

FOOTING FIXITY EFFECT ON PIER DEFLECTION

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ABSTRACT: The rotational restraint coefficient at the top of a pier and the rotational restraint coefficient at the bottom of the pier (that is, the degree of fixity in the foundation of the pier) are used to determine the effective length factor of the pier. Moreover, the effective length factor of a pier is used to determine the slenderness ratio of the pier, while the degree of fixity in the foundation of a pier is used to perform the first-order elastic analysis in order to compute the pier deflection. Finally, the slenderness ratio of the pier is used to determine if the effect of slenderness shall be considered in the design of the pier, while the magnitude of the pier deflection resulting from the first-order analysis is used to determine if the second-order force effect (the $p-\Delta$ effect) shall be considered in the design of the pier. The computations of the slenderness ratio and the deflection of a pier, however, have conventionally been carried out by assuming that the base of the pier is rigidly fixed to the footing, and the footing in turn, is rigidly fixed to the ground. Other degrees of footing fixity have been neglected by the conventional approach. In this paper, two examples are demonstrated for the slenderness ratio computation and the first-order deflection analysis for bridge piers with various degrees of footing fixity (including footings anchored on rock, footings not anchored on rock, footings on soil, and footings on multiple rows of end-bearing piles) recommended by the AASHTO LRFD Bridge Design Specifications. The results from the examples indicate that the degree of footing fixity should not be neglected since it significantly affects the magnitude of the slenderness ratio and the deflection of the pier.

KEYWORDS: Bridge design; Deflection; Foundations; Piers; Lateral loads.

1 INTRODUCTION

A single pier, as shown in *Fig. 1*, and a bent with multiple piers, as shown in *Fig. 2*, are the substructures of bridges. The single pier and the bent with multiple piers are considered as a sway cantilever element and a sway frame, respectively, in the longitudinal direction. For piers not braced against sidesway, the slenderness effects

must be considered where the slenderness ratio is 22 or larger [1]. The slenderness ratio is calculated as Kl_u/r

where : K is the effective length factor of a pier,
 l_u is the unbraced length of a pier, and
 r is the radius of gyration of the cross section of a pier.

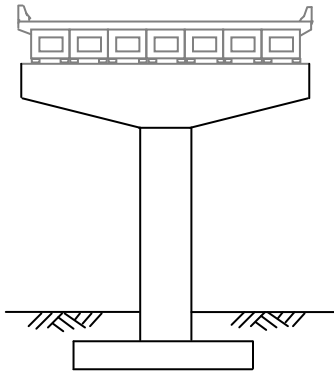


Figure 1. Single pier

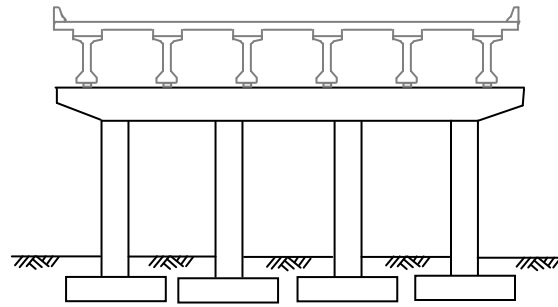


Figure 2. Bent with multiple piers

The effective length factor (K) of a cantilever pier in the longitudinal direction, as shown in *Fig. 1*, or a pier in a sway frame in the longitudinal direction as shown in *Fig. 2* can be obtained by using the Alignment Chart for Determining Effective Length Factor, K , for Unbraced Frames, presented in the AASHTO LRFD Bridge Design Specifications [1] or by using the following two equations [2]:

$$\text{For } K < 2, K = 4 - \frac{1}{1 + 0.2G_a} - \frac{1}{1 + 0.2G_b} - \frac{1}{1 + 0.01G_a G_b} \quad (1)$$

$$\text{For } K \geq 2, K = \frac{2\pi a}{0.9 + \sqrt{0.81 + 4ab}} \quad (2)$$

in which: $a = \frac{G_a G_b}{G_a + G_b} + 3$

$$b = \frac{36}{G_a + G_b} + 6$$

An alternate equation [3] to compute the K value is

$$K = \sqrt{\frac{1.6G_a G_b + 4.0(G_a + G_b) + 7.5}{G_a + G_b + 7.5}} \quad (3)$$

where:

$$G = \frac{\sum \left(\frac{E_c I_c}{L_c} \right)}{\sum \left(\frac{E_g I_g}{L_g} \right)} \quad (4)$$

in which: G is the rotational restraint coefficient at the column (pier) end [the subscripts a and b refer to the two ends of the column (pier) under consideration],

E_c is the modulus of elasticity of the column,

I_c is the moment of inertia of the column,

L_c is the unbraced length of the column,

E_g is the modulus of elasticity of the beam,

I_g is the moment of inertia of the beam, and

L_g is the unsupported length of the beam.

When computing the G value, I_c should be replaced by $0.70 I_{gc}$ and I_g should be replaced by $0.35 I_{gg}$ [4,5,6], in order to take into account the influence of axial loads, the presence of cracked regions in the member, and the effects of the duration of the loads. Note that I_{gc} is the moment of inertia of the gross concrete section of the column and I_{gg} is the moment of inertia of the gross concrete section of the beam.

