained by using different cable models. Numerical simulations are devoted to analyze the influence of the main physical characteristics of the bridge, on the maximum load-carrying capacity and related collapse mechanisms. A nonlinear procedure for finding the initial geometry of the bridge and prestress distribution under dead load is incorporated in the model. The strong role of nonlinear cables response, in coupling with the notable influence of the relative girder stiffness on the stability bridge behavior is analyzed. For the self-anchored combined cable-stayed-suspension bridges the influence of the dead load distribution factor on the limit load evaluation is also accounted.

KEYWORDS: Nonlinear cable behavior, Finite element model, Stability.

1 INTRODUCTION

Due to their ability to overcome long spans, during last decades cable supported bridges received notable attention. Several applications are proposed in the framework of both suspension and cable stayed bridge typologies. The erection procedures for a typical cable stayed bridge, due to the free cantilever arms growing to the half length of the main span, produce dangerous conditions because large displacements and rotations are observed [1]. Contrarily, the construction processes in the suspension bridges, are very safe, because the main cable guarantees stability during girder erection procedure also for long spans [1]. Moreover, the cable-stayed bridges with respect to the suspension systems denote, under live loads, enhanced stiffness properties. Consequently, the combination of the two systems seems able to provide notable advantages especially in the context of long span bridges, providing stable and safe erection processes due to the suspension cable system and reduced deformability of the girder due to the stiffening effect of additional stay elements.

In general, self-anchored long-span bridges exhibit a remarkable nonlinear behavior. Nonlinear effects in cable supported bridges may arise from different sources, including the nonlinear behavior of a single cable due to the sag effect induced by self-weight, changes in geometrical configurations due to large deflections effects (usually large rotations but small strains) in both towers and girder due to their slenderness, the geometrical instability effect of the axial compression induced in the towers and girder, as well as the interactions between cables, deck and pylons nonlinearities [2, 3].

Other nonlinear effects may be related to the constitutive behavior of materials [4-5] or to the coupling between torsion and bending of the girder. Considerable attention has been devoted in the literature to the nonlinear structural behavior problem of cable supported bridges [2-5, 6,7]. In order to reduce the complexity of the highly non-linear problem, most studies available in the literature have introduced some reasonable assumptions in their formulations including one or more of sources of nonlinearities. For instance, pylons nonlinearities arising from beam-column effect are often neglected assuming a high flexural stiffness in pylons. An in-plane analysis is typically carried out excluding out-of-plane and torsional deformation modes and their interaction [2, 5], which can be usually not important in absence of eccentric load also when a three-dimensional bridge model is developed [3]. Moreover, in the framework of the stability analysis of the long span cable supported bridges, the prebuckling behavior is often assumed to evolve linearly with the load parameter, thus leading to a linear eigenvalue problem for the critical load [3, 6]. Although most nonlinear analyses have focused on geometrical nonlinear effects some analyses have involved both geometric and material nonlinearities and analyzed the ultimate behavior and load capacity of a cable-stayed bridge [4, 5, 7].

Due to its inherently nonlinear behavior a conventional analysis of the cable supported bridge, based on linear assumptions is often not applicable especially for long span bridge for which the main girder has the tendency to become more slender and lighter. Existing models which do not take into account appropriately for the softening stay behavior, such as those based on the equivalent tangent modulus of elasticity or those assuming that the cable resists only tensile axial force increment with no stiffness against axial compression increment may lead to a notable underestimation of the maximum load carrying capacity of the bridge for specific loading conditions. Moreover, the actual prebuckling behavior of the bridge may notably deviate from the linear assumptions and a nonlinear limit point analysis should be carried out in place of a linear stability analysis [5]. As a consequence, a more realistic nonlinear structural analysis accounting for the most important geometrically nonlinear effects should be adopted in conjunction with a nonlinear stability one. To this aim this contribution proposes a numerical investigation on the nonlinear static behavior of long span cable supported bridges (both fan-shaped cable-stayed bridges and combined cable-stayed-suspension bridges are analyzed), by considering the nonlinear behavior of cables in coupling with the instability effect of the axial compression in both girder and pylons. The study is carried out by introducing a general nonlinear model for both the analyzed cable-stayed and combined bridges, after an introductory analysis illustrating the general features of the buckling and post-buckling behavior of the cable supported bridges. Therefore a nonlinear three-dimensional finite element model of the bridge is formulated to accurately determine the influence of nonlinear effects on the structural bridge behavior and on its maximum load carrying capacity. The cable system is modeled according to the multi element cable system approach, where each cable is discretized using multiple truss element and large deformations are accounted by using Green-Lagrange strains. The bridge is modeled by means of a 3D assembly of beam elements and the connections between cables and girder have been obtained by using constraint equations. A nonlinear procedure for finding the bridge initial shape is incorporated in the analysis in order to determine properly the initial geometry and pre-stress distribution under dead load.

