International Journal of Bridge Engineering (IJBE), Vol. 13, No. 1, (2025), pp. 53-65

HARMONIC MEAN OPTIMIZATION APPROACH IN THE CRACK CONTROL OF A REINFORCED CONCRETE T-BEAM

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ABSTRACT: This study focuses on using an optimization technique to control the crack propagation in reinforced concrete T-beams by employing multi-objective optimization and numerical simulations to enhance the shear strength. Four key factors (concrete density, Young's modulus of elasticity of concrete, steel density, and Young's modulus of elasticity of steel) were considered within a predefined range to predict the ductility and the elastic shear strain for the structural member. Box-Behnken design sampling method was utilized to prepare 27 numerical models using the ABAQUS finite element program. The outputs of the elastic shear strain and the maximum principal stress were collected in conjunction with MATLAB codes to solve complex matrices. The regression coefficients were determined via the least squares method which are needed to build the regression equations for prediction. The harmonic mean method was applied as a multi-criteria optimization strategy to unify conflicting objectives into a single optimization problem. The reliability of the regression equations was validated by comparing them with the numerical simulation results which resulted in achieving 100% accuracy with R^2 values of 1 for both the maximum principal stress and the elastic shear strain. The results highlight that the shear strength capacity of the reinforced concrete T-beams can be effectively optimized, enabling precise design control.

KEYWORDS: Regression equation; Interaction term; Elastic shear strain; Box-Behnken design; Multi-criteria optimization.

1 INTRODUCTION

Reinforced concrete is one of the most widely used construction materials due to its ability to withstand both compressive and tensile forces when steel reinforcement is integrated. Among the various reinforced concrete elements, T-beams are particularly important in structural systems such as bridges and buildings. Their flange-and-web geometry offers improved flexural performance and efficient material usage. However, these structural members are also vulnerable to shear cracking, especially under high loading or poor design. Shear cracks often occur before flexural failure, making their control essential to ensuring long-term structural integrity and safety. Crack control in reinforced concrete is a critical area of study because cracks affect not just appearance but also stiffness, durability, and serviceability. Two important indicators in this context are elastic shear strain, which reflects how much the material deforms under shear in the elastic range, and stress, which measures internal forces per unit area. A thorough understanding of how stress and strain evolve within T-beams under different loading conditions is crucial to preventing structural damage or failure. In civil engineering, especially in structural analysis and design, optimization techniques are increasingly used to improve performance, minimize material usage, and reduce cost.

Multi-objective optimization is particularly useful when different performance goals such as reducing stress and strain simultaneously conflict with each other. Instead of favoring one goal over the other, multi-objective methods seek a balance between them. One powerful technique for multi-objective problems is the harmonic mean method. This approach transforms a set of conflicting objectives into a single optimization function by calculating the harmonic mean of the objectives. Unlike the arithmetic mean, the harmonic mean emphasizes lower values, which is desirable when minimizing stress or strain, since a high value in one objective severely affects the combined result. This technique enables a balanced solution that avoids extremes and ensures more reliable structural performance. The harmonic mean has been used in several engineering disciplines, and its application in structural optimization is gaining traction for its simplicity and effectiveness in balancing trade-offs [1,2,3]. To support optimization, numerical modeling and surrogate models are used to predict structural behavior efficiently.

Among various experimental designs, the Box-Behnken method is a preferred approach for generating high-order regression models with fewer experimental runs [4,5,6]. This makes it ideal for structural simulations where testing is time-consuming or expensive. When combined with finite element analysis (FEA) tools like ABAQUS, engineers can simulate how reinforced concrete T-beams respond under different loads, material combinations, and geometries. This modeling approach captures both stress distributions and strain evolution, providing a strong basis for optimization algorithms. By collecting simulation outputs such as maximum stress and shear strain, researchers can build regression models using software like MATLAB. These surrogate models are used in optimization frameworks like genetic algorithms (GA) [7] or optimization (PSO) [8] to find the best design variables (e.g., concrete density, steel modulus) that result in minimum stress and strain while satisfying all safety and material constraints. Numerous studies have been conducted to enhance the structural performance of RC T-beams.

