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A MATHEMATICAL RESEARCH ON CABLES WITH FIXED OR MOVABLE ANCHORAGES

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ABSTRACT: For the analytical study of cables with dampers or movable anchorages, finite series of sinusoidal form are usually employed. This paper aims to determine the eigenfrequencies and shape functions of stay-cables with fixed or movable anchorages in order to make the application of the modal superposition method easier and more accurate.

KEYWORDS: stay-cables, dynamic behavior, movable anchorages, dampers

1 INTRODUCTION

Cable stayed bridges have been known since the beginning of the 18th century, but they have been of great interest only in the last fifty years, particularly because they have a special shape and also provide an alternative solution to suspension bridges for long spans. The main reasons for this delayed application were the difficulties in their static and dynamic analysis, the various non-linearities, the limited computational capabilities, and the lack of high strength materials and construction techniques. There are a great number of studies, concerning the static behavior [1-8], the dynamic analysis [9-18], and the stability of cable-stayed bridges [19-22].

A significant problem, which arises from the praxis, is the cables' rain-wind induced vibrations. Large amplitude Rain-Wind-Induced-Vibrations (RWIV) of stay cables is a major problem in the design of cable-stayed bridges. Such phenomena were first observed in the Meikonishi-bridge in Nagoya, Japan [23] and also later in other such bridges, as for instance on the fully steel Erasmus-bridge in Rotterdam, Netherlands (1996) and the Second Severn Crossing, connecting England and Wales [24]. It was found that the cables were stable only under wind action, while they were oscillating under a combined influence of rain and wind, leading to large amplitude motions, even for light-to-moderate simultaneous rain and wind action. The frequency of the observed vibrations was much lower than the critical one of the vortex-induced vibrations, while it was also perceived that the cable oscillations took place in the vertical plane mostly in single mode; for increasing cable length however, higher modes (up to the 4th) appeared. Most importantly, during the oscillations a water rivulet appeared on the lower surface of the cable, which was characterized by a leeward shift and vibrated in circumferential directions [23,25,26].

The produced by such a way, vibrations can reduce the life of the cable and its connection due to fatigue or rapid progress of the corrosion.

Several methods, including aerodynamic or structural means, have been investigated in order to control the vibrations of bridge's stay cables. Aerodynamic methods, such as change of the cables' roughness were effective only for certain classes of vibration. Another method is the coupling of the stays with secondary wires, in order to reduce their effective length and thereby to avoid resonance. This method changes the bridge's aesthetics.

Another widely applied method is this of external dampers attached transversely to the stay-cables. Many researchers have proposed passive control of cables using viscous dampers.

The last method consist of a system with movable anchorage by using a Friction Pendulum Bearing or an Elastomeric bearing to replace the conventional fixed support of stay cables.

For the analytical study of the two last methods are usually used finite series of sinus form.

This paper aims to determine the eigenfrequencies and shape functions of stay-cables with end conditions like the ones of the cables showed in figure 1, in order to make the application of the modal superposition method easier and more accurate.

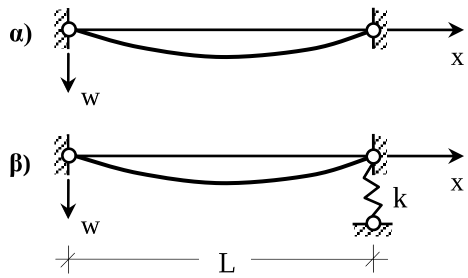

Figure 1. a) Cable with fixed ends b) Cable with a movable end

2. ASSUMPTIONS

a. The studied cable has, under the dead and live loads, the catenary shape elastic line, with displacements w_o and tensile forces T_o (see fig.1). Because of its shallow form, the above line can be replaced by a parabola of second order.

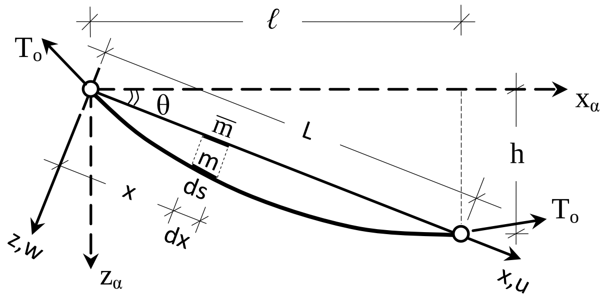

Figure 2. Cables and reference axes

b. Under the action of the dynamic loads $p_y(x,t)$ and $p_z(x,t)$, the cable takes the shape of figure 3, with additional displacements u_d, υ_d, w_d and tensile forces T_d.

c. The static and dynamic tensile forces are connected with the following relations:

$$\left.\begin{aligned} T(t) &= T_o + T_d(t) \\ H(t) &= H_o + H_d(t) \end{aligned}\right\} \qquad (1a,b)$$

where H, is the projection of T on x-axis.

