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BRIDGE WITH UN-BONDED POST-TENSIONED PIERS – DYNAMICS SIMULATION UNDER BASE EXCITATION

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ABSTRACT: The dynamics of bridges with un-bonded post-tensioned piers under ground motion is considered. The actual bridge structure is modeled as a 3-DOF with 2-masses excited at the base by a simulated seismic motion. The effect of the parameters controlling the system dynamics is captured and important conclusions for structural design purposes are drawn.

KEY WORDS: Bridges, un-bonded post-tensioned piers, base excitation.

1 INTRODUCTION

Following the large number of strong seismic events striking large urban centers around the world the last 30 years, modern societies have been pushing towards the establishment of modern seismic design approaches which favor/require construction techniques (e.g. pre-casting), and innovative technologies (e.g. seismic isolation) aiming to limit damage, and limit cost of repairing and downtime after a seismic event. Traditional design approaches based on the development of ductile mechanisms within the structure cannot adequately satisfy those requirements. For infrastructure systems and especially for bridges the interruption of functionality as well as the repair costs resulting from earthquake damages are of great concern, and have lead engineers to propose modern design approaches for these structures. One such modern design approach is the utilization of un-bonded post–tensioned piers to support the deck [1-7]. This approach limits the strength of the structure in an attempt to limit the accelerations in the deck and accordingly minimize the base or pier shear forces. Although such reduction of acceleration response is beneficial, it comes with the increase of deformations which could become unacceptable causing structural instability.

To the authors' knowledge, this quite novel and intuitively correct approach lacks theoretical validation, as far as its dynamic instability and bifurcations are concerned, in the context of a strictly non-autonomous vector-field formulation, dictated by the theory of dynamical systems. The main control parameters of the aforementioned problem, the influence of the variation of which will be studied, are the masses and the characteristics of the springs. The strongly coupled dynamic equations of motion are tackled within the framework of non-autonomous vector fields.

2 PROPOSED MODEL: FEATURES - MOTION EQUATIONS

2.1 Model description and properties

The considered 3-degrees-of-freedom, two-mass system is depicted in Figure 1, and proposed herein for simulating the dynamics of bridges with un-bonded post-tensioned piers.

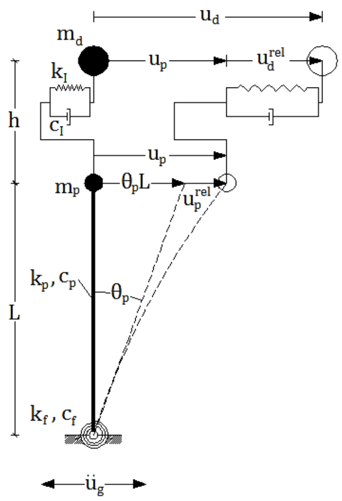

Figure 1. Proposed 3-DOF model

This simplified (stick) model consists of a concentrated mass representing the deck, supported by a linear viscoelastic spring representing the bridge bearings (expansion or seismic isolation with mild nonlinearities). This pier mass is connected to the foundation, via a massless flexible column (modeling the pier), which is pin connected to the ground with an additional linear rotational spring, simulating the connection of the pier to the foundation and characterized by a mild energy dissipation, coming from a linear viscous rotational dashpot. In the aforementioned illustration the subscripts “d”, “p”, “I” and “f” refer to the deck, the pier, the isolation system and the foundation respectively, while “m”, “k” and “c” represent the related masses, spring stiffnesses and damping coefficients of each of the above components, where applicable.

2.2 Equations of motion and local trivial instability

Two translational and one rotational generalized coordinates describe the dynamic response of the above model, which under simulated earthquake base excitation is governed by the system of strongly coupled linear non-autonomous equations of motion that follow ($\dot{} = d/dt$)

$$m_d\left(\ddot{\theta}_P L + \ddot{u}_P^{rel} + \ddot{u}_d^{rel}\right) + k_I u_d^{rel} + c_I \dot{u}_d^{rel} = m_d \ddot{u}_g(t) = F_1(t) \tag{1a}$$

$$m_P\left(\ddot{\theta}_P L+\ddot{u}_P^{rel}\right)-k_I u_d^{rel}-c_I\dot{u}_d^{rel}+k_p u_P^{rel}+c_P\dot{u}_P^{rel} = m_P\ddot{u}_g(t)=F_2(t) \tag{1b}$$

$$m_d\left(\ddot{\theta}_P L+\ddot{u}_P^{rel}+\ddot{u}_d^{rel}\right)(L+h)+m_P\left(\ddot{\theta}_P L+\ddot{u}_P^{rel}\right)L+k_f\theta_P+c_f\dot{\theta}_P =(L+h)F_1(t)+LF_2(t) \tag{1c}$$