The AASHTO LRFD Bridge Design Specifications [1] recommends that in the absence of a more refined analysis, the following G values can be used for various footing conditions: $G = 1.5$ for footings anchored on rock, $G = 3.0$ for footings not anchored on rock, $G = 5.0$ for footings on soil, and $G = 1.0$ for footings on multiple rows of end-bearing piles. In addition, the Specifications specify that the moment on a compression member may be increased by multiplying the moment by a moment magnification factor if the sidesway (Δ) of the compression member due to factored lateral or gravity loads calculated by conventional first-order elastic frame analysis is greater than $l_u/1500$. Similar to the computation of the G value, $0.70 I_{gc}$

should be used for the moment of inertia of the column and $0.35 I_{gg}$ should be used for the moment of inertia of the beam [6, 7] for the first-order elastic frame analysis. The aforementioned moment magnification factor may be taken as:

$$\delta = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_e}} \quad (5)$$

where: δ is the moment magnification factor,
 P_u is the factored axial load,
 ϕ_K is the stiffness reduction factor (0.75 for concrete members), and
 P_e is the Euler buckling load.

The Euler buckling load shall be determined as:

$$P_e = \frac{\pi^2 EI}{K^2 l_u^2} \quad (6)$$

where: E is the modulus of elasticity,
 I is the moment of inertia about the axis under consideration,
 K is the effective length factor in the plane of bending, and
 l_u is the unsupported length of a compression member.

In lieu of a more precise calculation, EI in Eq.(6) shall be taken as the greater of:

$$EI = \frac{\frac{E_c I_g}{5} + E_s I_s}{1 + \beta_d} \quad (7)$$

or

$$EI = \frac{E_c I_g}{1 + \beta_d} \quad (8)$$

where: E_c is the modulus of elasticity of concrete,
 I_g is the moment of inertia of the gross concrete section about the centroidal axis,
 E_s is the modulus of elasticity of longitudinal steel,

- I_s is the moment of inertia of longitudinal steel about the centroidal axis, and
- β_d is the ratio of the maximum factored permanent load moment to the maximum factored total load moment.

The determination of the effective length factor and the computation of the deflection of a pier, however, have been conventionally carried out by assuming that the base of the pier is rigidly fixed to the footing, and the footing in turn, is rigidly fixed to the ground. The degree of fixity in the foundation has been neglected by the conventional approximate approach. The accuracy of the approximate approach, therefore, is questionable.

Two approaches for the determination of the effective length factor and the computation of the deflection of piers are presented in this paper. Approach I is an approximate approach which assumes the bases of the piers are rigidly fixed to the ground, while Approach II is a refined approach which considers the degree of fixity in the foundation of the piers. Results from both approaches are then compared with each other.

2 APPROACH I – AN APPROXIMATE APPROACH

Considering the single pier in the longitudinal direction, as shown in *Fig. 1*, Approach I (an approximate approach) treats the base of the pier as rotationally fixed and translationally fixed, and, additionally, treats the top of the pier as rotationally free and translationally free, as shown in *Fig. 3*. In addition, considering the piers in the bent in the longitudinal direction, as shown in *Fig. 2*,



Figure 3. Ideal condition for a cantilever single pier in the longitudinal direction

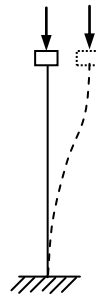


Figure 4. Ideal condition for piers in a bent in the longitudinal direction

Approach I treats the bases of the piers as rotationally fixed and translationally fixed, and, additionally, treats the tops of the piers as rotationally fixed and translationally free as shown in *Fig. 4*. The design value of the effective length factor, K , therefore, is 2.1 and 1.2 for the pier in *Figs. 3* and *4*, respectively [1]. In Approach I, the idealized conditions of the pier(s), as shown in *Figs. 3* and *4*, are also used to compute the deflection of a cantilever single pier and the piers in a bent, respectively, in the longitudinal direction.

3 APPROACH II – A REFINED APPROACH

In this approach, the degree of fixity in the pier's foundation is considered by using an appropriate G value (corresponding to the degree of fixity in the pier's foundation) at the bottom of the pier. In addition, the rotational rigidity at the top of the pier in the pier cap of a bent in the longitudinal direction is also considered. In order to achieve the appropriate G value at the bottom of a cantilever single pier in the longitudinal direction, a tie beam is introduced to the base of the pier, as shown in *Fig. 5(a)*. Similarly, in order to achieve the appropriate G values at the foundations of the piers in a bent in the longitudinal direction, a continuous tie beam is introduced to the bases of the piers, as shown in *Fig. 6(a)*. The structure shown in *Figs. 5(a)* and *6(a)* can be treated as the substructure shown in *Figs. 5(b)* and *6(b)*, respectively. The pinned ends of the tie beam shown in *Figs. 5(a)* and *6(a)* are located at the inflection points of the tie beam in the swing structure shown in *Figs. 5(b)* and *6(b)*, respectively. The inflection point in the tie beam in each bay is assumed to be located at the midspan of the bay width. In Approach II, *Figs. 5(a)* and *6(a)* are used as the structural systems to compute the deflection of a cantilever single pier and a bent with multiple piers, respectively, in the longitudinal direction.