2 OVERVIEW OF NONLINEAR BRIDGE BEHAVIOR

Cable behavior is one of the most relevant sources of nonlinear elastic behavior of cable supported bridges. In particular, for cable-stayed systems the axial stiffness of stays must be accurately evaluated in order to avoid inappropriate predictions of the actual load carrying capacity. To this end, it must be observed that for large stress increments the secant modulus of the stress-strain relationship should be adopted instead of the tangent one.

The stress increment $\Delta\sigma$ in the stay may be written in the form:

$$\Delta\sigma = E_s^*\left(\sigma_0, \Delta\varepsilon\right)\Delta\varepsilon\,, \tag{1}$$

where E_s^* is the secant modulus of the stay which is a nonlinear function of the axial strain increment $\Delta\varepsilon$, measured along the stay chord, and of the initial tension σ_0.

If a parabolic shape of the stay deformed configuration is assumed, the well-known Dischinger theory can be employed to model the secant and the tangent elastic moduli E_s^* and E_t^*, respectively:

$$E_s^* = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{E}{1+\dfrac{\gamma^2 l_0^2 E}{12\sigma_0^3}\,\dfrac{1+\beta}{2\beta^2}},\quad \beta = 1+\frac{\Delta\sigma}{\sigma_0},\qquad E_t^* = \lim_{\beta\to 1} E_s^* = \frac{E}{1+\dfrac{\gamma^2 l_0^2 E}{12\sigma_0^3}} \tag{2}$$

where E is the Young modulus of the material cable, γ the cable weight per unit volume, l_o the horizontal projection of the stay length, and σ_0. the initial stress in the stay. It must be observed that the tangent modulus is related to small stress increments from the initial configuration (i.e. $\beta\rightarrow 1$). In this case, the cable equivalent modulus can be considered constant during load increment and the nonlinear cable response can be approximated by the tangent linearized one. Assuming that when shortening occurs the cable stiffness vanishes leads to the tension only approximation of stay behavior (see for instance [2,6]):

$$\Delta\sigma / \Delta\varepsilon = E_t^* \quad if \;\; \Delta\varepsilon \geq 0; \qquad \Delta\sigma / \Delta\varepsilon = 0 \quad if \;\; \Delta\varepsilon \leq 0. \tag{3}$$

The effects of the above assumptions on the nonlinear cable behavior, can be analyzed qualitatively by considering a complete fan shaped and self-anchored cable-stayed bridge scheme with the girder not horizontally constrained and the load uniformly applied on the central span. Generally speaking, assuming a linear prebuckling behavior, the girder compression produces an equilibrium bifurcation when the load reaches the critical value.

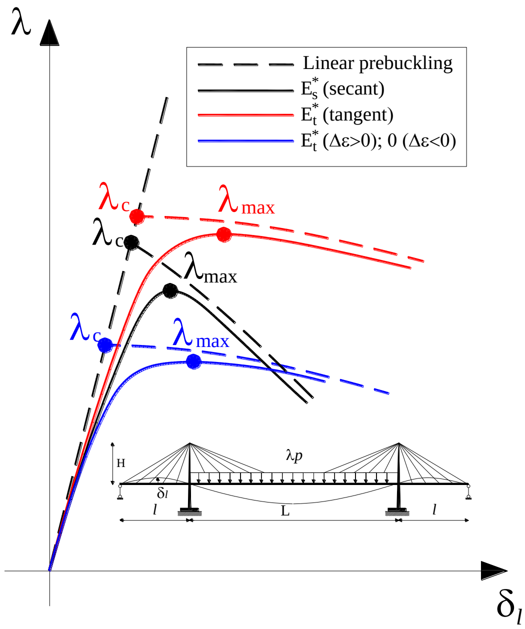

Figure 1. Nonlinear bridge behavior: load parameter versus lateral midspan deflection.

The postbuckling behavior strictly depends on the shape of the buckling mode and may show a decreasing behavior due to the softening cable response in compression (see the dashed line curve in Fig. 1). The actual bridge behavior taking into account the nonlinear prebuckling effects, doesn't exhibit an equilibrium bifurcation and is qualitatively shown by means of a continuous line in Fig. 1. In particular, when the secant modulus is adopted for the cable response a strong snap buckling behavior is expected with the maximum load parameter λ_{max} significantly below the critical load λ_c and post-buckling behavior of asymmetric unstable type.

This behavior is mainly attributed to the softening behavior of stays under shortening. On the other hand, when the stay behavior is modeled by means of the tangent modulus a non conservative prediction can be obtained since the limit load is larger than that based on the secant modulus formulation and a mild snap buckling occurs as in a symmetric unstable bifurcation. The magnitude of the critical load, depending exclusively on the tangent modulus distribution along the stays, changes slightly with respect to the secant modulus formulation. For instance, the critical load should remain unchanged provided that for each stay the stress at bifurcation is adopted as initial stress in the tangent modulus formula. Note that when buckling occurs at high load level, as in the case of uniform loading on the entire bridge, a value near to E can be assumed for the tangent modulus. A similar behavior occurs when the tension only truss model is adopted but the maximum load may be notably lower than the more accurate prediction obtained using the secant modulus model, thus leading to a conservative prediction of the maximum load-carrying capacity.