In the right hand sides of these equations the ground motion is simulated, for the cases of remote and near earthquakes, via an acceleration function envelope $\ddot{f}(t)$ with its characteristics shown in Figure 2.

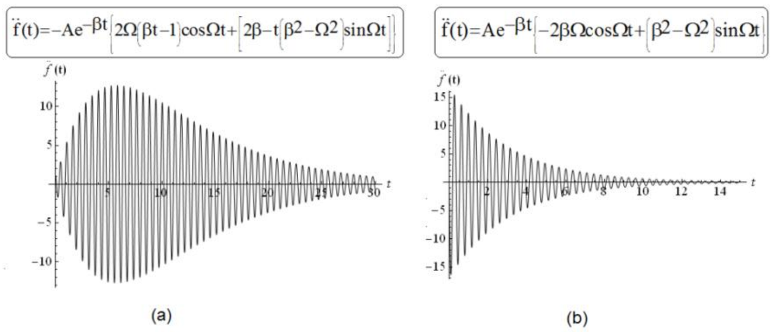

Figure 2. Typical plots of simulated ground motion for (a) remote and (b) near earthquakes

Evidently, for these simulations, the range of the parameters involved is:

$$\begin{aligned} &\text{Case (a)}: 0.01 \le A \le 0.10 \;\;,\;\; 0.05 \le \beta \le 0.30 \;\;,\;\; 5 \le \Omega \le 20 \\ &\text{Case (b)}: 0.01 \le A \le 0.10 \;\;,\;\; 0.15 \le \beta \le 0.50 \;\;,\;\; 1 \le \Omega \le 12 \end{aligned} \tag{2}$$

Introducing the following dimensionless parameters

$$\left.\begin{aligned} &\tau = t\sqrt{\frac{k_p}{m_p}} \;\;,\;\; u_1 = \frac{u_p^{rel}}{L} \;\;,\;\; u_2 = \frac{u_d^{rel}}{L} \;\;,\;\; \sigma = \frac{h}{L} \;\;,\;\; m = \frac{m_d}{m_p} \;\;,\;\; \theta_p(t)=\phi(\tau) \\ &\alpha = m(1+\sigma)+1 \\ &k_1 = \frac{k_I}{k_p} \;\;,\;\; k_2 = \frac{k_f}{k_p L^2} \;\;,\;\; c_1 = \frac{c_I}{\sqrt{k_p m_p}} \;\;,\;\; c_2 = \frac{c_p}{\sqrt{k_p m_p}} \;\;,\;\; c_3 = \frac{c_f}{L^2\sqrt{k_p m_p}} \end{aligned}\right\} \tag{3}$$

the system of motion equations 1(a-c) is written in dimensionless form as:

$$m\varphi''(\tau) + u_1''(\tau) + u_2''(\tau) + k_1 u_2(\tau) + c_1 u_2'(\tau) = m\gamma(\tau) \tag{4a}$$

$$\varphi''(\tau)+u_1''(\tau)+c_2u_1'(\tau)-c_1u_2'(\tau)+u_1(\tau)-k_1u_2(\tau)=\gamma(\tau) \tag{4b}$$

$$\varphi''(\tau)+u_1''(\tau)+(\alpha-1)u_2''(\tau)+\frac{c_3}{\alpha}\varphi'(\tau)+\frac{k_2}{\alpha}\varphi(\tau)=\gamma(\tau) \tag{4c}$$

where prime indicates differentiation with respect to τ, while the excitation function $\gamma(\tau)$ is the dimensionless base acceleration according to Figure 2.