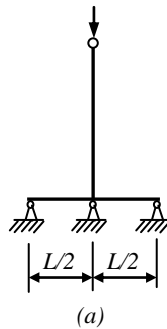


Figure 5. Modified structural system for a cantilever single pier in the longitudinal direction

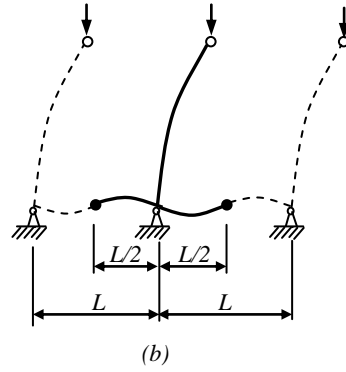


Figure 5. Modified structural system for a cantilever single pier in the longitudinal direction

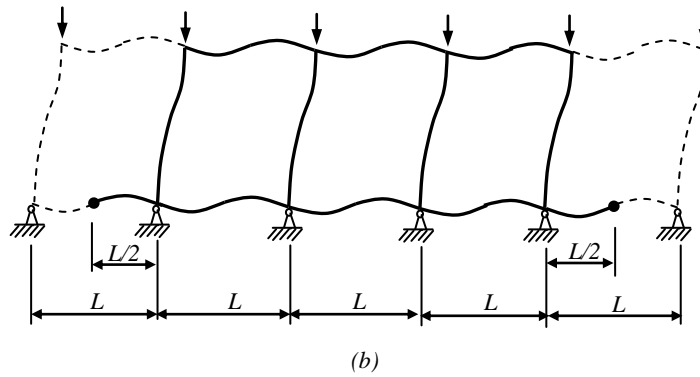
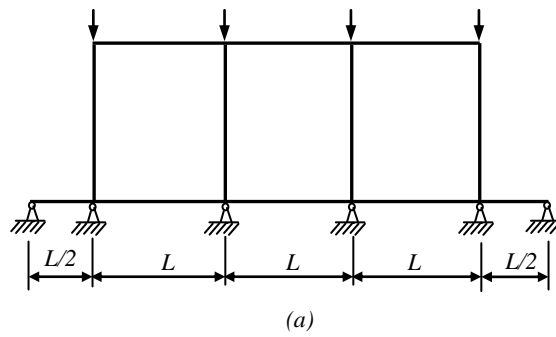


Figure 6. Modified structural system for a bent with multiple piers in the longitudinal direction

4 PIER DEFLECTION COMPUTATION EXAMPLES USING APPROACHES I & II

Two examples are demonstrated for the computation of pier and bent deflections in the longitudinal direction due to lateral and gravity loads. Concrete with a modulus of elasticity, $E = 25,100 \text{ MPa}$, is used for the pier and the bent.

Example 1: Compute the deflection in the longitudinal direction of the pier subjected to the factored lateral force (270 kN) and gravity load (6000 kN), as shown in *Fig. 7*, for the following four different footing conditions: (1) the footing anchored on rock, (2) the footing not anchored on rock, (3) the footing on soil, and (4) the footing on multiple rows of end-bearing piles.

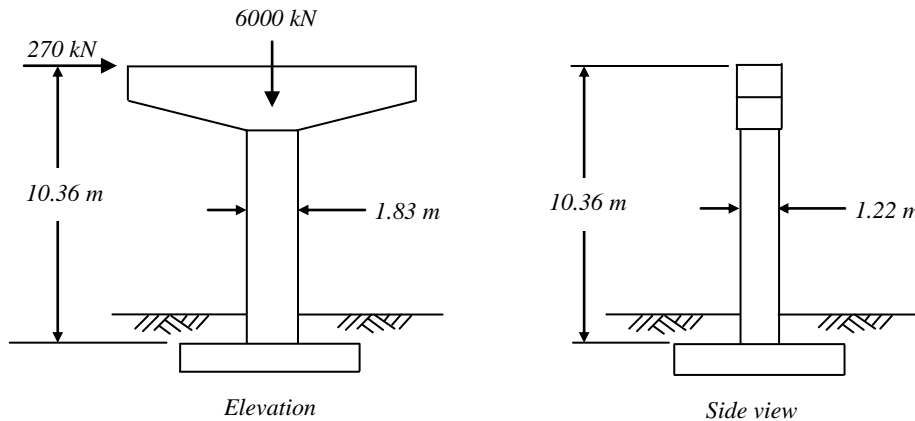


Figure 7. Loaded single pier illustrated for Example 1

Approach I: Referring to the pier in the longitudinal direction shown in *Fig. 7*, the base of the pier is treated as rotationally and translationally fixed for all four different footing conditions, while the top of the pier is treated as rotationally and translationally free, as shown in *Fig. 3*. Therefore, the design value of the effective length factor of the pier is 2.1. The pier height is 10.36 m. For a rectangular compression member, the radius of gyration (r) may be taken as 0.30 times the overall dimension in the direction in which stability is being considered [1].