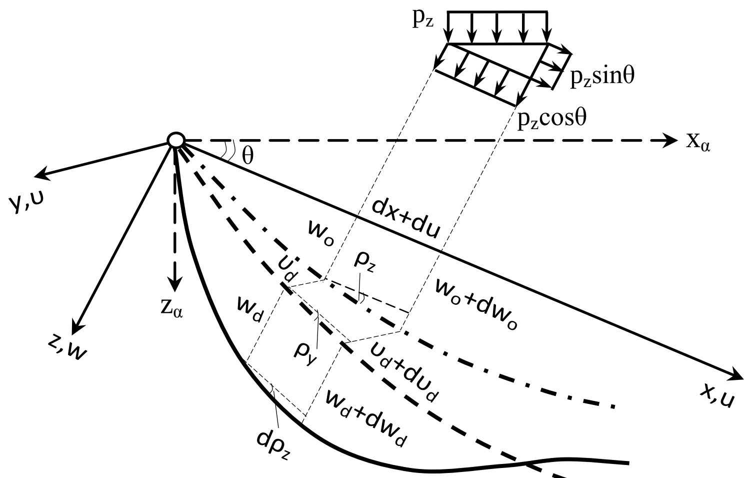

Figure 3. Deformation of the cable

d. The $\overline{m}(x)$ and m(s) of figure 2, are connected through the relation

$$\overline{m}(x) = m(s)\frac{ds}{dx} \qquad (2)$$

e. We neglect the flexural rigidity of the cables, (as it is proved in [2]).
f. The studied cables are referred to the inclined axis system 0-xyz of fig.3.

3 THE EQUATIONS OF MOTION

3.1 Projection on xoz-plane

For a shallow form of the cable, the following relations are valid:

$$\left.\begin{aligned} &\cos\rho_z = dx/ds \cong 1 \\ &\sin\rho_z = dw/ds \\ &\sin d\rho_z \cong 0 \end{aligned}\right\} \qquad (3.1a,b)$$

3.1.1 Equilibrium of horizontal forces

Taking the equilibrium of horizontal forces in xoz-plane we obtain:

$-T\frac{dx}{ds} + T\frac{dx}{ds} + \partial\left(T\frac{dx}{ds}\right) + p_x ds - c\dot{u}ds - m\ddot{u}ds = 0$, or finally:

$$\frac{\partial H}{\partial s} - c\dot{u} - m\ddot{u} = -p_x(x,t) \qquad (3.2)$$

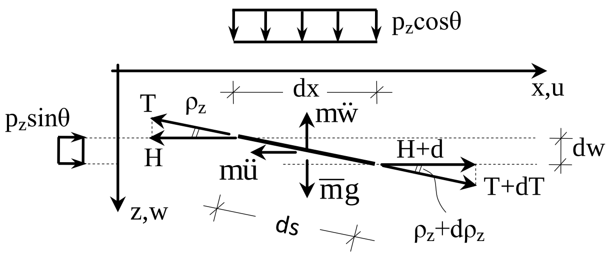

Figure 4. Projection on xoz-plane

3.1.2 Equilibrium of vertical forces

Taking the equilibrium of vertical forces in xoz-plane we obtain:

$$-T\frac{\partial w}{\partial s} + T\frac{\partial w}{\partial s} + \partial\left(T\frac{\partial w}{\partial s}\right) + \overline{m}gds + p_z ds - c\dot{w}ds - m\ddot{w}ds = 0 \qquad (3.3a)$$

Since static equilibrium is valid, i.e

$$\frac{\partial}{\partial s}\left(T_o\frac{\partial w_o}{\partial s}\right) = -\overline{m}g, \quad \text{and also: } w = w_o + w_d \text{ and } \frac{\partial w_o(x)}{\partial t} = 0$$

equation (2.3a) becomes:

$$T_o \frac{\partial^2 w_d}{\partial x^2} + T_d \left(\frac{\partial^2 w_o}{\partial x^2} + \frac{\partial^2 w_d}{\partial x^2} \right) - c\dot{w}_d - m\ddot{w}_d = -p_z(x,t) \qquad (3.3b)$$

3.2 Projection on xoy-plane

Taking the equilibrium of vertical forces in xoy-plane through a similar process like the one of §3.1.2 we obtain:

$$T_o \frac{\partial^2 \upsilon_d}{\partial x^2} + T_d \frac{\partial^2 \upsilon_d}{\partial x^2} - c\dot{\upsilon}_d - m\ddot{\upsilon}_d = -p_y(x,t) \qquad (3.4)$$

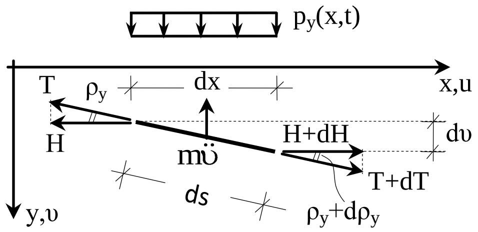

Figure 5. Projection on xoy-plane

4 THE PARABOLA APPROACH

The parabola is widely used especially for shallow forms of cables (see §2.a) as it is very close to the catenary one.

The equation of a parabola passing from the points (0,0), (L,0) and having $w_o'' = -\frac{\overline{m}g}{H_o}$, is given by the following formula:

$$w_o(x) = \frac{\overline{m}g}{2H_o} x(L-x) \qquad (4)$$

5 CABLES WITH FIXED ENDS

In order to determine the eigenfrequencies and shape functions, we have to find the tensile T_d.