It results that bridge stability is mainly a consequence of two competing nonlinear effects in the tangent stiffness expression: the instability of axial compression in both girder and pylons and the stabilizing one due to the tangent stiffness of stays attached to the left and right pylons. The former generally increases with the load parameter λ, the latter may increase or decrease depending on the actual deformed configuration of the bridge due to the softening cable behavior under shortening.

3 BASIC ASPECTS OF BRIDGE MODELS

3.1 Cable-stayed bridge

Here a cable stayed bridge scheme with a fan-shaped arrangement of stays is analyzed, based on a diffused stays arrangement ($\Delta/L \ll 1$). The bridge model is able to predict the static behavior of cable stayed bridges taking into account the nonlinear behavior of stays, adopting the Dischinger’s fictitious modulus and taking into account the instability effect due to the axial compression in the girder [8].

The analyzed bridge scheme is illustrated in the Fig. 2. The girder is supported by stays joining at the tower tops. The two lateral couple of stays, called anchor stays, assure the bridge equilibrium and are anchored by means of two vertical supports; the girder is not constrained in the longitudinal direction.

It is assumed that the erection method is such that the deck final configuration is practically straight and free from bending moments. The bridge static response when the live load λp increasing with the parameter λ is applied, is now considered starting from the straight equilibrium configuration of the bridge deck corresponding to the application of the dead load g and cables pre-stress. As a matter of fact, the horizontal equilibrium of the bridge requires shear forces to be the same at the pylon top sections and this ensures that displacements of the pylon tops will always be equal and opposite. Similarly due to rotational equilibrium considerations about the y-axis, the pylon tops torsional rotations will always be equal and opposite.

The vertical, horizontal and torsional equilibrium equations for the girder respectively are:

$$EIv^{IV} + \left[\left(N^g + \Delta N(w) \right) v' \right]' - q_v = \lambda p$$
$$EAw^{II} + q_h = 0 \tag{4}$$
$$C_t \theta^{II} = -m_t - \lambda pe$$

where EI is the girder flexural stiffness, N^g is the axial force in the girder due to the dead load g. $\Delta N(w)=EAw'$ indicates the axial force increment in the girder due to live loads p; E, A and C_t are respectively the Young modulus, the cross section area and the torsional stiffness of the girder. Moreover q_v and q_h are the vertical and the horizontal components of the stays-girder interaction, respectively, whereas m_t denotes the stays-girder torsional couple interaction.

The two components of the stays-girder interaction depend on E^*_{SL} and E^*_{SR}, representing the Dischinger's secant modulus for the cables, respectively applied on the left (L) and on the right (R) stays with respect to the y-z plane (see the right of Fig.2). Note that E^*_{SL} and E^*_{SR} depend on the additional axial strain $\Delta\varepsilon$ in the cables produced by the additional displacements v, w, u, θ and ψ and that the initial stress of eqn (2) here represents the stress in the cables under the dead loads g.

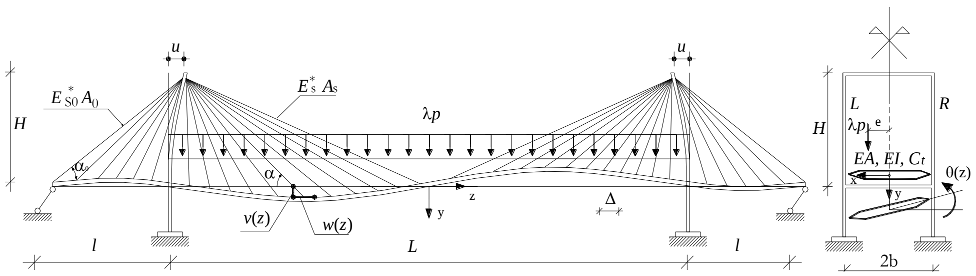

Figure 2. The long-span cable stayed bridge structural scheme.

Usually the stays cross section area A_s of the left or right curtains of stays is designed so as dead loads produce constant tension in all stays and this leads to $A_S = g\Delta/(2\sigma_g \sin\alpha)$ in which σ_g is defined as a function of the allowable stress σ_a as $\sigma_g=g\sigma_a/(p+g)$ assuming that the stress increment in the stays are proportional to the design live load p. For the anchor stays the cross sectional area A_0 is designed in such a way that the allowable stress σ_a is obtained for live loads p applied to the central span only, leading to:

$$A_0 = \frac{gl}{4\sigma_{g0}} \left[1 + \left(l/H \right)^2 \right]^{1/2} \left[\left(L/(2l) \right)^2 - 1 \right], \quad \sigma_{g0} = \sigma_a \left[1 + \frac{p}{g} \frac{\left(L/(2l) \right)^2}{\left(L/(2l) \right)^2 - 1} \right]^{-1} \tag{5}$$

where σ_{g0} is the initial tension in the anchor stays under dead load g.