Therefore, the radius of gyration of the pier about the transverse axis can be computed as $1.83 \text{ m} \times 0.3 = 0.549 \text{ m}$. The transverse axis slenderness ratio of the pier, therefore, can be computed as:

$$\frac{Kl}{r} = \frac{1 \times (0.36 \text{ m})}{0.549 \text{ m}} = 39.6$$

Since the slenderness ratio is larger than 22, the slenderness effects on the pier must be considered. From the first-order elastic analysis using $0.70 I_{gc}$ as the moment of inertia for the pier bending about the transverse axis, the deflection at the top of the pier can be computed as:

$$\Delta = \frac{Fh^3}{3EI} = \frac{70 \text{ kN} \times (0.36 \text{ m})^3}{3 \left(25.1 \frac{\text{kN}}{\text{mm}^2} \right) \left(0.70 \left[\frac{(220 \text{ mm}) \times (830 \text{ mm})^3}{12} \right] \right)} = 9.14 \text{ mm}$$

where: F is the factored lateral load,
h is the height of the pier,
E is the modulus of elasticity of the pier, and
I is the moment of inertia of the pier.

Since the deflection, 9.14 mm, is larger than $l_u/1500 (= 6.91 \text{ mm})$, the deflection (which corresponds to the bending moment at the base of the pier) due to the factored lateral load must be multiplied by the moment magnification factor, δ . From Eq. (5), the moment magnification factor can be computed as:

$$\delta = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_e}} = \frac{1}{1 - \frac{6000 \text{ kN}}{0.75 \times (31 \times 10^5 \text{ kN})}} = 1.065$$

where $P_e = \frac{\pi^2 EI}{(l_u)^2} = \frac{\pi^2 \left(26 \times 10^{12} \text{ kN} \cdot \text{mm}^2 \right)}{[1 \times (0.36 \text{ m})]^2} = 1.31 \times 10^5 \text{ kN}$

Assuming that Eq. (8) controls over Eq. (7) and $\beta_d = 0$ (there is no factored permanent load moment about the transverse axis of the pier), one has:

$$EI = \frac{E_c I_g}{1 + \beta_d} = \frac{\left(25.1 \frac{\text{kN}}{\text{mm}^2}\right) \left[\frac{\left(220\text{mm} \times 830\text{mm}\right)}{12} \right]}{1 + 0} = 6.26 \times 10^{12} \text{ kN} - \text{mm}^2$$

Therefore, the final deflection of the pier in the longitudinal direction due to the slenderness effects is:

$$\Delta = (1.065) (9.14 \text{ mm}) = 9.73 \text{ mm}$$

Table 1 summarizes the results obtained from Approach I. Note that this table includes all of the four footing conditions: (1) the footing anchored on rock, (2) the footing not anchored on rock, (3) the footing on soil, and (4) the footing on multiple rows of end-bearing piles.

Table 1. Deflections of the pier in Example 1 computed by Approach I

| footing condition | effective length factor | slenderness ratio | pier base rotation (rad) | deflection calculated by first-order analysis | moment magnification factor, δ | final deflection of the pier |
|-------------------|-------------------------|-------------------|--------------------------|---|---------------------------------------|------------------------------|
| all | 2.1 | 39.6 | 0 | 9.14 mm | 1.065 | 9.73 mm |

Approach II: Referring to the pier in the longitudinal direction shown in *Fig. 7*, assign $G_a = 1.5$ for footings anchored on rock, $G_a = 3.0$ for footings not anchored on rock, $G_a = 5.0$ for footings on soil, and $G_a = 1.0$ for footings on multiple rows of end-bearing piles as per the G values recommended by the AASHTO LRFD Bridge Design Specifications [1]. Since the top of the pier is rotationally free about the transverse axis of the pier and translationally free along the longitudinal direction of the pier, $G_b = \infty$ at the top of the pier. From Eq. (2) or the Alignment Chart for Determining Effective Length Factor, K , for Unbraced Frames, presented in the AASHTO LRFD Bridge Design Specifications [1], one has: $K = 2.5$ for the footing anchored on rock, $K = 2.9$ for the footing not anchored on rock, $K = 3.4$ for the footing on soil, and $K = 2.3$ for the footing on multiple rows of end-bearing piles. Since the transverse axis slenderness ratios (Kl_u/r) of the pier with four different footing conditions are all larger than 22, the slenderness effects on the pier must be considered for all of the four different footing conditions.

In order to use the $0.70 I_{gc}$ value as the moment of inertia for the pier bending about its transverse axis, a reduced cross section of $1.68 \text{ m} \times 1.12 \text{ m}$ (as shown in *Fig. 8*)

is used to replace the original cross section of 1.83 m × 1.22 m for the pier. Also, referring to Fig. 5(a), a modified structural system with different lengths of added tie beams is made, as shown in Fig. 8, in order to achieve the appropriate G values at the base of the pier for the four different footing conditions.