The following relations are valid:

$$\left. \begin{aligned} ds^2 &= dx^2 + dw_o^2 \\ (ds+\Delta ds)^2 &= (dx+\Delta dx)^2 + (\Delta d\upsilon)^2 + (dw_o + \Delta dw_o)^2 \end{aligned} \right\} \qquad (5.1)$$

From this last, neglecting the higher order terms we get:

$\Delta dx = \Delta ds \frac{ds}{dx} - \Delta w_o \frac{dw_o}{dx}$, and remembering that $\Delta dw_o = dw_d$, we obtain:

$$\Delta dx = \frac{\Delta ds}{ds} \cdot \frac{ds}{dx} \cdot \frac{ds}{dx} dx - \frac{dw_d}{dx} \cdot \frac{dw_o}{dx} dx \tag{5.2}$$

On the other hand we have: $\sigma = \varepsilon \cdot E$ or

$$\varepsilon = \frac{\sigma}{E} = \frac{T_d}{EA} \tag{5.3a}$$

Because of (5.3), equation (5.2) becomes:

$\Delta dx = \frac{T_d}{EA} \cdot \left(\frac{ds}{dx}\right)^2 \cdot dx - \frac{dw_d}{dx} \cdot \frac{dw_o}{dx} \cdot dx$ and having the condition $\int_0^L \Delta dx = 0$,

the above gives:

$$\frac{T_d}{EA} \int_0^L \frac{dx}{\cos^3 \rho_z} - \int_0^L \frac{dw_d}{dx} \cdot \frac{dw_o}{dx} \cdot dx = 0 \tag{5.3b}$$

From eq. (5.3b) and after integration by parts with boundary conditions $w_d(0) = w_d(L) = 0$ we obtain:

$$\left.\begin{aligned} T_d &= -\frac{w''_o}{L_o} \int_0^L w_d dx \\ \text{and: } L_o &= \int_0^L \frac{dx}{EA \cos^3 \rho_z} \end{aligned}\right\} \tag{5.4a,b}$$

Therefore the equations of free motion can be written as follows:

$$\left.\begin{aligned} & T_o w''_d + T_d w''_o = c\dot{w}_d + m\ddot{w}_d \\ & T_o \upsilon''_d = c\dot{\upsilon}_d + m\ddot{\upsilon}_d \\ & T_d = -\frac{w''_o}{L_o} \int_0^L w_d dx \\ & \text{where: } L_o = \int_0^L \frac{dx}{EA \cos^3 \rho_z} \end{aligned}\right\} \tag{5.5a,b,c,d}$$

5.1 The vertical motion

Equation (5.5a) because of (5.5c) is written:

$$T_o w''_d - c\dot{w}_d - m\ddot{w}_d - \frac{w''^2_o}{L_o}\int_0^L w_d dx = 0 \tag{5.6}$$

We are searching for a solution of separate variables under the form:

$$w_d(x,t) = W(x)\cdot\Phi(t) \tag{5.7}$$

Therefore, equation (5.6) becomes:

$$\frac{T_o W'' - \frac{w''_o}{L_o}\int_0^L W dx}{mW} = \frac{\ddot{\Phi} + \frac{c}{m}\dot{\Phi}}{\Phi} = -\omega_w^2 \tag{5.8}$$

From equation (5.8) we get the following uncoupled equations:

$$\left.\begin{aligned} & W'' + \frac{m\,\omega_w^2}{T_o} W = \frac{w''^2_o}{T_o L_o}\int_0^L W dx \\ & \ddot{\Phi} + \frac{c}{m}\dot{\Phi} + \omega_w^2 \Phi = 0 \end{aligned}\right\} \tag{5.9a,b}$$

Equation (5.9a) has the solution:

$$\left.\begin{aligned} & W(x) = c_1 \sin\lambda_w x + c_2 \cos\lambda_w x + \frac{w''^2_o}{T_o L_o}\int_0^L W dx \\ & \text{where:}\quad \lambda_w^2 = \frac{m\omega_w^2}{T_o} \end{aligned}\right\} \tag{5.10a,b}$$

Equation (5.10a) is an integral-differential equation with degenerate kernel, i.e. the classic equation of Hammerstein. Integrating it, we finally obtain:

$$\int_0^L W dx = \frac{\lambda_w T_o L_o}{\lambda_w^2 T_o L_o - w''^2_o \cdot L}\left[c_1(1 - \cos\lambda_w L) + c_2 \sin\lambda_w L\right]$$

And equation (5.10a) becomes:

$$\left.\begin{aligned} & W(x) = c_1\left[\sin\lambda_w x + G(1 - \cos\lambda_w L)\right] + c_2\left[\cos\lambda_w x + G\sin\lambda_w L\right] \\ & \text{where:}\quad G = \frac{w''^2_o}{\lambda_w(\lambda_w^2 T_o L_o - w''^2_o \cdot L)} \end{aligned}\right\} \tag{5.11a,b}$$

The boundary conditions are:

$$W(0) = W(L) = 0 \tag{5.12a,b}$$

Introducing equation (5.11a) into the above equations we get:

$$\left.\begin{array}{l} G(1-\cos\lambda_w L)\cdot c_1 + (1+G\sin\lambda_w L)\cdot c_2 = 0 \\ \left[\sin\lambda_w L + G(1-\cos\lambda_w L)\right]c_1 + \left[\cos\lambda_w L + G\sin\lambda_w L\right]c_2 = 0 \end{array}\right\} \quad (5.13a,b)$$