The horizontal and torsional equilibrium equations of the pylons, involving the effects of the stays-girder interaction, lead to integral equations [9]. If the left pylon equilibrium is considered, the following equation is obtained:

$$\int_{-(l+L/2)}^{0} q_h dz + Ku + S_L^0 + S_R^0 = 0, \quad \int_{-(l+L/2)}^{0} m_f dz + Kb^2\psi + \left(S_L^0 - S_R^0\right)b = 0\,, \tag{6}$$

where K and m_f are the pylon tops flexural stiffness and the horizontal flexural couple per unit length acting on the left side of the bridge (z<0), respectively. Moreover, in equations (6) S^0_L and S^0_R are the horizontal components of the anchor stays axial forces, for the left and right curtains of stays with respect to the vertical *yz* plane, respectively, and ψ is the torsional rotation of the tower top. A similar equation is obtained for the right pylon.

In order to analyze the main parameters governing the bridge behavior, the following dimensionless quantities are introduced:

$$a = \frac{\gamma^2 E H^2}{12{\sigma_g}^3}, \quad \varepsilon = \sqrt[4]{\frac{4I\sigma_g}{H^3 g}}, \quad \varepsilon_A = \frac{A\sigma_g}{Hg}, \quad \tau = \sqrt{\frac{C_t\sigma_g}{Eb^2Hg}}\,. \tag{7}$$

The parameters ε, ε_A and τ, respectively represent a measure of the relative bending, axial and torsional stiffness between the girder and stays, whereas a is related to the stay deformability accounting for the cable sag effect. Additional details on the continuum bridge formulation can be found in [8]. It is worth noting that the above formulation does not take into account for buckling in the horizontal plane (out-of-plane buckling) and is strictly valid for the H- shape pylons. These restrictions will be removed in the discrete bridge model.

3.2 Combined cable-stayed suspension bridge

The structural model of the combined bridge, as shown in Fig.3, is based both on stayed and suspension cable systems, arranged in a self-anchored scheme, where the connection between suspension main cable and pylon is assumed to be a frictionless saddle system. Moreover, the bridge scheme is consistent with a fan-shaped system, in which the stays are perfectly constrained to the pylons and simply supported constraints are considered at girder pylon connections and at the left and right girder cross section ends. The bridge model is founded on the assumption of a uniform distribution of stays along the deck. In particular, the stay spacing Δ is small in comparison to the bridge central span L. As a result, the self-weigh loads produce negligible bending moments on the girder with respect to that raised by the live loads.

A proper erection procedure is supposed to guarantee that the girder position under self-weight is practically coincident with the undeformed configuration and, consequently, free from bending moments. In particular, the initial stress distribution produces tension in the cable systems and compression in both girder and pylons. The overall geometrical parameters of the combined cable system are governed by the self-weight loading condition (Fig.3). In particular, the stay and hanger cross sections are designed in such a way that the self-weight loads produce constant tension over all distributed elements and equal to a fixed value, namely σ_g.

The hanger elements follow a linear elastic behavior and are characterized by negligible weight in comparison to that involved by the main cable or the girder. These assumptions are defined consistently with some formulations recoverable from the literature [1]. In particular, it is supposed that the hanger elements remain always in tension, due to fact that a proper pre-stress system applied during the erection procedure is able to prevent compression state. Moreover, a proper erection procedure is assumed to distribute the girder self-weight load with a rate r ($0<r<1$), namely dead load distribution factor. This hypothesis is in agreement with the main theory on combined supported bridges, which considers the dead load distribution subdivided into equivalent loads depending on the amount required of the cable steel quantity involved in the cable system [1].

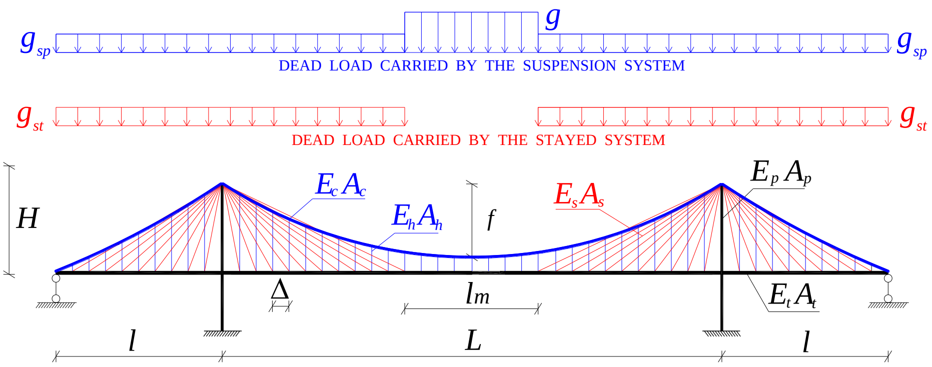

Figure 3. The long-span combined cable-stayed-suspension bridge structural scheme.