The system is considered perfect and at rest before the initiation of the ground motion. Since no external loading exists, the trivial state represents the only valid equilibrium configuration. It would thereafter be of major importance to seek out the local stability of this state, by exploring the nature of the roots of the corresponding 6^{th}-order characteristic polynomial (which in fact are the eigenvalues of the trivial equilibrium), via the possible violation of one or more of the Liénard-Chipart conditions [8]. Employing advanced symbolic manipulations in *Mathematica* [9], the above polynomial, equal to:

$$G(\rho)=\rho^6+\alpha_1\rho^5+\alpha_2\rho^4+\alpha_3\rho^3+\alpha_4\rho^2+\alpha_5\rho+\alpha_6 \tag{5}$$

possesses coefficients α_i (i=1,…,6) given by:

$$\alpha_6=\frac{k_1^2k_2}{(m-1)m(1+\sigma)(1+m+m\sigma)}\ ,\ \alpha_5=\frac{k_1\left(c_3k_1+(c_1-c_2)k_2\right)}{(m-1)m(1+\sigma)(1+m+m\sigma)} \tag{6a,b}$$

$$\alpha_4=\frac{-c_1c_2k_2+k_1^2(1+m+m\sigma)+k_1(1+c_1c_3-c_2c_3+m+m\sigma)}{(m-1)m(1+\sigma)(1+m+m\sigma)} \tag{6c}$$

$$\alpha_3=\frac{\begin{pmatrix} c_1\left(1-c_2c_3+k_1-2k_2+m+k_1m+(1+k_1)m\sigma\right) \\ -c_2\left(k_2+k_1(1+m+m\sigma)\right)\end{pmatrix}}{(m-1)m(1+\sigma)(1+m+m\sigma)} \tag{6d}$$

$$\alpha_2=\frac{-1+c_1c_2+m(1+\sigma)+k_1\left(m^2(1+\sigma)\right)+\dfrac{2c_1c_3+c_2c_3+k_2}{1+m(1+\sigma)}}{(m-1)m(1+\sigma)} \tag{6e}$$

$$\alpha_1=\frac{(m-1)(c_1+c_2+c_2m)+c_2m^2\sigma-\dfrac{c_3}{1+m(1+\sigma)}}{(m-1)m(1+\sigma)} \tag{6f}$$

Rational values of the involved parameters will be used in obtaining numerical results. For moderate dimensions of deck and piers and commonly used values of damping and stiffness, the following range shall be utilized, in seeking possible unstable trivial situations as well as forced dynamic responses:

$$0.1 \le k_1 \le 1 \ , \ 0.002 \le k_2 \le 0.005 \ , \ \sigma \le 0.10 \ , \ 2 \le m \le 10$$
$$0.02 \le c_1 \le 4 \ , \ 0.04 \le c_2 \le 0.1 \ , \ 0.005 \le c_3 \le 0.08 \qquad (7)$$

Utilizing the powerful *FindMinimum* and *Reduce* commands embedded in *Mathemetica*, it was found that only coefficient α_3 may be less or equal to zero within the range given in (7). This fact however leads to at least one eigenvalue of the trivial state with positive real part, implying local instability, i.e. the possibility, for an infinitesimal disturbance, of the system to exhibit at least a divergent motion, and, under ground motion, the occurrence of complicated dynamic phenomena. Two typical situations of such a possibility are given in the contents of Table 1.These two cases represent actual bridge structures; e.g. a typical 2-span seismic isolated highway overpass.

Table 1. Two characteristic cases of local trivial instability

Case No	m	σ	k_1	k_2	c_1	c_2	c_3	eigenvalue(s) with positive real part(s)
1	3	0.05	0.5	0.0035	0.05	0.05	0.04	0.004±0.37i
2	1.5	0.03	0.1	0.004	1	0.07	0.08	0.18±1.14i

Solving numerically the system of equations 4(a-c) for $\gamma(\tau)=0$ and $\varphi'(\tau)=0.0001$, for both cases shown in the above Table, an unbounded motion response was found, validating the unexpected theoretical prediction described earlier. This response is depicted in the phase-plain portraits $[u_1(\tau),\ u'_1(\tau)]$ of Figure 3.