In order for the above system to have a non-trivial solution, the determinant of the coefficients of the unknowns must be zero. This condition concludes to the following eigenfrequencies equation:

$$2G\cos\lambda_w L - \sin\lambda_w L - 2G = 0 \quad (5.14)$$

With λ_w from equ. (5.10b).
Finally, from equs. (5.13), (5.14) and (5.111au), one can determine the following form of the shape functions:

$$W_n(x) = c_1\cdot\left[\sin\lambda_{wn}x - \frac{G_n(1-\cos\lambda_{wn}L)}{1+G_n\sin\lambda_{wn}L}\cdot\cos\lambda_{wn}x + \frac{G_n(1-\cos\lambda_{wn}L)}{1+G_n\sin\lambda_{wn}L}\right] \quad (5.16)$$

Easily one can prove that the following orthogonality condition is valid:

$$\int_0^L W_n W_k\,dx = \begin{cases} 0 & \text{for } n\neq k \\ \Gamma_n & \text{for } n=k \end{cases} \quad (5.17)$$

5.2 The lateral motion

Equation (5.5b) is written:

$$T_o\upsilon''_d - c\dot{\upsilon}_d - m\ddot{\upsilon}_d = 0 \quad (5.18)$$

Following the previous procedure and searching for a solution of the form $\upsilon_d(x,t) = V(x)R(t)$, we conclude to the following equations:

$$\left.\begin{array}{l} T_o V'' + m\omega_\upsilon^2 V = 0 \\ \ddot{R} + \dfrac{c}{m}\dot{R} + \omega_\upsilon^2 R = 0 \end{array}\right\} \quad (5.19a,b)$$

Equation (5.19a) has the solution:

$$\left.\begin{array}{l} V(x) = d_1\sin\lambda_\upsilon x + d_2\cos\lambda_\upsilon x \\ \text{where:}\quad \lambda_\upsilon^2 = \dfrac{m\,\omega_\upsilon^2}{T_o} \end{array}\right\} \quad (5.20a,b)$$

With boundary conditions $V(0) = V(L) = 0$ we conclude to the following expressions:

$$\left.\begin{aligned} &V_n(x) = d_1 \sin\frac{n\pi x}{L} \\ &\text{and}: \quad \omega_{\upsilon n}^2 = \frac{n^2\pi^2 T_o}{m L^2} \end{aligned}\right\} \qquad (5.21a,b)$$

6 CABLES WITH MOVABLE END

In order to determine the tensile T_d we are starting from equation (5.3b):

$$\frac{T_d}{EA}\int_0^L \frac{dx}{\cos^3\rho_z} - \int_0^L \frac{dw_d}{dx}\cdot\frac{dw_o}{dx}\cdot dx = 0 \qquad (6.1)$$

From this last, after integration by members and with boundary condition $w_d(0) = 0$ we obtain:

$$\left.\begin{aligned} &T_d = \frac{1}{L_o}\left(w_o'(L) w_d(L) - w_o'' \int_0^L w_d dx \right) \\ &\text{with}: \quad L_o = \int_0^L \frac{dx}{EA\cos^3\rho_z} \end{aligned}\right\} \qquad (6.2a,b)$$

Therefore the equations of free motion can be written as follows:

$$\left.\begin{aligned} &T_o w_d'' + T_d w_o'' = c\dot{w}_d + m\ddot{w}_d \\ &T_o \upsilon_d'' = c\dot{\upsilon}_d + m\ddot{\upsilon}_d \\ &T_d = \frac{1}{L_o}\left(w_o'(L) w_d(L) - w_o'' \int_0^L w_d dx \right) \\ &\text{where}: \quad L_o = \int_0^L \frac{dx}{EA\cos^3\rho_z} \end{aligned}\right\} \qquad (6.3a,b,c,d)$$

6.1 The vertical motion

Equation (6.3a), because of (6.3c) gives:

$$T_o w_d'' - c\dot{w}_d - m\ddot{w}_d = -\frac{w_o''\cdot w_o'(L)}{L_o}\cdot w_d(L) + \frac{w_o''^2}{L_o}\int_0^L w_d dx = 0 \qquad (6.4a)$$

Taking into account equation (4), the above becomes:

$$T_o w_d'' - c\dot{w}_d - m\ddot{w}_d = \frac{\overline{m} g L w_o''}{2H_o L_o}\cdot w_d(L) + \frac{w_o''^2}{L_o}\int_0^L w_d dx = 0 \qquad (6.4b)$$

We are searching for a solution of the form:

$$w_d(x,t) = W(x)\Phi(t) \tag{6.5}$$

Introducing (6.5) into (6.4b) we get:

$$\frac{T_o W'' - \dfrac{\overline{m}\, g L w_o''}{2H_o L_o} W(L) - \dfrac{w_o''}{L_o}\int_0^L W dx}{mW} = \frac{\ddot{\Phi} + \dfrac{c}{m}\dot{\Phi}}{\Phi} = -\omega_w^2$$

