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THE DYNAMICS OF A VERTICALY RESTRAINED ROCKING BRIDGE

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ABSTRACT: This paper investigates the rocking response and stability analysis of an array of slender columns caped with a rigid beam which are vertically restrained. This simplified system describes models a rocking bridge. The nonlinear equation of motion is formulated in which the stiffness and the prestressing force of the tendons are treated separately. In this way, the post-uplift stiffness of the vertically restrained rocking bridge can be anywhere from negative to positive depending on the axial stiffness of the vertical tendons. The paper shows that the tendons are effective in suppressing the response of rocking bridges with small columns subjected to long period excitations. As the size of the columns, the frequency of the excitation or the weight of the cap-beam increases, the vertical tendons become immaterial.

KEY WORDS: Rocking, uplifting, negative stiffness, earthquake engineering

1 THE DYNAMICS OF THE VERTICALLY RESTRAINED ROCKING FRAME

The main motivation for this study is to establish the dynamics of the vertically restrained rocking frame which emerges as a most promising alternative design concept for tall bridges [1]. Our analysis goes beyond the one bay configuration introduced in [1], which essentially represents the transverse motion of the bridge system as shown in Figure 1 (top left) and examines the planar rocking response of an array of *N* identical vertically restrained columns capped with a rigid beam that is clamped with the vertical restrainers. This configuration, shown in Figure 1, idealizes the longitudinal motion of a multi-span bridge.

When the elasticity, *EA*, of the restrainer is small compared to the weight of the rocking columns, $m_c g$, upon uplifting the lateral stiffness of the systems remains negative as in the free rocking case. As the elasticity, *EA*, of the restrainer increases, the lateral stiffness of the rocking frame increases gradually from negative to positive as shown in Figure 1 (bottom-left).

Assuming that the rocking column will not topple, it will recenter, impact will happen at the new pivot point and subsequently it will rock with opposite

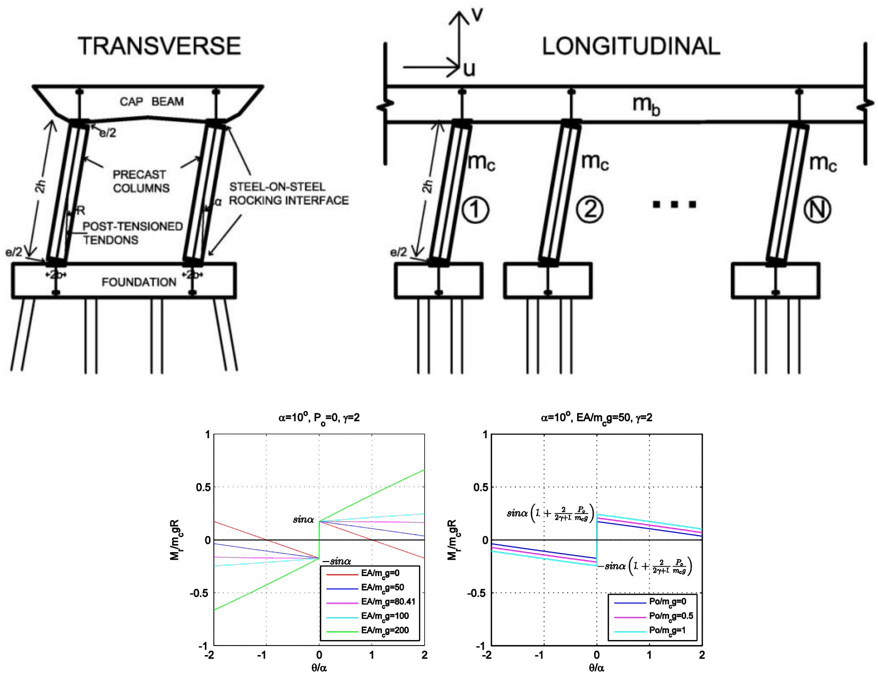

Figure 1. Transverse (top left) and longitudinal (top right) sections of the vertically restrained rocking frame together with the moment-rotation diagrams for various values of the dimensionless stiffness of the tendon EA/m_cg (bottom-left) and the dimensionless pretestressing force P_o/m_cg (bottom right.)