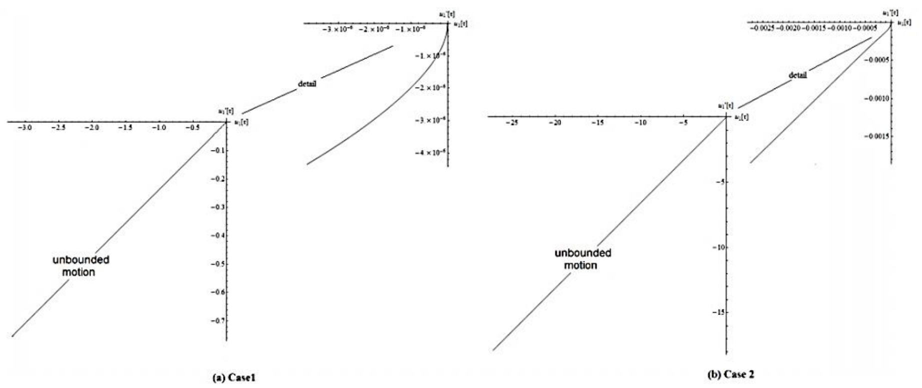

Figure 3. Unbounded motion exhibited by the system at its trivial state (for both Cases shown in Table 1), for an infinitesimal initial disturbance in the absence of base excitation

2.3 Some aspects of non-autonomous formulation

The forced oscillations of the model governed by eqs.(4) can also be treated as a linear non-autonomous 6^{th}-order vector field, within the context of the theory of dynamical systems [10]. For such a type of systems, a variety of responses have been reported, and as far dynamic stability is concerned, these may vary from unbounded motions and simple periodic orbit bifurcations to complicated resonance phenomena, without excluding strange or chaotic behaviour [11]. Evidently, to perform a rigorous non-autonomous formulation regarding the multi-parameter foregoing system would be very intriguing, but for reasons of space limitation such an approach is not included herein, but proposed for future research. Instead, a straightforward dynamic analysis will be hereafter adopted, which as it will be demonstrated may produce very important qualitative results.

3 NUMERICAL RESULTS AND DISCUSSION

For typical remote and near earthquake simulations, it would be of great interest to compare between the dynamic responses of the system corresponding to cases related to stable and unstable trivial configurations. In what follows, Cases 1 and 2, as in Table 1, will be considered for the unstable situation, while two more combinations of parameters (for which the trivial state is stable) will be used, namely Cases 3 and 4, with details given in Table 2.The parameters for these cases are also representative of actual bridge structures.

Table 2. Two characteristic cases of local trivial stability

Case No	m	σ	k_1	k_2	c_1	c_2	c_3
3	10	0.05	0.1	0.004	0.2	0.07	0.05
4	6	0.02	0.3	0.004	0.8	0.02	0.05

In obtaining numerical results, the following simulated ground motion parameters were used for assessing the excitation function $\gamma(\tau)$:

- Remote earthquake simulation : A=0.05, β=0.20, Ω=5
- Near earthquake simulation : A=0.05, β=0.15, Ω=2

The straightfarward dynamic analysis lead to two totally different responses, which are illustrated throughout Figures 3 and 4, corresponding to iniatially unstable and stable trivial states respectively, in terms of phase – plain plots $[u_1(\tau),\ u_1'(\tau)]$. More specifically, the system under both remote and near eartquake simulated excitation was found to exhibit small amplitude vibrations, which decay to zero after the end of the forcing function, i.e. to finally rest at its trivial stable equilibrium; the free vibration was not depicted in Figure 3 for clarity.

On the other hand, the dynamics of the intially unstable system configurations were related to an unbounded motion, leading to very large chatastrophic displacements. Same qualitative results were also obtained for other combinations of parameters regarding the ground motion (within the afore mentioned ranges) not shown herein for brevity. It is postulated that the above findings are directly dependent on the nature of the stability of the trivial state, a fact implying that during design this should be taken inti account, in order to avoid potential unfavorable and rather unexpected dynamic behavior.