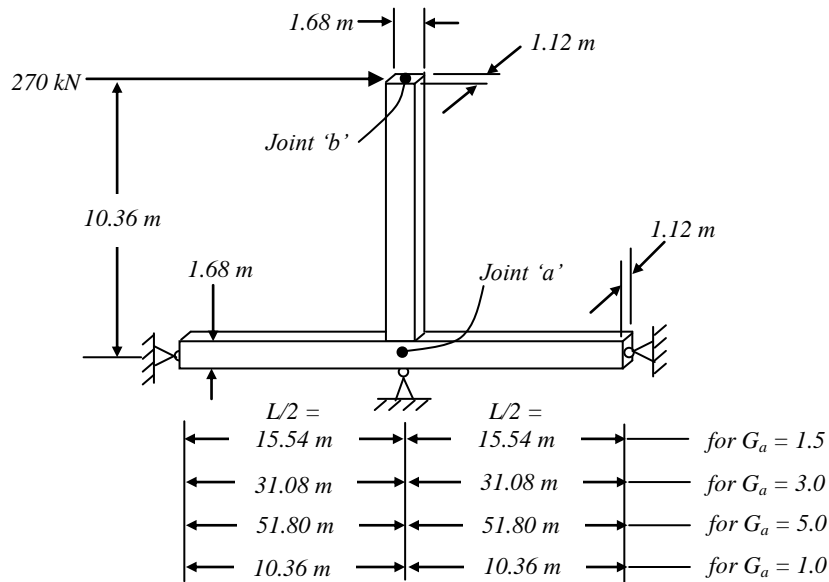


Figure 8. Modified structural system for Approach II in Example 1

For example, as shown in Fig. 5(b) and Fig. 8, if $L/2 = 15.54$ m for the tie beam, the span length of the tie beam will be $L = 31.08$ m on each side of joint 'a.' Therefore, the G_a value at the base of the pier can be computed using Eq. (4) as:

$$G_a = \frac{\sum \left(\frac{E_c I_c}{L_c} \right) \left[\frac{1.12 \text{ m} \times 1.68 \text{ m}}{12} \right]}{\sum \left(\frac{E_g I_g}{L_g} \right) \left[\frac{1.12 \text{ m} \times 1.68 \text{ m}}{12} \right]} = \frac{10.36 \text{ m}}{31.08 \text{ m}} = 1.5$$

Considering the pier with $G_a = 1.5$ for the footing anchored on rock, the deflection at the top of the pier due to the factored lateral force (270 kN) along the longitudinal direction of the bent can be computed using the following steps:

- (1) Referring to *Fig. 9(a)*, the bending moment at the base of the pier (at joint 'a') can be computed as: $M_{ab} = 270 \text{ kN} \cdot 10.36 \text{ m} = 2797 \text{ kN-m}$.
- (2) The end moments at joint 'a' of the tie beam therefore can be computed as: $M_{ad} = M_{ac} = M_{ab} / 2 = 2797 \text{ kN-m} / 2 = 1399 \text{ kN-m}$.
- (3) Referring to *Fig. 8*, the moment of inertia about the transverse axis of the pier and the tie beam can be computed as: $I = (1120 \text{ mm})(1680 \text{ mm})^3 / 12 = 4.426 \times 10^{11} \text{ mm}^4$.
- (4) Referring to *Fig. 9(b)*, the rotation at joint 'a' of the tie beam can be computed as $\theta_a = [(M_{ad})(I_{ad})] / [3(E_{ad})(I_{ad})] = [(1,399,000 \text{ kN-mm})(15,540 \text{ mm})] / [3(25.1 \text{ kN/mm}^2)(4.426 \times 10^{11} \text{ mm}^4)] = 0.000652 \text{ rad}$.
- (5) Referring to Approach I, when the base of the pier is considered rigidly fixed to the ground, the deflection at the top of the pier was computed as 9.14 mm. Since the rotation at the base of the pier is $\theta_a = 0.000652 \text{ rad}$ for the condition in which the footing is anchored on rock ($G_a = 1.5$), the total deflection (including the deflection caused by the rotation at the base of the pier) can be computed as: $\Delta = 9.14 \text{ mm} + (10,360 \text{ mm})(0.000652) = 15.89 \text{ mm}$.

Since the deflection, 15.89 mm, is larger than $l_u/1500$ (= 6.91 mm), the deflection due to the factored lateral load must be multiplied by the moment magnification factor. Similar to the procedure shown in Approach I, the moment magnification factor can be computed by using Eq. (5) as:

$$\delta = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_c}} = \frac{1}{1 - \frac{6000 \text{ kN}}{0.75 (2.1 \times 10^4 \text{ kN})}} = 1.095$$

Note that $K = 2.5$ has been used for the computation of P_c .

Therefore, the final deflection of the pier in the longitudinal direction due to the slenderness effects is:

$$\Delta = (1.095) (15.89 \text{ mm}) = 17.40 \text{ mm}$$

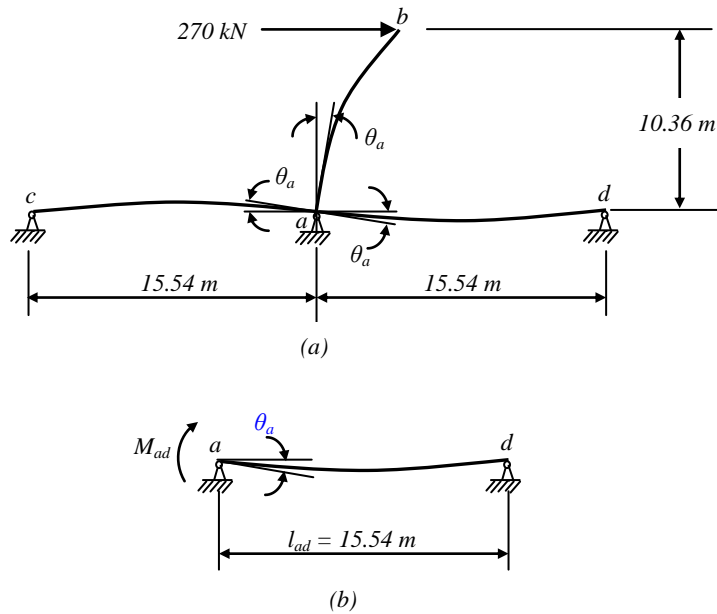


Figure 9. Joint rotation at the base of the pier with $G_a = 1.5$

The deflections in the longitudinal direction of the pier for four different footing conditions are computed using Approach II and are summarized as shown in Table 2, where the footing conditions are (1) the footing anchored on rock, (2) the footing not anchored on rock, (3) the footing on soil, and (4) the footing on multiple rows of end-bearing piles.

Table 2. Deflections of the pier in Example 1 computed by Approach II

| footing condition | effective length factor | slenderness ratio | pier base rotation (rad) | deflection calculated by first-order analysis | moment magnification factor, δ | final deflection of the pier |
|-------------------|-------------------------|-------------------|--------------------------|---|---------------------------------------|------------------------------|
| (1) | 2.5 | 47.2 | 0.000652 | 15.89 mm | 1.095 | 17.40 mm |
| (2) | 2.9 | 54.7 | 0.001305 | 22.66 mm | 1.132 | 25.66 mm |
| (3) | 3.4 | 64.2 | 0.002174 | 31.66 mm | 1.191 | 37.72 mm |
| (4) | 2.3 | 43.4 | 0.000435 | 13.65 mm | 1.079 | 14.73 mm |

Example 2: Compute the deflection in the longitudinal direction of the bent subjected to the factored lateral and gravity loads, as shown in *Fig. 10*, for the following four different footing conditions: (1) footings anchored on rock, (2) footings not anchored on rock, (3) footings on soil, and (4) footings on multiple rows of end-bearing piles.

Approach I: Referring to the bent in the longitudinal direction shown in *Fig. 10*, the bases of the piers are treated as rotationally and translationally fixed for all four different footing conditions, while the tops of the piers are treated as rotationally fixed and translationally free (since the cap of the piers is treated as a rigid element), as shown in *Fig. 4*. Therefore, the design value of the effective length factor of the piers is 1.2.

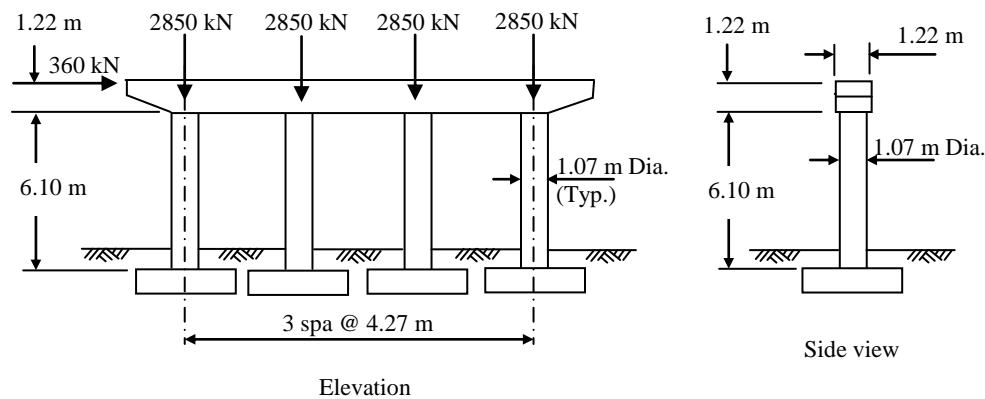


Figure 10. Loaded bent with multiple piers illustrated for Example 2

The unbraced length of each of the piers is 6.10 m. For a circular compression member, the radius of gyration (r) may be taken as 0.25 times the diameter [1]. Therefore, the radius of gyration of each of the piers can be computed as $1.07 \text{ m} \times 0.25 = 0.268 \text{ m}$. The transverse axis slenderness ratio of each of the piers, therefore, can be computed as:

$$\frac{Kl_u}{r} = \frac{(1.2)(6.10\text{m})}{0.268\text{m}} = 27.3$$

Since the slenderness ratio is larger than 22, the slenderness effects on the piers must be considered. Using $0.70 I_{gc}$ as the moment of inertia for each of the piers bending about the transverse axis, the deflection at the top of each of the piers in the longitudinal direction of the bent can be computed by using the first-order elastic analysis as:

$$\Delta = \frac{Fh^3}{12EI} = \frac{(60\text{kN}/4)(100\text{mm})^3}{12\left(25.1\frac{\text{kN}}{\text{mm}^2}\right)(0.70)\left[\frac{\pi(35\text{mm})^4}{4}\right]} = 1.51\text{mm}$$

where: F is the factored lateral load applied to each of the piers,
 h is the height of each of the piers (measured from the top of the footing to the bottom of the pier cap),
 E is the modulus of elasticity of the piers, and
 I is the moment of inertia of each of the piers.