, which concludes to the following uncoupled equations:

$$\left.\begin{aligned} & W'' + \frac{m\omega_w^2}{T_o} W = \frac{\overline{m}\, g L w_o''}{2H_o L_o T_o} W(L) + \frac{w_o''^2}{T_o L_o}\int_0^L W dx \\ & \ddot{\Phi} + \frac{c}{m}\dot{\Phi} + \omega_w^2 \Phi = 0 \end{aligned}\right\} \tag{6.6a,b}$$

The solution of equation (6.6a) is:

$$W(x) = c_1 \sin\lambda_w x + c_2 \cos\lambda_w x + \frac{\overline{m}\, g L w_o''}{2H_o L_o T_o \lambda_w^2} W(L) + \frac{w_o''^2}{T_o L_o \lambda_w^2}\int_0^L W dx \tag{6.7}$$

For x=L, the above (6.7), brcomes:

$$W(L) = c_1 \sin\lambda_w L + c_2 \cos\lambda_w L + \frac{\overline{m}\, g L w_o''}{2H_o L_o T_o \lambda_w^2} W(L) + \frac{w_o''^2}{T_o L_o \lambda_w^2}\int_0^L W dx ,$$

which finally gives:

$$W(L) = \frac{2H_o L_o T_o \lambda_w^2}{2H_o L_o T_o \lambda_w^2 - \overline{m} g L w_o''} \cdot \left(c_1 \sin\lambda_w L + c_2 \cos\lambda_w L + \frac{w_o''^2}{T_o L_o \lambda_w^2}\int_0^L W dx \right) \tag{6.8}$$

Because of (6.8), equation (6.7) becomes:

$$\left.\begin{aligned} & W(x) = c_1(\sin\lambda_w x + G_1 \sin\lambda_w L) + c_2(\cos\lambda_w x + G_1 \cos\lambda_w L) + (G_2 + G_3) + \int_0^L W dx \\ & \text{where:}\quad G_1 = \frac{\overline{m} g L w_o''}{2H_o L_o T_o \lambda_w^2 - \overline{m} g L w_o''}, \quad G_2 = \frac{w_o''^2}{L_o T_o \lambda_w^2}, \quad G_3 = G_1 \cdot G_2 \end{aligned}\right\}$$

(6.9a,b,c,d)

The above eq. (6.9a), of Hammerstein type, following the procedure of §5.1, finally gives:

$$\int_0^L W dx = \frac{1}{1-(G_2+G_3)L} \cdot \left\{ c_1\left(\frac{1-\cos\lambda_w L}{\lambda_w} + G_1 L \sin\lambda_w L \right) + c_2\left(\frac{\sin\lambda_w L}{\lambda_w} + G_1 L \sin\lambda_w L \right) \right\}$$

Therefore, equation (6.9a) becomes:

$$\left.\begin{aligned} &W(x) = c_1(\sin\lambda_w x + D_1) + c_2(\cos\lambda_w x + D_2) \\ &\text{where:}\quad D_1 = G_1 \sin\lambda_w L + \frac{G_2 + G_3}{1-(G_2+G_3)}\left(\frac{1-\cos\lambda_w L}{\lambda_w} + G_1 L \sin\lambda_w L\right) \\ &\qquad\qquad D_2 = G_1 \cos\lambda_w L + \frac{G_2 + G_3}{1-(G_2+G_3)}\left(\frac{\sin\lambda_w L}{\lambda_w} + G_1 L \cos\lambda_w L\right) \end{aligned}\right\} \tag{6.10a,b,c}$$

The boundary conditions are:

$$\left.\begin{aligned} &W(0) = 0 \\ &T_o W'(L) + k_s W(L) = 0 \end{aligned}\right\} \tag{6.11a,b}$$

Introducing equation (6.10a) into the above conditions we obtain:

$$\left.\begin{aligned} &c_1 D_1 + c_2(1 + D_2) = 0 \\ &c_1\left[\lambda_w T_o \cos\lambda_w L + k_s(\sin\lambda_w L + D_1)\right] + c_2\left[-\lambda_w T_o \sin\lambda_w L + k_s(\cos\lambda_w L + D_2)\right] = 0 \end{aligned}\right\} \tag{6.12a,b}$$

In order for the above system to have a non-trivial solution, the determinant of the coefficients of the unknowns must be equal to zero. This condition concludes to the following eigenfrequencies equation:

$$D_2\left[D_1 - (1+D_2)k_s\right] + \left[D_1 k_s - (1+D_2)T_o\lambda_w\right]\cos\lambda_w L - \left[(1+D_2)k_s + D_1 T_o \lambda_w\right]\sin\lambda_w L = 0 \tag{6.13}$$

Finally from equations (6.12), (6.13) and (6.10a), one can determine the following form of the shape functions:

$$W_n(x) = c_1\left[(\sin\lambda_w x + D_1) + \frac{\lambda_w T_o \cos\lambda_w L + k_s(\sin\lambda_w L + D_2)}{\lambda_w T_o \sin\lambda_w L - k_s(\cos\lambda_w L + D_2)}\cdot(\cos\lambda_w x + D_2)\right] \tag{6.14}$$