rotations. During rocking, the dependent variables *u(t)* and *v(t)* of the center of mass of the cap-beam with mass m_b are given for $\theta(t)<0$ and $\theta(t)>$ by the following expressions:

$$\ddot{\theta}(t) = -\frac{1+2\gamma}{1+3\gamma}p^2\left[\sin(\alpha\,\mathrm{sgn}\theta-\theta)+\frac{\ddot{u}_g}{g}\cos(\alpha\,\mathrm{sgn}\theta-\theta)\right]- -\frac{2}{1+3\gamma}p^2\sin\alpha\sin\theta\left[\underbrace{\frac{EA}{m_c g}\tan\alpha}_{elasticity}+\underbrace{\frac{P_o}{m_c g}\frac{1}{\sqrt{2-2\cos\theta}}}_{prestressing}\right] \quad (1)$$

Where $p=\sqrt{3g/4R}$ and $\gamma = m_b/Nm_c$

The first bracket in equation (1) describes the dynamics of the free standing rocking frame [2,3] whereas the second bracket describes the contribution of the vertical tendons. The only damping in the system comes from energy loss during impact. The ratio of the kinetic energy of the rocking frame after and before impact [2] is:

$$r = \frac{\dot{\theta}_2^2}{\dot{\theta}_1^2} = \left(\frac{1 - \frac{3}{2}\sin^2\alpha + 3\gamma\cos 2\alpha}{1 + 3\gamma} \right)^2 \tag{2}$$

2 COMMENTS ON THE EFFECT OF THE RESTRAINING TENDONS AND THE MASS OF THE CAP BEAM

The mathematical structure of equation (1) offers some valuable information regarding the effectiveness of the vertical restrainers in association with the size of the columns and the weight of the cap-beam. The term associated with the first bracket of equation (1) expresses the dynamics of the free-standing rocking frame ([2], [3]). The quantity $\hat{p} = \sqrt{\frac{1+2\gamma}{1+3\gamma}}\,p$ is the frequency parameter of the free-standing rocking frame showing that its dynamic response is identical to the rocking response of a single free-standing column with the same slenderness; yet with larger size – that is a more stable configuration.

The term associated with the second bracket of equation (1) expresses the contribution of the vertical restrainers. As the size of the columns increases (smaller p), the effectiveness of the vertical restrainers is suppressed with p^2; while the effectiveness of the restrainers further reduces as the weight of the cap-beam increases (large γ). Simply stated, the combination of a heavy deck atop tall columns enhances the seismic stability of the free-standing rocking frame while it reduces the effectiveness of the vertical restrainers.

On the other hand, at the limiting case of a massless rigid cap-beam (γ=0), equation (1) indicates that the vertically restrained rocking frame experiences an apparent restraining stiffness that is 4 times larger and an apparent prestressing force that is 2 times larger than the corresponding values of the solitary rocking column that is vertically restrained with the same tendon (same $EA/m_c g$ and same $P_o/m_c g$, [4]).

3 FROM NEGATIVE TO POSITIVE STIFFNESS

In the vertically restrained rocking frame, the negative stiffness originates from the fact that as the rotation increases the restoring weight vectors of the columns and the cap-beam approach the pivot point; whereas, the positive stiffness originates from the presence of the vertical elastic restrainers which offer an increasing restoring moment.

Without loss of generality we concentrate in the case of positive rotations ($\theta(t)$>0). Equation (1) indicates that the rotation-dependent restoring moment is

$$M(\theta) = m_c gR\left[\sin(\alpha-\theta) + \frac{2}{1+2\gamma}\sin\alpha\sin\theta\left(\tan\alpha\frac{EA}{m_c g} + \frac{1}{\sqrt{2}\sqrt{1-\cos\theta}}\frac{P_o}{m_c g}\right)\right] \quad (3)$$

Figure 1 (bottom) plots the expression given by equation (3) for various values of the dimensionless elastic force $\frac{EA}{m_c g}$ for a column with slenderness α=10^o. Linearization of (3) gives:

$$\frac{M(\theta)}{m\,gR} = \sin\alpha\left[1 + \frac{2}{2\gamma+1}\frac{P}{m\,g} + \theta\left(\frac{2}{2\gamma+1}\tan\alpha\frac{EA}{m\,g} - \cot\alpha\right)\right] \quad (4)$$