Traditional reinforcement techniques like stirrups and longitudinal bars offer basic crack control but are limited in their long-term effectiveness [4, 9]. To overcome these challenges, researchers have applied FEM, GA, and PSO to optimize reinforcement layouts and material properties [7,10,1]. Surrogate models based on coupled FEA–stochastic methods have also proven effective in damage prediction under dynamic conditions such as explosions [10,8]. Geometry has a major in- fluence on shear performance. Studies show that flange width and thickness [11], as well as the shear span-to-depth ratio [12], significantly affect crack propagation.

Likewise, innovative reinforcement strategies like near-surface-mounted (NSM) techniques improve shear capacity and delay failure [13,14]. Research on lightweight concrete T-beams also highlights the influence of material type on shear performance [15]. Advanced modeling techniques such as artificial neural networks (ANN) and support vector machines (SVM) have emerged as powerful tools for predicting shear strength and crack propagation using large FEA datasets [16,17]. Hybrid strategies that combine simulated annealing (SA) or differential evolution (DE) with FEM have improved efficiency and accuracy in structural optimization [18]. In terms of reinforcement materials, studies have explored the performance of GFRP-reinforced T-beams under shear, showing notable improvements in crack control [19]. Meanwhile, FEA-based design assessments under static and dynamic shear loading continue to refine our understanding of crack behavior and strengthening techniques [20,21].

Despite these contributions, many current methods still lack practical and cost-effective optimization strategies tailored to real-world applications. This gap underscores the need for advanced yet computationally efficient solutions. In this study, we aim to integrate the harmonic mean optimization approach into the design of reinforced concrete T-beams to simultaneously optimize elastic shear strain and stress behavior. By employing a combination of finite element modeling, regression analysis, and numerical optimization, our objective is to enhance crack control and improve the structural response under loading, offering a robust and efficient method for engineering high-performance T-beams.

2 MULTI CRITERIA OPTIMIZATION METHOD

The multi-criteria optimization approach is employed to improve four distinct parameters, each constrained within specific range values as specified in Table 1. The regression equations, which possess established boundaries, are subjected to further analysis to as- certain the minimum and maximum values for both elastic shear strain and stress within the reinforced concrete T-beam [2]. The optimized values for each factor are rigorously verified to guarantee their compliance with the specified range limits, in conjunction with the stipulation that they do not surpass the maximum values delineated by the regression equations. This approach ensures the optimization process is effectively directed towards the design of the shear strength of the structural system. The endeavor to optimize multiple objectives, particularly when they exhibit conflict, presents significant challenges. The resolution necessitates the identification of a compromise that concurrently fulfills all objectives. A variety of methodologies have been proposed to tackle multi-objective optimization dilemmas by amalgamating the objectives into a singular entity while pre- serving the constraints. In this study, the harmonic mean method is introduced for this purpose (see Section 3.3).

2.1 Box-Behnken design method

The Box-Behnken designs are employed to produce higher-order response surfaces while utilizing a reduced number of experimental runs compared to conventional factorial methodologies [4]. This methodology, in conjunction with central composite approaches, efficiently reduces the number of experimental runs, thereby maintaining the fidelity of the characterization of higher-order surfaces. The Box-Behnken design integrates twelve mid-edge nodes in addition to three central nodes to sufficiently approximate a second-order polynomial equation. The amalgamation of central composite and Box-Behnken designs results in a comprehensive factorial design, augmented by three additional samples collected at the central point. Box-Behnken designs strategically position experimental points at the midpoints of the edges within the cubical design domain, in addition to placing points at the center. The Box-Behnken experimental designs facilitate the modeling of the response surface. These designs are not predicated on traditional full or fractional factorial frameworks. The designated design points are meticulously positioned at the centroids that correspond to the subregions located within the dimensional framework of k-1, thereby ensuring a precise calibration of the experimental layout. In a particular context that encompasses three distinct factors, for instance, these strategically arranged points are established at the midpoints along the edges that outline the experimental space, which is visually represented in Figure 1. This careful arrangement not only enhances the clarity of the experimental design but also facilitates a more re- fined understanding of the interactions among the variables under consideration, thereby contributing significantly to the overall robustness of the empirical investigation.