From an analytical point of view r corresponds to a dead loads distribution factor, equal to the fraction of the total girder dead load taken by the suspension system in the regions where both suspension and cable stayed systems are present. Therefore, the girder self-weight amounts applied to the cable stayed (st) and the suspension (sp) systems, gst and gsp respectively, are defined by the following expressions:

$$g_{st} = (1-r)\,g, \qquad g_{sp} = rg, \tag{8}$$

where g represents the girder self-weight loads per unit length. The cross section area of a generic stay or hanger, and of the the anchor stays are dimensioned by means of the following equations:

$$A_s=\frac{g_{st}\Delta}{\sigma_g \sin\alpha}, \quad A_h=\frac{g_{sp}\Delta}{\sigma_g}, \quad A_0=\frac{g_{st}l}{2\sigma_{g0}}\left[1+\left(l/H\right)^2\right]^{1/2}\left[\left(L/2l\right)^2-1\right] \tag{9}$$

where σ_g and σ_{go} are the self-weight design tension before defined for the cable-stayed bridge scheme, Δ is the stay and hangers spacing step, α is the stay orientation angle with respect to the horizontal direction. Similarly for the anchor stays, the geometric area is determined in such a way that the corresponding allowable stress, i.e. σ_a, is reached in the static case, for live loads applied on the central span only.

From a practical point of view, the design parameter r is an indicator of the ratio between the suspension system steel quantity and that involved in the combined bridges. As an example, assuming that r is equal to 0 or 1, the combined bridge tends to a perfect cable stayed or suspension bridge scheme, respectively. The cable stayed system, especially for long spans, is affected by high stiffness reduction due to Dischinger effects. As a consequence, the stays are supposed to be distributed on a reduced portion of the main span, namely *2l+L-lm* (Fig.3), which can be estimated, approximately, by the following relationship [1]:

$$l_m = L - L\left[1+\left(\left(g_{sp}+g_c\right)/\left(g+g_c\right)\right)^{1/2}\right]^{-1} \tag{10}$$

where g_c is the self-weight main cable suspension distributed loads. In the framework of long span bridges, the ratio between sag and horizontal main cable projection length is usually small. Therefore, the initial main cable configuration y and the corresponding horizontal axial force H_g can be determined, utilizing a parabolic approximation of the cable profile, as:

$$y(z)=-M(z)/H_g, \quad H_g=\left[\left(g_c+rg\right)L^2/8+g_{st}l_m\left(L-l_m/2\right)/4\right]/f \tag{11}$$

where $M(z)$ is a fictitious bending moment due to distributed self-weight loads taken by the suspension system calculated as for a simply supported beam and f is the main cable sag. The cross sectional area of the main cable is given by [1]:

$$A_c = H_{g+p}/\left(\sigma_a \cos\phi\right), \tag{12}$$

where ϕ is the orientation angle formed by the main cable tangent at pylon intersection and the vertical direction and H_{g+p} is the horizontal main cable force related to live loads applied to the whole central span. The main cable position as well as the corresponding axial force horizontal projection, i.e. $\left(\bar{y}_C, H_t\right)$, are supposed to be expressed as the sum of contributions related to self-weight loading (y, H_g) defined by eqn (11), and corresponding ones produced by the live loads application, i.e. (v_c, h):

$$\bar{y}_C(z) = y(z) + v_C(z), \quad H_t(z) = H_g + h(z) \tag{13}$$

According to a continuum approach, the interaction forces q_s between main cable and girder, are expressed, analytically, for small hanger spacing, by means of a continuous function depending on the bridge kinematic, the main cable configuration and the hanger stiffness properties, as:

$$q_s = -EA_h(v - v_c)/(H - y)\Delta \tag{14}$$

in which v and v_c are the vertical displacements for the girder and the main cable, respectively. Consistently with a flat-sag based cable formulation, which admits a parabolic approximation of the main cable profile, the equilibrium equation along vertical direction is a function of the hangers interaction forces and can be expressed in terms of incremental quantities measured starting from the undeformed configuration:

$$\left[H_t v_C'\right]' = q_s\,. \tag{15}$$

The vertical equilibrium equation for the girder is in this case:

$$EIv^{IV} + \left[\left(N^g + \Delta N(w)\right)v'\right]' - q_v - q_s = \lambda p\,, \tag{16}$$

where q_v is the vertical component of the stays-girder interaction. Horizontal and torsional equilibrium equations for both girder and pylons can be expressed like in the case of cable-stayed bridges keeping in mind that here the torsional couple m_t includes both the stay-girder torsional interaction and the hanger-girder torsional interaction.