The factor of the rotation θ in equation (4) is the stiffness of the system upon uplifting; and therefore, the condition for the linearized system to exhibit a positive stiffness is

$$\frac{EA}{m_c g} > \left(\frac{1}{2}+\gamma\right)\frac{1}{\tan^2\alpha} \quad (5)$$

For instance, when α=10^o, according to expression (5) a vertically restrained rocking frame exhibits a positive stiffness if $EA/m_c g > 48.25$ for γ=1 and $EA/m_c g > 80.41$ for γ=2. When inequality (5) becomes an equality the vertically restrained rocking column exhibits a rigid-plastic behavior without enclosing any area.

4 ACCELERATION NEEDED TO INITIATE UPLIFT

The minimum uplifting acceleration of the vertically restrained rocking frame is

$$\ddot{u}_g^{up} = g\tan\alpha\left(1 + \frac{2}{2\gamma+1}\frac{P_o}{m_c g}\right) \quad (6)$$

Equation (6) indicates that as the ratio of the weight of the deck to the weight of the columns increases (larger $\gamma = m_b/Nm_c$), the effect of the prestressing force, P_o, reduces and the uplift acceleration tends to that of the free-standing rocking frame, that is gtanα [2,3].

5 ROCKING SPECTRA: SELF SIMILAR RESPONSE

The pulse excitation shown as an insert in the subplots of Figure 2 is the well known symmetric Ricker wavelet given by

$$\psi(t) = a_p(1 - \frac{2\pi^2 t^2}{T_p^2})e^{-\frac{1}{2}\frac{2\pi^2 t^2}{T_p^2}} \tag{7}$$

The first two terms in the right-hand-side of equation (1) express the response of the free standing rocking frame which is fully described by five independent dimensionless variables [2], $\Pi_\theta=\theta$, $\Pi_\omega=\omega_p/p$, $\Pi_a=\tan\alpha$, $\Pi_\gamma=\gamma=m_b/\mathrm{N}m_c$, $\Pi_g=a_p/g$ where a_p and $\omega_p=2\pi/T_p$ is the acceleration amplitude and the cyclic frequency of the excitation pulse.

The contributions of the elasticity, E and the prestressing force of the tendon, P_o, are entering into equation (1) in a dimensionless form, $\Pi_E=EA/m_cg$ and $\Pi_P=P_o/m_cg$.

With the seven dimensionless Π-terms established, the dynamic response of the vertically restrained rocking frame can be expressed as

$$\theta(t) = \varphi\left(\frac{\omega}{p}, \tan\alpha, \gamma, \frac{a}{g}, \frac{EA}{mg}, \frac{P}{mg}\right) \tag{8}$$

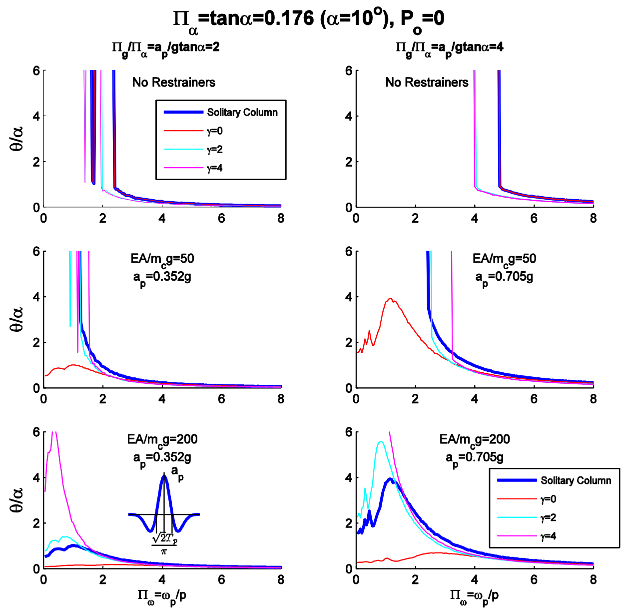

*Figure 2.*Rocking Spectra for different values of the dimensionless products $\Pi_g=a_p/g$, $\Pi_E=EA/m_cg$ and $\Pi\gamma=\gamma$ when a vertically restrained rocking frame with columns having slenderness $\alpha=10^o$ ($\Pi_\alpha=\tan\alpha=0.176$) is subjected to a symmetric Ricker wavelet. In tall rocking frames (large values of ω_p/p) the effect of the vertical restrainers is marginal.