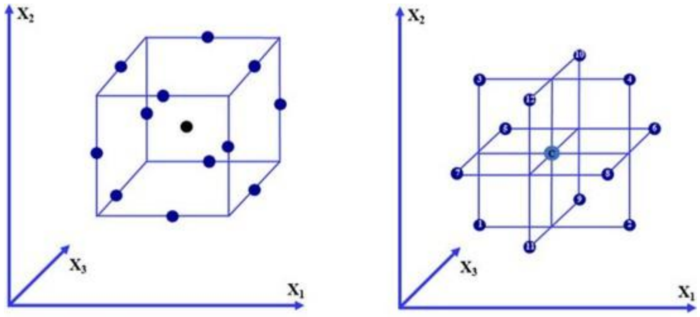

Figure 1. Box-Behnken sampling method (three factors) (Adapted from Ref. [4] with permission)

2.2 Response surface model

The regression model is constructed based on four independent variables: the density of concrete, the Young's modulus of elasticity for concrete, the density of steel, and the Young's modulus of elasticity for steel (refer to Table 1). The mathematical expressions used to estimate the elastic shear strain and the maximum principal stress in a reinforced concrete T-beam are provided in Equation 1 [7]:

$$y = f(x)\alpha + \epsilon \quad (1)$$

In this context:

- x denotes a vector comprising input variables xi, where i = 1, . . . , k, and f (x) is a function defined over k elements.
- α represents the vector of regression parameters.
- ϵ is the random error term assumed to have a mean of zero.

The estimation of the regression coefficients α is achieved using the least squares method, as shown in Equation 2:

$$\alpha = (X'X)^{-1}X' \quad (2)$$

Here, X' signifies the transpose of the design matrix X, and $(X'X)^{-1}$ denotes the inverse of the matrix product $X'X$ [7]. The function $f(x)$, which characterizes both the elastic shear strain and the maximum principal stress, incorporates linear, quadratic, and interaction terms derived from the aforementioned four influencing variables.

2.3 Materials and range data

The regression equations are developed based on four factors using the Box-Behnken design sampling method. A total of 27 models of reinforced concrete T-beams are created and analyzed using the ABAQUS finite element software. Numerical simulations are conducted to collect data on elastic shear strain and maximum principal stress after running the models. Table 1 presents the selected factors along with their respective range values.

Table 1. Factors and range data

Factor Symbol	Factor Description	Range Data
X_1	Concrete density (ρ_c) (kg/m^3)	2200–2600
X_2	Concrete Young's modulus (E_c) (GPa)	25–35
X_3	Steel density (ρ_s) (kg/m^3)	7800–8000
X_4	Steel Young's modulus (E_s) (GPa)	190–230

2.4 Finite element analysis

The finite element representation of the reinforced concrete T-beam spans a total length of 2.8 meters. The flange has a width of 0.3 meters and a thickness of 0.06 meters, whereas the web has dimensions of 0.08 meters in width and 0.22 meters in height. Reinforcement within the flange consists of four longitudinal steel bars, each with a diameter of 0.012 meters, accompanied by 19 evenly spaced transverse steel bars of the same diameter placed between adjacent longitudinal bars. Furthermore, the web is reinforced with two longitudinal steel bars, each measuring 0.012 meters in diameter, located at the bottom section. Stirrups are uniformly distributed throughout the beam, comprising 19 steel bars with a diameter of 0.012 meters.

The mechanical behavior of the steel is modeled using both elastic properties, namely Young's modulus and Poisson's ratio, and plastic characteristics, including yield stress and plastic strain. For the concrete, an elastic- plastic formulation is adopted: elastic properties such as Young's modulus and Poisson's ratio are defined, alongside a concrete damage plasticity model that captures plastic deformation, compressive response, and tensile cracking. A point load of 70,000 N is applied at a location 0.65 meters from the midpoint of the left support to evaluate the beam's shear performance (refer to Figure 2). Additionally, a uniformly applied load rate of 260 N/s over a duration of 100 seconds ensures the system remains within the elastic regime, facilitating comparative analysis across 27 simulation scenarios. Boundary conditions are defined such that translational movement in the longitudinal and transverse directions is permitted, while vertical displacements are restrained at both supports. The finite element mesh consists of 828 C3D8R hexahedral elements for the concrete domain and 1280 T3D2 linear elements to represent the reinforcement bars and stirrups.