Since the deflection due to the factored lateral load is 1.51 mm, which is less than $l_u/1500$ ($= 4.07$ mm), the deflection does not need to be increased by being multiplied by the moment magnification factor. Therefore, $\delta = 1.0$. Table 3 summarizes the results obtained from Approach I. Note that this table includes all of the four footing conditions: (1) footings anchored on rock, (2) footings not anchored on rock, (3) footings on soil, and (4) footings on multiple rows of end-bearing piles.

Table 3. Deflections of the bent in Example 2 computed by Approach I

| footing condition | K value of exterior piers | K value of interior piers | $\frac{Kl_u}{r}$ of exterior piers | $\frac{Kl_u}{r}$ of interior piers | bent deflection calculated by first-order analysis | moment magnification factor, δ | final deflection of the bent |
|-------------------|---------------------------|---------------------------|------------------------------------|------------------------------------|--|---------------------------------------|------------------------------|
| all | 1.2 | 1.2 | 27.3 | 27.3 | 1.51 mm | 1.0 | 1.51 mm |

Approach II: In order to use the $0.70 I_{gc}$ value as the moment of inertia for the piers bending about the transverse axis, a reduced cross section of 0.978 m dia. is used to replace the original cross section of 1.07 m dia. for the piers. Also, in order to use the $0.35 I_{gg}$ value as the moment of inertia for the pier cap bending about the transverse axis, a reduced cross section of 0.940 m \times 0.940 m is used to replace the original cross section of 1.22 m \times 1.22 m for the pier cap. In addition, referring to

Fig. 6(a), a modified structural system is made for the bent in the longitudinal direction, as shown in Fig. 11. In the modified structural system, a continuous tie beam is added to the bases of the piers in order to achieve the appropriate G values at the foundations of the piers in the bent in the longitudinal direction. The cross section of the added tie beam is $0.579 \text{ m} \times 0.579 \text{ m}$, $0.488 \text{ m} \times 0.488 \text{ m}$, $0.429 \text{ m} \times 0.429 \text{ m}$, and $0.643 \text{ m} \times 0.643 \text{ m}$ for $G_a = 1.5$, 3.0 , 5.0 , and 1.0 , respectively. For example, if the cross section of the tie beam is $0.579 \text{ m} \times 0.579 \text{ m}$, the G_a value at each of the pier base can be computed by using Eq. (4) as:

$$G_a = \frac{\sum \left(\frac{E_c I_c}{L_c} \right)}{\sum \left(\frac{E_g I_g}{L_g} \right)} = \frac{\frac{\pi \left(\frac{0.978 \text{ m}}{2} \right)^4}{4}}{2 \frac{\pi \left(\frac{0.579 \text{ m}}{2} \right)^4}{12}} = \frac{6.71 \text{ m}}{4.27 \text{ m}} = 1.5$$

Note that $G_a = 1.5$ is for footings anchored on rock, $G_a = 3.0$ is for footings not anchored on rock, $G_a = 5.0$ is for footings on soil, and $G_a = 1.0$ is for footings on multiple rows of end-bearing piles. Also note that the pier height shown in Fig. 11 is measured from the top of the footing to the mid-height of the pier cap.

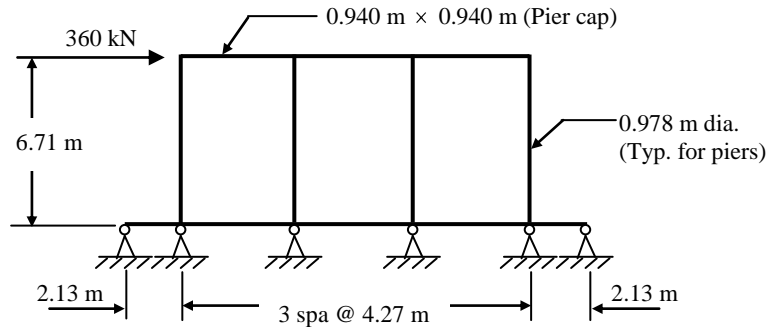


Figure 11. Modified structural system for Approach II in Example 2

Referring to *Fig. 11*, one has $G_b = 0.22$ at each of the tops of the interior piers and $G_b = 0.44$ at each of the tops of the exterior piers by using Eq. (4). With the G_a and G_b values of each pier in the bent determined, the effective length factor, K , of each pier in the bent can be determined by using Eq. (3) or the Alignment Chart for Determining Effective Length Factor, K , for Unbraced Frames, presented in the AASHTO LRFD Bridge Design Specifications [1]. The determined K values of the exterior and interior piers for the four different footing conditions are summarized in Table 4. Also, as shown in Table 4, since the slenderness ratios (Kl_u/r) of all of the piers with four different footing conditions in the direction in which stability is being considered are all larger than 22, the slenderness effects on all of the piers must be considered for all of the four different footing conditions.