4 3D-DISCRETE MODEL

In this section a 3D discrete FE bridge model is examined for both the case of cable-stayed and combined cable-stayed suspension bridges, taking into account the geometric nonlinear effects for the cable system under general loading conditions, in order to obtain accurate results. In particular, the actual stays (and hangers for combined bridges) spacing is considered in the model. This discrete model has been analyzed by means of a displacement-type finite element (FE) approximation, implemented in the commercial software COMSOL MULTIPHYSICS™ [10]. A three dimensional finite element model has been developed by using beam elements for the girder and the pylons and nonlinear truss elements for the cable system. Specifically, the bridge deck is replaced by a longitudinal spline with equivalent sectional and material properties and the pylons are composed by two columns connected at their top and at the level of the bridge deck by two horizontal beam elements. Moreover, the instabilizing effects produced in both girder and pylons by the axial compression force N has been accounted by adding the following weak contributions for pylons and girder, respectively, to the virtual work principle formulation:

$$-\left(\int_{L_e} N\theta_x\delta\theta_x\, dL + \int_{L_e} N\theta_z\delta\theta_z\, dL\right)_{Pylons}$$
$$-\left(\int_{L_e} N\theta_y\delta\theta_y\, dL + \int_{L_e} N\theta_x\delta\theta_x\, dL\right)_{Girder} \tag{17}$$

where N is the axial force, θ_x, θ_y and θ_z the denote rotations about the x, y, and z axes, and L_e is the element length.

The cable system is modeled according to the Multi Element Cable System (MECS) approach, where each cable is discretized using multiple truss element. The stiffness reduction caused by sagging is accounted for by allowing the cable to deform under applied loads. Large deformations are accounted by using Green Lagrange strains and the axial strain is calculated by expressing the global strains in tangential derivatives and projecting the global strains on the cable edge. Additional details about the approach here adopted to model nonlinear cable behavior can be found in [10]. It is worth noting that different approaches have been proposed in the literature to model the nonlinear cable behavior ranging from the simple equivalent modulus approach, according to which each cable is replaced by one bar element characterized by an equivalent tangent modulus [4,7] often with a tension only behavior [2, 5, 6], to more accurate techniques based on the elastic catenary results.

In the case of the simplified cable behavior simulated by using the tangent modulus for the cable-stayed bridges, a single truss element is adopted for each stay and the geometrical nonlinearities are deactivated. The constitutive behavior is simulated by introducing the expression shown in eqn (2) for the truss element constitutive modulus, with the initial stress derived from results obtained through the initial shape analysis formulated in the sequel.

The tension only behavior (always for the cable-stayed bridges) is modeled similarly except that a nonlinear constitutive behavior according to eqn (3) is incorporated in the constitutive relationship of the truss element representing the single stay. To this end the longitudinal modulus of the truss element is multiplied by a step function depending on the axial strain increment with respect to the initial configuration of the bridge under dead loading, in order to exclude any stiffness contribution of the cable under shortening. Therefore, the constitutive behavior of the truss element is (see [8] for additional details):

$$\Delta\sigma = E_t^* step(\Delta\varepsilon)\,\Delta\varepsilon,\;\; step(\Delta\varepsilon)=1 \;\; if\,\Delta\varepsilon \geq 0;\; step(\Delta\varepsilon)=0 \;\; if\,\Delta\varepsilon < 0 \tag{18}$$

The constraint condition between the girder and the stays (and the hangers for combined bridges) is modeled with offset rigid links to accommodate cable anchor points, by means of the extrusion coupling variable methodology (see [10] for additional details). The buckling and post-buckling behaviors have been investigated by using nonlinear analyses taking into account large deformation but small strains with linear stress-strain relationship and a solution strategy based on the damped Newton method has been adopted. A suitable modeling technique in this case, where the relationship between applied loads and displacements is highly nonlinear, is to use an algebraic equation that controls the applied live loads λp so that the generalized deflection of a control point reaches the prescribed values. In order to capture the typical snapping behavior of the load-displacement curve, a generalized deflection increasing monotonically with the evolution of the loading process is adopted, so that no snap-back are detected when the load-displacement curve is plotted in terms of the assumed control parameter. In the case of loading on the central span of the bridge, an appropriate choose to capture the snapping behavior is the lateral midspan deflection δ_l or the girder end in-plane rotation θ_l, although in some cases relevant to the TO model the central midspan deflection δ_c has been adopted. An initial shape analysis must be carried out to find the geometric configuration together with the associated pre-stress force distribution in cables satisfying both equilibrium and the design requirements of a straight initial bridge configuration. All geometric nonlinearities are taken into consideration in the initial shape analysis, namely geometric effects of axial compression in both girder and towers and cable sag nonlinearties induced by cable dead load.

5 NUMERICAL RESULTS

5.1 Cable-stayed bridge

Here numerical results for the cable-stayed bridge model are presented to examine the instabilizing effect produced by the axial force in the girder for increasing live loads λp. In particular two types of loadings are considered: a uniform load distributed on the whole bridge length and a uniform load applied on the central span only. The following dimensionless parameters are used in the numerical analyses:

$$L/2H = 2.5,\;\; l/H = 5/3,\;\; b/H = 0.1,\;\; \Delta/L = 1/105,\;\; \sigma_a/E = 7200\big/2.1\times10^6,\;\; K/g = 50$$

whereas the material properties assume the values that concern the usual case of steel girders and towers. The value of the dead load g is equal to 300,000 N/m, typical of a steel deck, whereas the cable unit volume weight has been assumed equal to γ =77.01 kN/m^3. The other parameters ε, τ, ε_A and a are used to define the bridge geometrical parameters according to eqn (7).