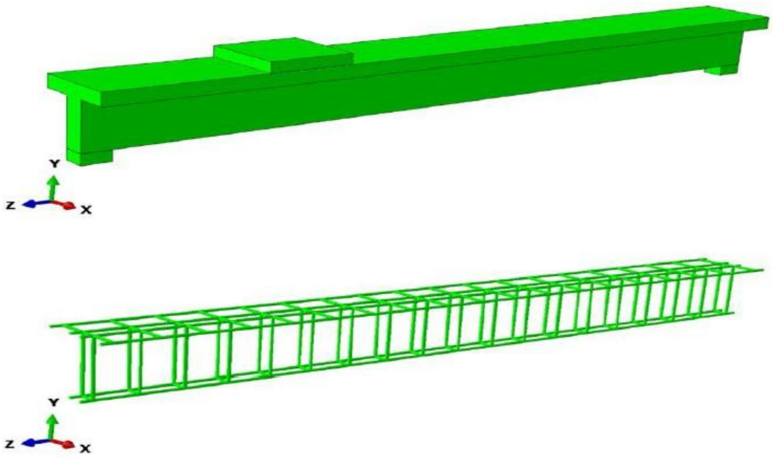

Figure 2. Finite element mesh configuration of the reinforced concrete T-beam

3 RESULTS AND DISCUSSION

The optimization algorithms successfully identified reinforcement configurations that resulted in reduced shear crack formation compared to traditional designs. Optimized beams demonstrated a 15-20% increase in shear strength, with a significant reduction in crack width under applied loads [8]. Figure 3 illustrates the maximum elastic shear strain observed in the optimized beam configurations, showing the areas of highest stress concentration and validating the improvements in shear resistance. FEA simulations revealed that strategic placement of stirrups and longitudinal reinforcement improved shear resistance and delayed crack initiation [22]. Material optimization also played a key role in enhancing shear crack control. Beams made with high-strength concrete and advanced steel alloys showed superior performance, reducing both the onset and propagation of shear cracks [18,6]. As shown in Figure 4, the distribution of maximum elastic shear stress further supports the enhanced performance of materials used in the optimized models. These optimized designs were not only more effective in controlling shear cracks but also proved to be cost-efficient when compared to conventional reinforcement solutions.

3.1 Regression equations

The regression models were formulated using MATLAB in conjunction with the Box- Behnken design of experiments. The model coefficients were estimated using the least squares approach. The elastic shear strain *(EE23)* and the maximum principal stress serve as predictive parameters for evaluating the shear strength and stress behavior of the T-beam when subjected to dynamic loading conditions, as characterized in the finite element analysis. The reliability of these regression equations was assessed through the coefficient of determination R^2, detailed in the supplementary material.