Table 4. Deflections of the bent in Example 2 computed by Approach II

| footing condition | K value of exterior piers | K value of interior piers | $\frac{Kl_u}{r}$ of exterior piers | $\frac{Kl_u}{r}$ of interior piers | bent deflection calculated by first-order analysis | moment magnification factor, δ | final deflection of the bent |
|-------------------|---------------------------|---------------------------|------------------------------------|------------------------------------|--|---------------------------------------|------------------------------|
| (1) | 1.31 | 1.27 | 29.8 | 28.9 | 3.94 mm | 1.0 | 3.94 mm |
| (2) | 1.46 | 1.41 | 33.2 | 32.1 | 4.87 mm | 1.048 | 5.10 mm |
| (3) | 1.59 | 1.54 | 36.2 | 35.1 | 5.75 mm | 1.057 | 6.08 mm |
| (4) | 1.25 | 1.21 | 28.5 | 27.5 | 3.52 mm | 1.0 | 3.52 mm |

Referring to *Fig. 11*, the deflection of the bent due to the factored lateral force (360 kN) along the longitudinal direction can be computed by using the computer software SAP2000 [8]. The results are shown in Table 4. Since both of the deflections (3.94 mm and 3.52 mm) of the bent with footing conditions (1) and (4) are less than $l_u/1500$ ($= 4.07$ mm), these deflections do not need to be increased by being multiplied by the moment magnification factor. Therefore, $\delta = 1.0$. However, since both of the deflections (4.87 mm and 5.75 mm) of the bent with footing conditions (2) and (3) are larger than $l_u/1500$, these deflections must be multiplied by the moment magnification factor, δ . Considering the bent with footing condition (2), the moment magnification factor can be computed by using Eq. (5) as:

$$\delta = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_c}} = \frac{1}{1 - \frac{4 \text{ (} \llcorner 850 \text{ kN} \llcorner)}{0.75 \llcorner \times 80,385 \text{ kN} + 2 \times 86,186 \text{ kN} \llcorner}} = 1.048$$

where

$$P_e = \frac{\pi^2 EI}{l_u^2} = \frac{\pi^2 (6.46 \times 10^{11} \text{ kN} \cdot \text{mm}^2)}{(4.46 \times 1000 \text{ mm})^2} = 80,385 \text{ kN (for exterior piers)}$$

and

$$P_e = \frac{\pi^2 EI}{l_u^2} = \frac{\pi^2 (6.46 \times 10^{11} \text{ kN} \cdot \text{mm}^2)}{(4.41 \times 1000 \text{ mm})^2} = 86,186 \text{ kN (for interior piers)}$$

Assuming that Eq. (8) controls over Eq. (7) and $\beta_d = 0$ (there is no factored permanent load moment about the transverse axis of the pier), one has:

$$EI = \frac{E_c I_g}{1 + \beta_d} = \frac{25.1 \frac{\text{kN}}{\text{mm}^2} \left[\frac{\pi \left(\frac{1070 \text{ mm}}{2} \right)^4}{4} \right]}{1 + 0} = 6.46 \times 10^{11} \text{ kN} \cdot \text{mm}^2$$

Therefore, the final deflection of the bent with footing condition (2) due to the slenderness effects is:

$$\Delta = (1.048) (4.87 \text{ mm}) = 5.10 \text{ mm}$$

Using the same approach, the moment magnification factor, δ , and the final deflection of the bent with footing condition (3) can be computed and the results are shown in Table 4.

5 CONCLUSIONS

The computation of the deflection of a single cantilever pier has conventionally been carried out by assuming that the base of the pier is rigidly fixed to the ground. Similarly, the computation of the deflection in the longitudinal direction of a bent with multiple piers has conventionally been carried out not only by assuming that the bases of the piers are rigidly fixed to the ground but also by assuming that the tops of the piers are rotationally fixed to the pier cap. The rotational restraint coefficient(s) at the base of a single cantilever pier, as well as at the bases and tops

of the piers in a bent in the longitudinal direction, have completely been neglected by the conventional approach (which is classified as Approach I in this paper). In this paper, two examples, one for a single cantilever pier and the other for a bent with multiple piers, are demonstrated for the slenderness ratio computation and the first-order deflection analysis for the pier(s) by using two approaches. Approach I is an approximate approach assuming the footings are rigidly fixed to the ground, while Approach II is a refined approach considering various degrees of footing fixity. The various degrees of footing fixity include (1) footings anchored on rock, (2) footings not anchored on rock, (3) footings on soil, and (4) footings on multiple rows of end-bearing piles, as specified by the AASHTO LRFD Bridge Design Specifications. Example 1 indicates that Approach II results in the largest moment magnification factor of $\delta = 1.191$ (for the footing on soil condition), while the δ value resulted from Approach I is only 1.065. That is, the percentage of moment increase that resulted from Approach II is 19.1 %, which is about 3 times of that (6.5 %) of the result from Approach I. Also, Approach II results in the largest deflection of 37.72 mm (for the footing on soil condition) which is about 4 times of that (9.73 mm) of the result from Approach I. Example 2 indicates that Approach II results in the largest moment magnification factor of $\delta = 1.057$ (for the footings on soil condition), while the moment magnification factor can be neglected in Approach I (that is $\delta = 1.0$). Also, Approach II results in the largest deflection of 6.08 mm (for the footings on soil condition), which is about 4 times of that (1.51 mm) of the result from Approach I. The results from the examples indicate that the degree of footing fixity should not be neglected since it significantly affects the magnitude of the bending moment(s) and the deflection(s) of the pier(s).

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