A parametric analysis is carried out by adopting the following values: $\varepsilon = 0.2$ or 0.3, a = 0.10 or 0.20 and p/g = 0.5 or 1, whereas ε_A has been assumed equal to 54.5. As far as the analysis carried out with the discrete model is concerned, the following additional parameters are needed:

$$\varepsilon_h = \sqrt[4]{4I_{yy}\sigma_g/b^3 g}, \qquad I_r = I_{pxx}/I$$

defining respectively the relative girder to stay stiffness for bending in the horizontal plane and the tower to girder bending stiffness ratio for bending in the vertical plane. The former parameter gives the bending stiffness EI_{yy}, whereas the latter the pylon bending stiffness EI_{pxx}. The towers stiffness I_{pzz} for out-of-plane bending has been assumed equal to EI_{pxx} and the ratio between the height of the pylon from pier bottom to bridge deck H_1 and H has been assumed equal to 0.25. The axial, bending and torsional stiffnesses of the beams connecting the two towers of the pylons have been assumed equal to the corresponding ones adopted for the towers. Moreover, the cross section area and the torsional stiffness of the towers have been assumed equal to those of the girder. These parameters allow to define the bridge characteristics for the 3D discrete model. On the other hand the following parameters have been assumed for the remaining parameters: τ=0.1, ε_h=5, t=0.1. The influence of the different stays response approaches introduced in section 2, on the bridge nonlinear behavior by using the more general 3D discrete model is here investigated. To this end the response of a single stay has been modeled by using the multiple truss element nonlinear formulation, which will be denoted as NLM, the tangent modulus linear model (denoted as LM in the sequel) and the tension only approximation (denoted as TO in the sequel).

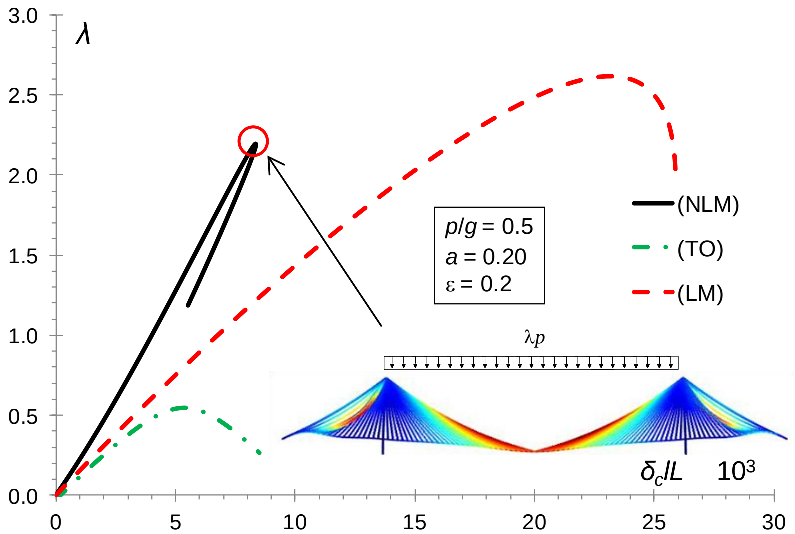

Figure 4. Cable stayed bridge: load parameter λ versus central midspan deflection δ_c.

Figure 4 shows for ε=0.2 , a=0.2 and p/g=0.5 the typical snapping for low values of λ due to the coupling between the instability effect of the axial compression in the girder and the softening behavior of stays response in the lateral span, occurring in the case of the central loading condition and the H pylon shape. As the load parameter increases while in the central span the instability effect of the axial compression is balanced by the stiffening stays in the lateral spans stays show a large stress reduction produced by the lateral spans deflection and an instability condition is rapidly reached producing a bound in the applied load. The load-displacement curves are represented in term of the central midspan deflection δ_c in order to better appreciate the differences between the three considered models, although the nonlinear analyses have been driven always by the lateral midspan deflection. In figure 4 the results obtained by using the linear tangent model (LM) and the tension only model (TO), are also shown in order to appreciate the influence of the nonlinear cable response modeling on the global bridge behavior.

5.2 Combined cable-stayed-suspension bridge

Here numerical results for the self-anchored combined cable-stayed suspension bridge model are presented. It's well known that for the combined bridges a distribution of the girder self-weight load with a rate *r* ($0<r<1$), namely dead load distribution factor, is assumed. This hypothesis is in agreement with the main theory on combined bridges, which considers the dead load distribution subdivided into equivalent loads depending on the amount required of the cable steel quantity involved in the cable system. In particular, *r* corresponds to a dead load distribution factor, equal to the fraction of the total girder dead load taken by the suspension system in the regions where both suspension and cable stayed systems are present. From a practical point of view, assuming that *r* is equal to 0 or 1, the combined bridge tends to a perfect cable stayed or suspension bridge scheme, respectively. In the numerical simulations three different values for *r* are considered, namely r=0 (cable stayed bridge), r=1 (suspended bridge) and r=0.5 (combined bridge).