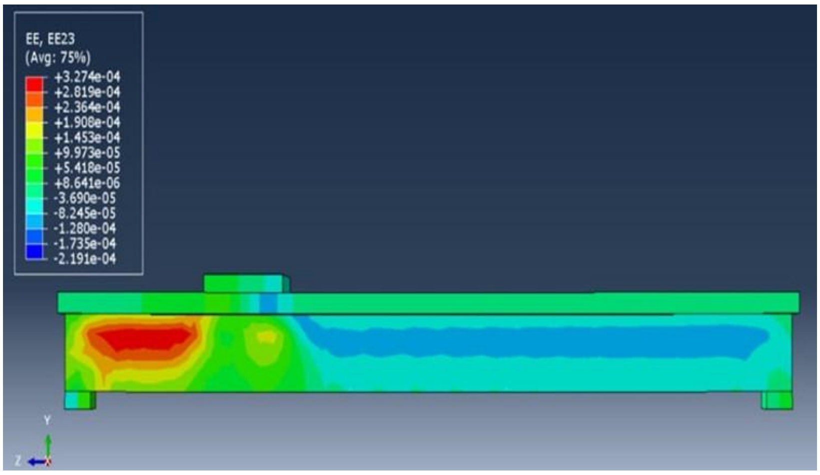

Figure 3. Maximum elastic shear strain

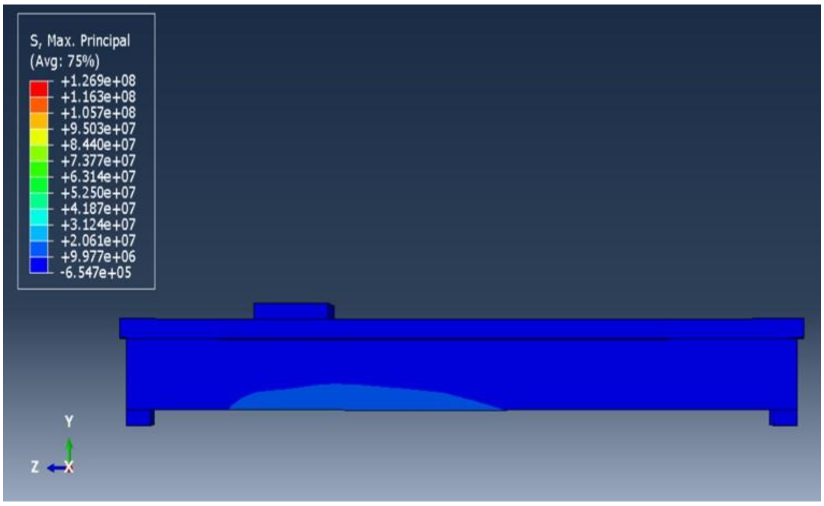

Figure 4. Maximum elastic shear stress

3.2 Coefficient of determination

The reliability of the regression equations was confirmed through R^2 values. For elastic shear strain, $R^2 = 1$ (Figure 5), signifying a perfect representation of the system's structural response. Similarly, $R^2 = 1$ for maximum principal stress (Figure 6), confirming the equation's reliability in predicting T-beam principal stress with 100% efficiency.

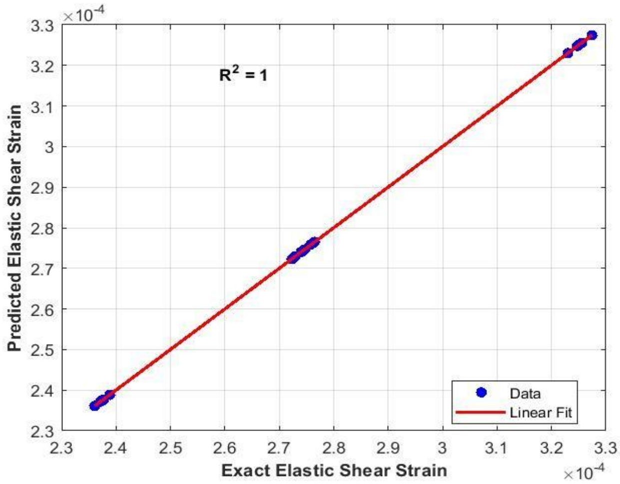

Figure 5. Coefficient of determination-elastic shear strain

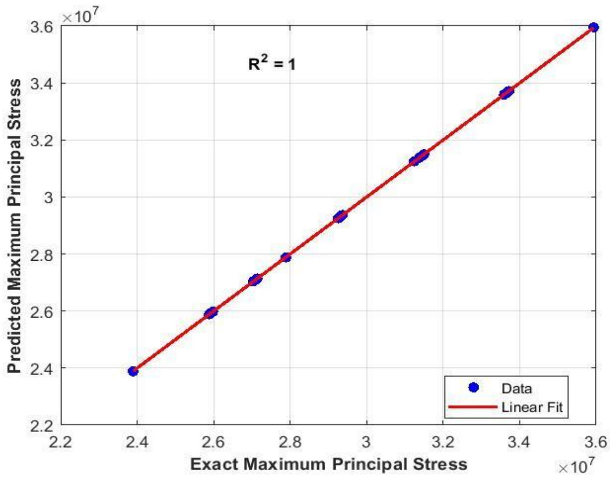

Figure 6. Coefficient of determination maximum principal stress

3.3 Harmonic approach for multi-criteria optimization problems

A part of optimization problems is nonlinear; in this study, the case is quadratic with constraints, which makes it suitable for modeling structural behavior under realistic physical limits. In [1], a methodology was introduced for addressing multi-criteria programming problems (MOPPs) under the presumption that the optimal values for each criterion exceed zero. Over the course of time, a plethora of strategies has been devised to manage these complex issues [2,18]. Among these methodologies, the harmonic mean was utilized to recast the multi-criteria problem as a single-objective optimization problem [3]. The objective function pertinent to our analysis is quadratic in nature and is constrained by specific limitations. Typically, such models are designed to concurrently optimize multiple objectives while adhering to certain constraints. The formulation of the multi-criteria quadratic programming problem (MOQPP) is articulated as follows:

$$\max f_i = c_i{}^T X + \frac{1}{2} X^T G_i X, \qquad i = 1,2,\ldots,r$$

$$\min f_i = c_i{}^T X + \frac{1}{2} X^T G_i X, \qquad i = r+1,\ldots,s, \tag{3}$$

subject to the constraints:

$$\mathrm{lb_i} \leq x_i \leq \mathrm{ub}_i, \quad X \geq 0.$$

In this context, X represents an n-dimensional vector of decision variables, c denotes an n-dimensional vector of constants, r signifies the count of objectives designated for maximization, and $s-r$ indicates the number of objectives intended for minimization. The matrix G is an $n \times n$ symmetric matrix composed of coefficients, while lb and ub correspond to the lower and upper bounds of the decision variables, respectively. The harmonic mean, denoted as H, of a dataset is defined as the reciprocal of the arithmetic mean of the reciprocals of the individual values. For n observations $(x_1, x_2, \ldots, x_n)$, it is expressed mathematically as:

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} \tag{4}$$

To amalgamate the objective functions, a common variable set is identified by reformulating the problem into a single-objective framework through the utilization of the harmonic mean. Let $max\ f_i = m_i$ for $i = 1,\ldots,r$ and $min\ f_i = m_i$ for $i = r+1,\ldots,s$. The unified objective function is articulated as:

$$\max \mathrm{g} = \sum_{k=1}^{r} \frac{\max f_k}{H} - \sum_{k=r+1}^{s} \frac{\min f_k}{H_1} \tag{5}$$

where H and H_1 represent the harmonic means corresponding to the maximized and minimized objectives, respectively. The combined criteria can be further simplified to:

$$\max \mathrm{g} = \sum_{k=1}^{r} \frac{\max f_k}{H} \tag{6}$$

The algorithm is delineated as follows:

1. Each objective $max\ f_k$ is solved utilizing the simplex method, followed by a feasibility check.
2. In the event of infeasibility, the dual simplex method is employed to

rectify the issue.

3. The harmonic mean H for $max\ f_i$ is computed, and the optimization of the combined objective in Equation (5) is executed under the same constraints.
4. The optimal point X^* is subsequently substituted into each individual objective to ascertain their optimal solutions.

For instance, the optimization yields $max\ f_1 = 26.184716298363242$ and $max\ f_2 = 5.552906220959551e + 06$. The harmonic mean is $H = 52.36943263222501$. Dividing the coefficients of each objective by H and summing them, we find the optimal point $X^* = (2200, 35, 7860.3365, 190)$, which lies within the feasible region. Substituting X^* into each objective, we obtain $f_1 = 2.385557750676666 \times 10^{-4}$ and $f_2 = 2.384022324502486e \times 10^{7}$.

Optimization techniques, including genetic algorithms (GA) and particle swarm optimization (PSO), are applied to adjust the reinforcement configuration, material selection, and beam geometry. The optimization process aims to minimize shear cracking while maintaining structural stability and minimizing costs. The effectiveness of each optimized design is evaluated using FEA simulations to measure shear strength, crack development, and overall performance.

4 CONCLUSIONS

This paper presents the potential of an optimization technique in enhancing shear crack control in reinforced concrete T-beams. This work contributes to the ongoing development of more resilient and efficient structural designs, providing engineers with valuable tools for the optimization of reinforced concrete T-beams. The use of computational modeling and optimization algorithms offers a practical solution for real-world engineering applications, ensuring the durability and safety of reinforced concrete structures. Furthermore, the harmonic mean method was successfully employed to transform the multi-objective optimization problem into a single-objective framework, enabling balanced optimization of the shear strain and principal stress behavior of the reinforced concrete T-beam. This approach ensured efficient and unified control over conflicting design objectives. The factors that are involving in the design process would be more reliable and safer after the guaranty of the optimization approach. Further research into alternative materials and more advanced optimization algorithms may lead to even greater improvements in shear crack control and structural performance.

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to **Dr.-Ing. Nazim Abdul Nariman,** a senior lecturer at **Tishk International University Sulaimani** for his invaluable support and guidance throughout the completion of this paper. His insights, encouragement, and constructive feedback were instrumental in shaping the final outcome of this work.

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