6.2 The lateral motion

Equation (5.5b) is written:

$$T_o \upsilon''_d - c\dot{\upsilon}_d - m\ddot{\upsilon}_d = 0 \tag{6.15}$$

Following the previous procedure and searching for a solution of the form $\upsilon_d(x,t) = V(x)R(t)$, we conclude to the following equations:

$$\left.\begin{aligned} & T_o V'' + m\omega_\upsilon^2 V = 0 \\ & \ddot{R} + \frac{c}{m}\dot{R} + \omega_\upsilon^2 R = 0 \end{aligned}\right\} \qquad (6.16a,b)$$

Equation (6.16a) has the solution:

$$\left.\begin{aligned} & V(x) = d_1 \sin\lambda_\upsilon x + d_2 \cos\lambda_\upsilon x \\ & \text{where:} \quad \lambda_\upsilon^2 = \frac{m\,\omega_\upsilon^2}{T_o} \end{aligned}\right\} \qquad (6.17a,b)$$

The boundary conditions are:

$$\left.\begin{aligned} & V(0) = 0 \\ & T_o V'(L) + k_s V(L) = 0 \end{aligned}\right\} \qquad (6.18a,b)$$

Introducing (6.17a) into the above we get d_2=0 and thus the equation of the eigenfrequencies is:

$$T_o\lambda_\upsilon \cos\lambda_\upsilon L + k_s \sin\lambda_\upsilon L = 0 \qquad (6.19)$$

Finally the shape functions are:

$$V_n(x) = d_1 \sin\lambda_\upsilon x \qquad (6.20)$$

with λ_υ from equation (6.17b).

7 NUMERICAL RESULTS AND DISCUSSION

Let us consider now a cable with the following characteristics:
L=250 m, m=7 kg/m, T_o=300000dN, E_c=2.08*10^{10} dN/m^2.
We study the following cases:

7.1 Cables with fixed ends

According to the formula (5.14) of §5.1, we determine the following eigenfrequencies:

$\omega_1 = 2.8186,\ \omega_2 = 5.2030,\ \omega_3 = 7.8130,\ \omega_4 = 10.4059,$

$\omega_5 = 13.0093,\ \omega_6 = 15.6089\ \sec^{-1}$

while using equation (5.16), we obtain the shape functions of figure 6.

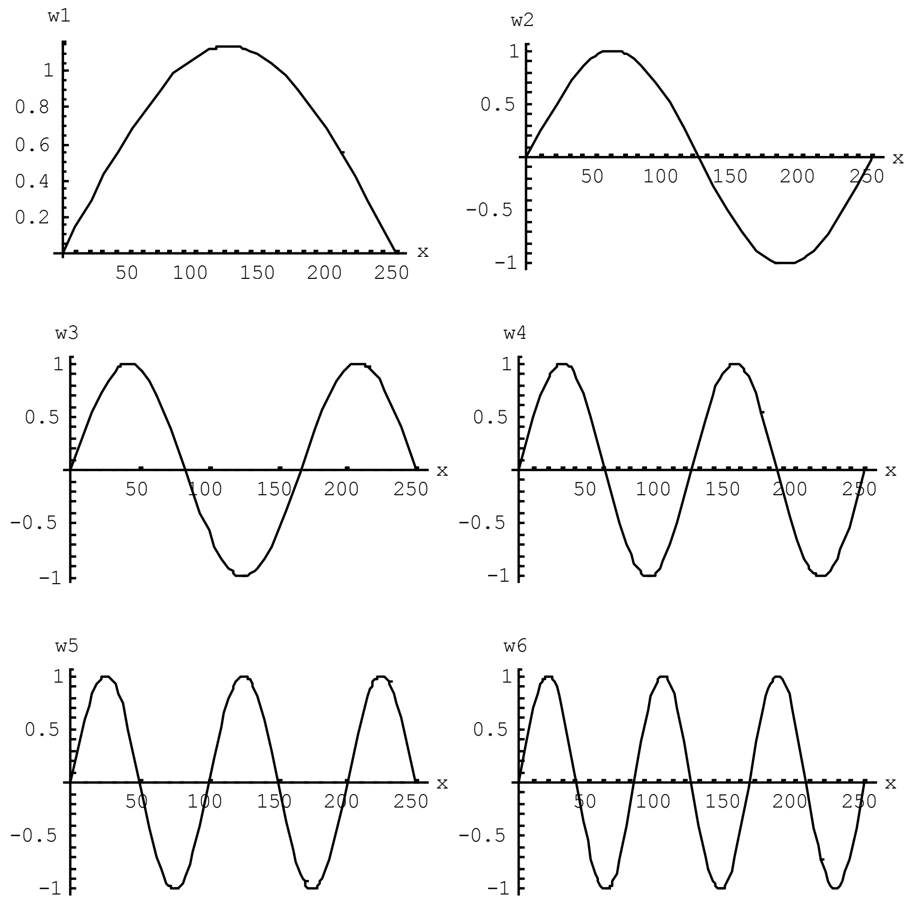

Figure 6. The first six shape functions

7.2 Cables with movable end

7.2.1 The vertical motion

According to the formula (6.13) of §6.1, and for different values of k_s, we determine the following Table 1 showing the first six eigenfrequencies.