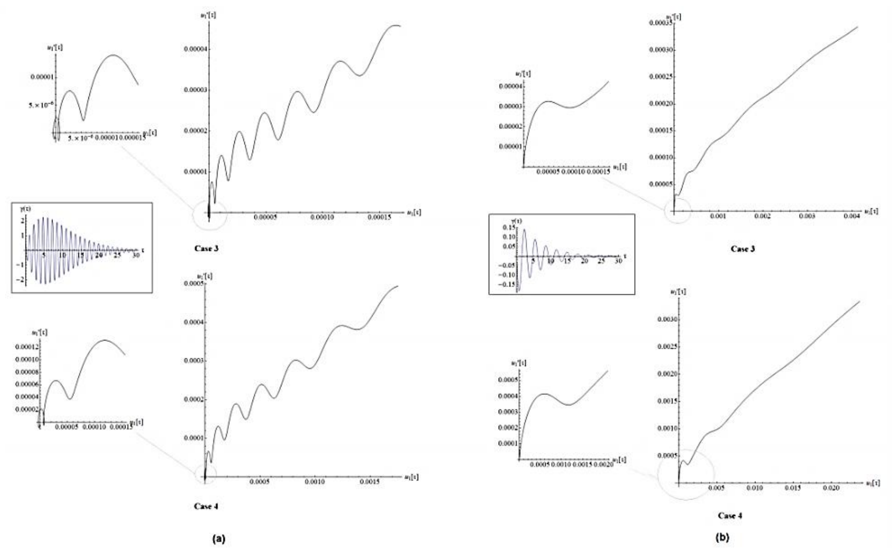

Figure 4. Dynamic response of the initially stable system under (a) remote and (b) earthquake simulations

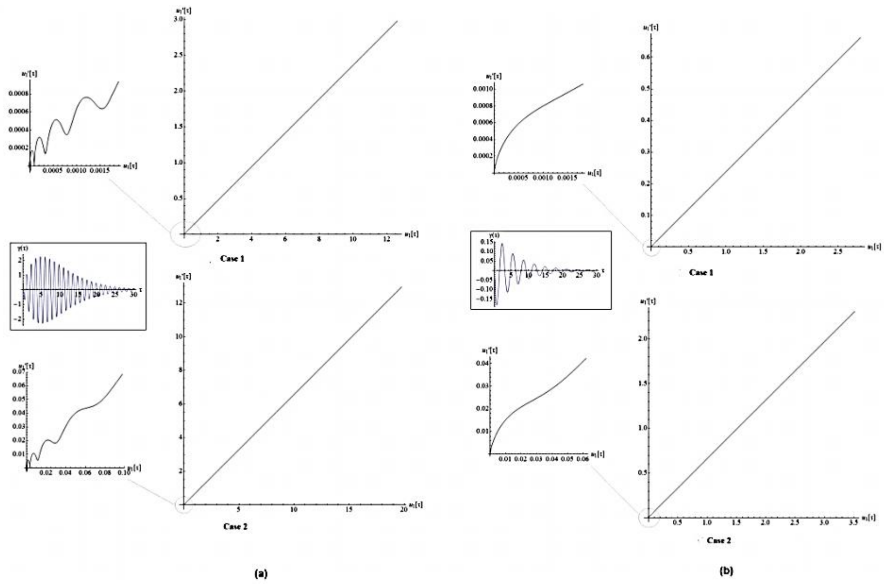

Figure 5. Dynamic response of the initially unstable system under (a) remote and (b) earthquake simulations

4 CONCLUSIONS

The non-dimensional formulation of the equations of motion of a bridge structure supported by un-bonded post-tensioned piers is presented. The behavior of the springs of the model is assumed linear. This assumption is considered valid as long as the potential nonlinearities of the springs representing bridge bearings and pier to foundation connection are small. It was shown that there are cases which could represent real bridge structures where the dynamic behavior can become unstable. These phenomena are not common but they might become catastrophic and as such the bridge designers should be aware and take appropriate actions even at the preliminary design phases. Considering the assumptions and limitations of this study, additional work is required, focusing on the following issues, in order to reach a much more integrated prediction of the real structural response:(a) Adopt a more detailed non-linear model to capture bearing and pier-foundation springs behavior, (b) Seek the dynamics of thebridge model under real (recorded) seismic excitations, and (c) Employ perturbation techniques and capture possible resonance phenomena associated with both the linear model utilized in this study and the non-linear model for the future study.

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