6 THE CONTINGENCY OF RESONANCE

Equation (4) indicates that the linearized rotational stiffness of the vertically restrained rocking column is given by

$$K_r = m_c g R \sin\alpha \left(\frac{2}{2\gamma+1} \tan\alpha \frac{EA}{m_c g} - \frac{1}{\tan\alpha} \right) \tag{9}$$

When ${}^{EA}\!/\!_{m_c g}$ is sufficiently large and satisfies inequality (5), K_r is positive and upon uplifting ($\theta \neq 0$) the rotational frequency of the vertically restrained rocking frame (see equation (20)) is (for $\sin\alpha \approx \tan\alpha$)

$$\omega_r = \sqrt{\frac{1+2\gamma}{1+3\gamma}}\, p \sqrt{\frac{2}{2\gamma+1} \tan^2\alpha \frac{EA}{m_c g} - 1} \tag{10}$$

At resonance, $\omega_p = \omega_r$ and this happens when

$$\frac{\omega_p}{p} = \sqrt{\frac{1+2\gamma}{1+3\gamma}} \sqrt{\frac{2}{2\gamma+1} \tan^2\alpha \frac{EA}{m_c g} - 1} \tag{11}$$

Figure 2 shows rocking spectra of a vertically restrained rocking frame for two levels of the ground excitation (Π_g/Π_α=2 and 4) and three different values of the elasticity of the tendon (free-standing=$EA/m_c g$=0, 50 and 200) as the weight of the cap-beam (deck) increases (γ=0, 2 and 4). The ground excitation is the symmetric Ricker pulse expressed by equation (7). Clearly, as the weight of the cap-beam increases (larger γ), for a given value of the elasticity of the restrainers, the lateral stiffness of the rocking frame decreases (see eq. 4).

Figure 3 shows rocking spectra of a vertically restrained frame with slenderness α=10^o and γ=4 for different values of the dimensionless products Πg, Π_E and Π_p when subjected to a symmetric Ricker pulse. All plots show that at small values of ω_p/p (rocking frames with short columns or long duration pulses), the vertically restrained frames exhibit large rotations-overturning; whereas, when the stiffness is positive, they exhibit the expected amplification in the neighborhood of resonance. On the other hand, as ω_p/p increases(larger columns or shorter duration pulses) the response from all configurations reduces to a single curve showing that the effect of the vertical restrainers becomes marginal compared to the seismic resistance that originates from the mobilization of the rotational inertia of the columns.

At this point it is worth translating the dimensionless products of Figure 3 to physical quantities of typical bridges. First we consider 9.6m tall piers with width $2b$=1.6m (R=4.87, p=1.23rad/s and tanα=1.6/9.6=0.166). These are typical dimensions of bridge piers of highway overpasses and other smaller bridges in Europe and the USA. Let us assume that this rocking frame with p=1.23rad/s, tanα=0.166 and γ=4 is excited by the Ricker pulse that approximates the strong 1992 Erzincan Turkey record (a_p=0.35g, T_p=1.44sec). This gives $\Pi_\omega=\omega_p/p=2\pi/pT_p$=3.54. Figure 3 (left) that is for a_p=0.352g, shows that at ω_p/p=3.54 the effects of the restrainers is marginal and that the free-standing rocking frame experiences approximately the same uplift as the vertically restrained rocking frame with EA=200m_cg. Figure 3(center) indicates that when the acceleration amplitude of the 1.44s long Ricker pulse is increased to a_p=0.53g (that is a most strong excitation) the free- standing rocking frame is at the verge of overturning; however, its stability is appreciably enhanced even