Table 1

k_s	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
5000	2.3108	4.4900	6.8850	9.4048	11.9368	14.5048
10000	2.5052	4.7439	7.1430	9.6287	12.1334	14.6755
50000	2.7104	5.1082	7.6368	10.1758	12.7181	15.2673
100000	2.7390	5.1658	7.7257	10.2899	12.8588	15.4298
150000	2.7487	5.1851	7.7563	10.3298	12.9088	15.4888
200000	2.7536	5.1950	7.7718	10.3500	12.9343	15.5190
∞	2.8186	5.2030	7.8130	10.4059	13.0093	15.6089

Using equation (6.14) of §6.1, and for different values of k_s, we determine the following plots of figures 7 to 9, showing the first three eigenshapes in relation to k_s.

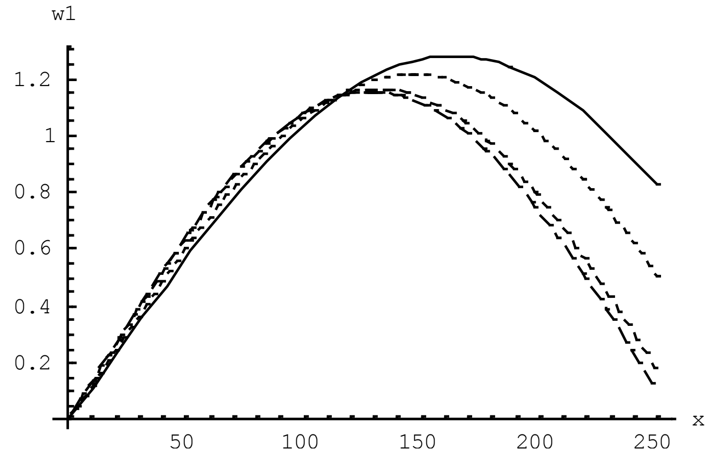

Figure 7. The first eigenshape for k_s=5000 ____ 10000 …… 50000 - - - 200000 _ _ _

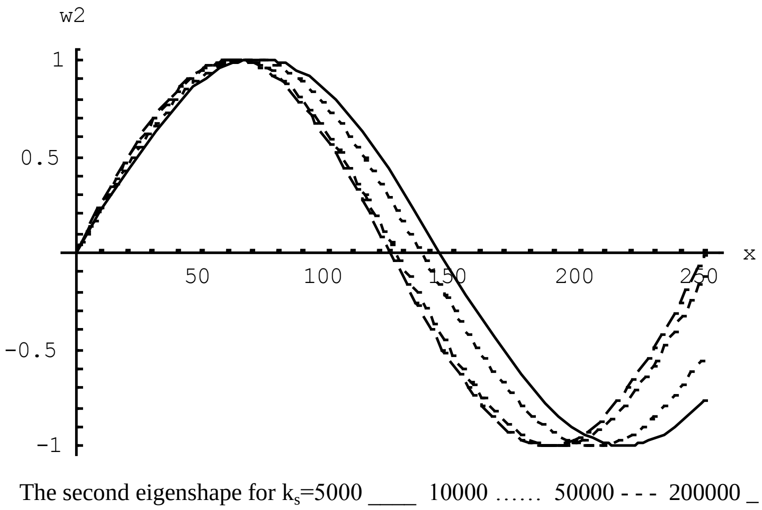

Figure 8. The second eigenshape for k_s=5000 ____ 10000 …… 50000 - - - 200000 _ _ _

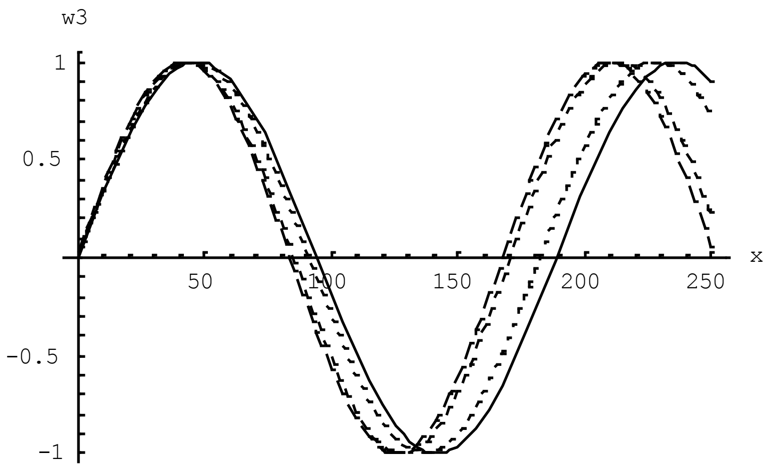

Figure 9. The third eigenshape for k_s=5000 ____ 10000 …… 50000 - - - 200000 _ _ _

7.2.2 The lateral motion

According to the formula (6.19) of §6.2, and for different values of k_s, we determine the following Table 2 with the first six eigenfrequencies.

Table 2

k_s	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
5000	2.1415	4.4485	6.8882	9.3966	11.9396	14.5016
10000	2.3317	4.7072	7.1400	9.6205	12.1349	14.6722
50000	2.5406	5.0819	7.6244	10.1686	12.7151	15.2640
100000	2.5706	5.1414	7.7130	10.2834	12.8549	15.4268
150000	2.5808	5.1617	7.7426	10.3236	12.9047	15.4859
200000	2.5859	5.1719	7.7580	10.3440	12.9301	15.5162
∞	2.6015	5.2030	7.8045	10.4059	13.0074	15.6089

Finally using equation (6.20) of §6.2, and for different values of k_s, we determine the following plots of figures 10 to 12, showing the first three eigenshapes in relation to k_s.