In this section the influence of the coupling stayed-suspension parameter *r* on the maximum load parameter for the self-anchored combined cable-stayed suspension bridge is analyzed. The load condition corresponding to a uniform load applied on the central span only is considered in the analysis.

The geometrical configuration is characterized by L=1500 m, l=500 m and f=214 m. The value of the dead load *g* is equal to 300,000 N/m, typical of a steel deck. The stays and hangers spacing Δ is assumed equal to L/30, whereas two different value for *p/g* are considered (p/g=0.25 and 0.50). For the combined bridges the multiple truss element nonlinear formulation is used to model the single cable response (main cable and stays). For the hangers a single linear truss element is employed. Figure 5 shows for the analyzed geometrical configuration (L=1500 m, l=500 m, f=214 m) and p/g=0,25; 0.50 the load-displacement curves for the three bridge schemes corresponding to the distribution dead load factor *r* (r=0, 0.5, 1). In particular, the typical snapping behavior for relatively low values of λ occurring in the case of the central loading condition can be observed. The load-displacement curves are represented in term of the lateral midspan deflection δ_l (used like control parameter to drive the nonlinear analyses) and show the differences between the three considered bridge schemes. The diagrams evidence the conservative prediction of the limit load in the case of suspension bridge (r=1) with respect to the cable-stayed system (r=0). Moreover, it's possible to observe how combination of the two systems provides stabilizing effect in terms of maximum load parameter ($0 < r < 1$).

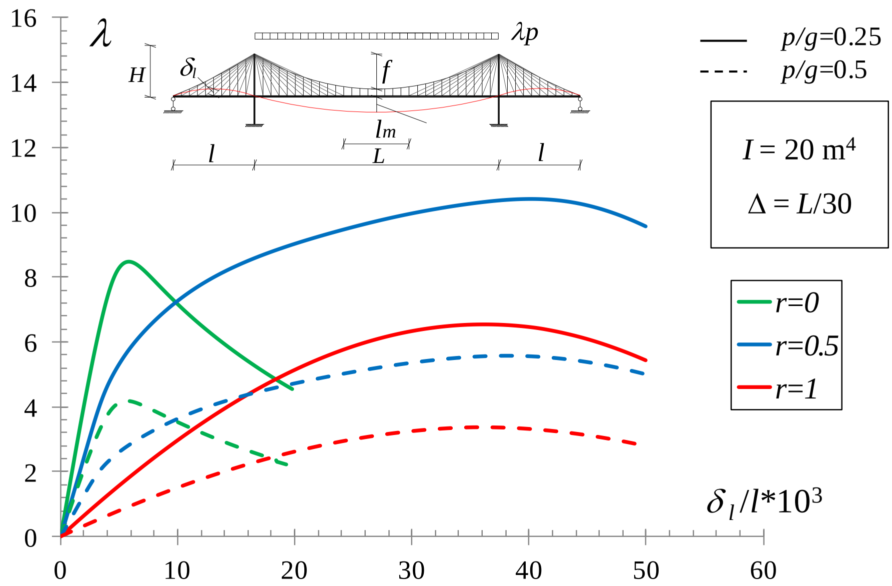

Figure 5. Combined bridge: load parameter λ versus lateral midspan deflection δ_l.

6 CONCLUDING REMARKS

A numerical investigation on the geometrically nonlinear static behavior of self-anchored long span cable supported bridges is carried out by adopting a tridimensional finite element model for the bridge. Both long span cable-stayed bridges, with a fan-shaped arrangement of stays and self-anchored combined cable-stayed-suspension bridges are considered in the numerical computations. For the first bridge model analyzed numerical investigations are devoted to study the influence of three different stays response approaches on the bridge nonlinear behavior by using a more general discrete approach. In particular, the mechanical response of a single stay has been modeled by using the multiple truss element nonlinear formulation (denoted as NLM), the tangent modulus linear model (denoted as LM) with the initial stress derived from the initial shape analysis and the tension only approximation (denoted as TO) according to which cable assumes a vanishing stiffness under shortening.

Numerical results show the typical snapping behavior for high values of the load parameter λ due to the coupling between the instability effect of the axial compression in the girder and the softening behavior of stays response in the lateral span, occurring in the case of the central loading condition. Moreover, from numerical investigations it was found that in the case of uniform loading on the whole bridge length, contrarily to that found for central loading condition, as the load parameter increases the effect of softening in stays tangent stiffness is negligible occurring for a very small group of cables and for larger load level. For this loading condition it is possible to appreciate the conservative behavior of the LM and TO models with respect to the NLM, in terms of the maximum load parameter λ_{max}. In the case of the self-anchored combined cable-stayed-suspension bridges the influence of the coupling stayed-suspension parameter r on the maximum load for the bridge is analyzed. Results evidence the conservative prediction of the limit load in the case of suspension bridge (r=1) with respect to the cable-stayed system (r=0). Moreover, numerical investigations show that combination of the two cable systems provides stabilizing effect in terms of maximum load parameter.

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Received: Sep 15, 2014 Accepted: Oct 2, 2014