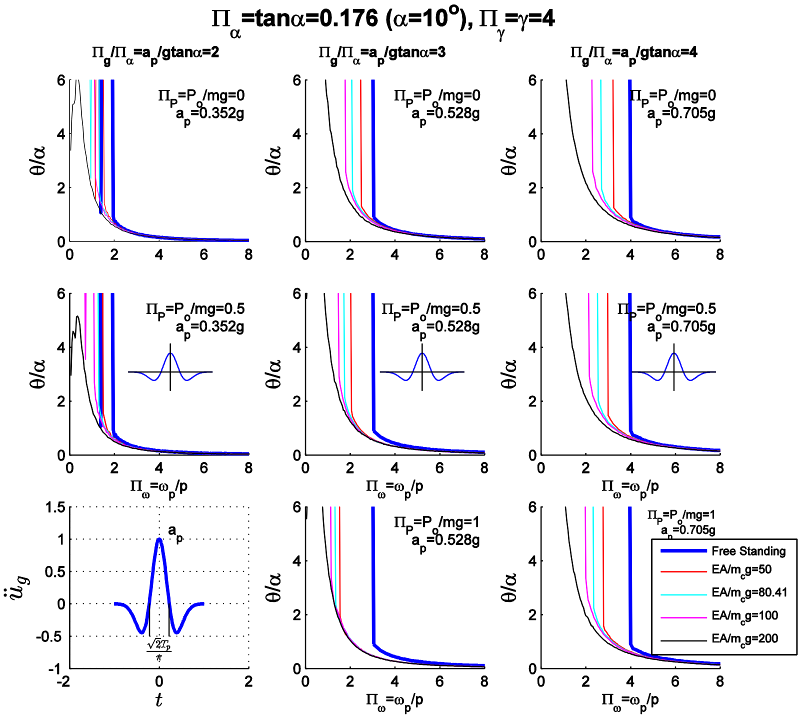

*Figure 3.*Rocking spectra for different values of the dimensionless products. Π_g=a_p/g, Π_E=EA/m_cg and Π_P=P_o/m_cg when the vertically restrained rocking frame with column slenderness α=10^o (Π_α=tanα=0.176) and Π_γ=γ=4 is subjected to a symmetric Ricker wavelet. For values of Πω=ω_p/p>4, the response of the free standing rocking frame is essentially identical to the response of the restrained frame showing that for tall rocking frames the effect of the vertical restrainers is marginal.

with the use of relative flexible restrainers (say EA=50m_cg) which maintains a negative lateral stiffness

We now consider a 24m tall bridge pier with 2b=4.0m (R=12.17m, p=0.778rad/s and tanα=4/24=0.166). Such tall piers are common in valley bridges. Let us again assume that this rocking frame with p=0.778rad/s, tanα=0.166 and γ=4 is excited by a Ricker pulse with a_p=0.35g and T_p=1.44s. This gives $\Pi_\omega=\omega_p/p=2\pi/pT_p$=5.61. For such value of ω_p/p, the free standing rocking frame with (24m)×(4m) piers survives the 1.44s long acceleration pulse even when its acceleration amplitude is as high as a_p=0.705g as shown in Figure

3 (right). The main conclusion that emerges from Figure 3 is that as the size of the columns (or the frequency of the excitation) increases, the effect of the vertical restrainers becomes immaterial given that most of the seismic resistance originates from the mobilization of the rotational inertia of the columns.

7 CONCLUSIONS

This paper investigates the rocking response and stability analysis of an array of slender columns caped with a rigid beam which are vertically restrained with elastic prestressed tendons that pass through the center lines of the columns. While the lateral stiffness of a free standing rocking frame is negative, the lateral stiffness of a vertically restrained rocking frame can be anywhere from negative to positive depending on the axial stiffness of the restraining tendons.

The paper shows that the restraining tendons are effective in suppressing the response of rocking frames with small columns when subjected to long period excitations. As the size of the columns, the frequency of the excitation or the weight of the cap beam increases, the vertical restraining tendons become immaterial given that most of the seismic resistance of tall rocking frames originates primarily from the mobilization of the rotational inertia of their columns.

The paper shows that for up to medium-size rocking frames, where the concept of vertical restrainers may be attractive there is engineering merit for the vertical tendons to be flexible enough so that the overall lateral stiffness of the rocking frame remains negative. In this way, the pivot points are not overloaded with high compressive forces; while at the same time the rocking structure enjoys ample seismic stability by avoiding resonance.

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