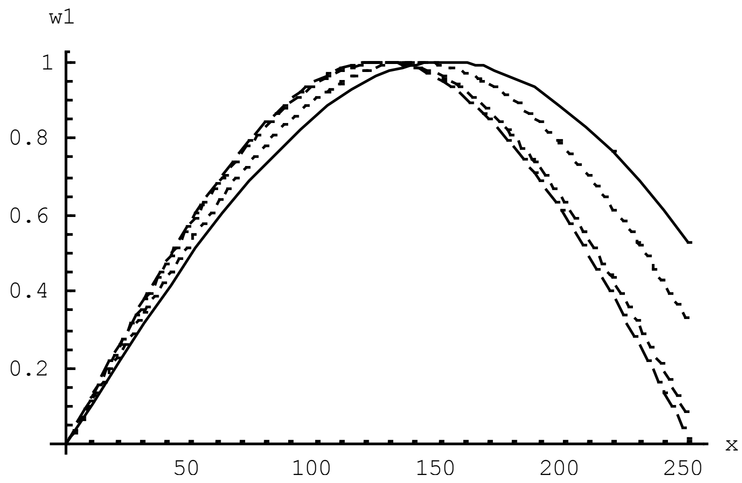

Figure 10. The first eigenshape for k_s=5000 ____ 10000 …… 50000 - - - 200000 _ _ _

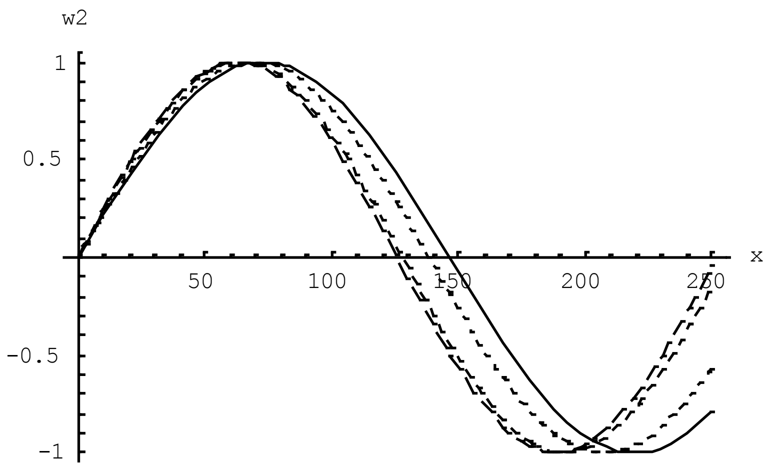

Figure 11. The second eigenshape for k_s=5000 ____ 10000 …… 50000 - - - 200000 _ _ _

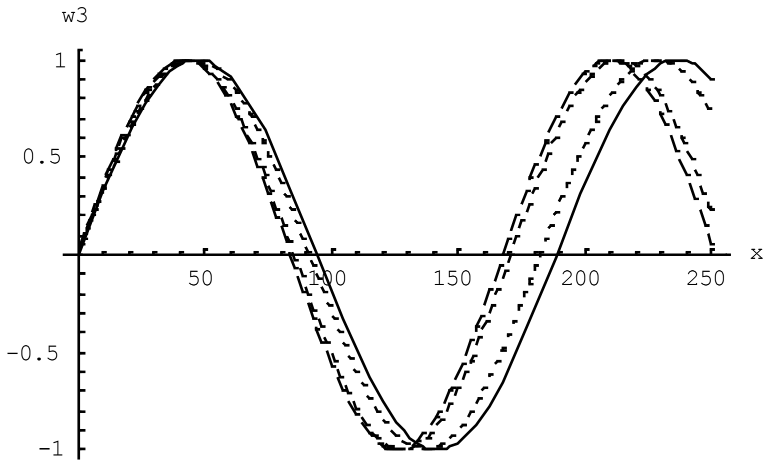

Figure 12. The third eigenshape for k_s=5000 ____ 10000 …… 50000 - - - 200000 _ _ _

8 CONCLUSIONS

On the basis of the chosen cable models, we may draw the following conclusions:

1. A mathematical model for determining and studying the eigenfrequencies and eigenshapes of a cable fixed at both ends or fixed at the one and elastically joined at the other end has been presented.
2. Using the classic equations of a cable and determining the related equations for a cable with moving end we conclude to an integral-differential equation which is solved.
3. The case of a cable with movable end is more complicated than the fixed.
4. For the vertical motion, we see that the eigenfrequencies depend on the value of k_s, and compared to the eigenfrequencies of the corresponding cable with fixed ends, we establish that the differences amount from 18% for soft springs to 2.3% for hard ones.
5. For $k_s > 100000\,\mathrm{dN/m}$, we observe that the cable behaves rather as a cable with fixed ends.
6. The eigenshapes are strongly affected for soft springs and slightly for springs with $k_s > 100000\,\mathrm{dN/m}$.
7. Similar conclusions are drawn for the lateral motion.

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