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KARUSH-KUHN-TUCKER CONSTRAINED OPTIMIZATION MODEL FOR THE FLEXURAL CRACK PREDICTION IN PRE-STRESSED CONCRETE BEAMS

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ABSTRACT: In this paper, we utilize a multi-objective approach with numerical simulations models for the optimization of the flexural strength for a pre-stressed concrete beam. The optimization approach is conducted utilizing five factors that are (concrete Young's modulus, concrete density, steel Young's modulus, steel density, and pre-stressing) which have a limited range. The surrogate model that predicts both the strain and the deflection for the pre-stressed concrete beam is constructed using least square tool in MATLAB. Twenty one numerical simulations are generated using ABAQUS finite element programs using the experimental design method of Latin Hypercube. The surrogate model's reliability has been tested by comparing the outputs from the numerical models with the surrogate models. The coefficient of determination (R^2) value for both (the maximum principal strain and the maximum deflection outputs) was 1 which indicates 100% accuracy. A non-linear multi-objective optimization with constrained factor range was conducted based on the Karush-Kuhn-Tucker (KKT) method for the surrogate models. At first, we scalarized two objectives into single one using harmonic mean, then we used the KKT method which worked very well for optimizing the two surrogate models of prediction.

KEYWORDS: Surrogate model; Coefficient of determination; Latin hypercube method; Multi-objective optimization; Maximum principal strain; Maximum deflection.

1 INTRODUCTION

The pre-stressing technique has been widely and extensively used in the design of civil engineering construction especially in buildings and bridges. The pre-stressing can be adopted in several aspects for design optimization and aesthetic use in structures by architectural engineers with execution efforts by civil engineers. The general aim for pre-stressing is to control the generation of cracks in the concrete members and decrease the deflection in the structures. The global objective of pre-stressing is to increase the safety of the construction during the presence of static loads, live loads, dynamic loads of earthquakes, and wind loads.

Many research studies have been undertaken to study the pre-stressed reinforced concrete structures by the use of theoretical and experimental studies to enhance and improve the performance and serviceability of the constructions such as buildings, bridges and other civil engineering structures. Bhawar et al, (2015) [1] has studied the pre-stressed reinforced concrete beams (bridge girders) by optimizing (minimizing) the overall cost of the design by considering many design variables related to the concrete and the steel. They used optimtool in MATLAB program for the optimization study. They concluded a cost optimization approach proposal for the minimization of the cost in feasible design for the pre-stressed reinforced concrete girders. Nariman et al, (2022) [2] presented factorial approach to optimize the flexural strength of a reinforced concrete beam. They applied surrogate modeling supporting on Box-Behnken sampling method to arrange the models of the simulations for prediction analysis. They used ATENA program to obtain the exact data of the structural behavior of the system. They detected that the optimization approach has efficiently controlled and optimized the design for the R C beam. Piatek and Siwowski (2022) [3] studied the utilization of CFRP strips in reinforced concrete beams for flexural strength. They analyzed the cracks and the yielding of the structural member. They recognized that the pre-stressing of the reinforcement has positively enhanced and increased the performance and strength of the reinforced concrete beam.

Radnic et al, (2015) [4] investigated the behavior of the pre-stressed concrete beam. They considered many factors to optimize the performance of the structure with the use of actual experimental specimens and laboratory tests. They realized that the pre-stressing magnitude affected positively the performance and the serviceability of the reinforced concrete beams and they mentioned the necessity for optimizing this behavior in separate research studies to control the cracks and the deflection of the structural member. Bischoff et al, (2018) [5] they studied the effect of moment of Inertia factor in predicting the deflection and the crack of partial adopted pre-stressed concrete beams and slabs. They presented a new approach to predict the performance of the structural members. They recommended the use of trilinear approach in the calculation of deflections in cracked pre-stressed members.

It has been extremely proven that the pre-stressing technique is greatly beneficial for building bridges and all structures to increase the strength of the structural system and enhance and ensure the safety of the constructions. Despite the fact that many researches have been undertaken to use optimization of pre-stressing aspect, but still many approaches of optimizations haven't been applied to enrich the design consideration by civil engineers. In this paper, we will apply a multi-objective optimization approach with Karush-Kuhn-Tucker approach to optimize the flexural behavior of the pre-stressed concrete beam members. The optimization process would be applied by the utilization of numerical analysis and simulation by ABAQUS finite element program and MATLAB codes. The Latin Hypercube experimental design method would be dedicated for the process to generate the models and the construction of two surrogate models of the cracks prediction for the pre-stressed concrete beam.

2 RESPONSE SURFACE MODEL

The general formulation of an optimization problem and all of its elements are presented in, where the mathematical formulation was drawn from.
The general mathematical formulation of an optimization case is:

$$Max(min).\, f_i\ (x), \qquad i = 1, 2, \dots, M$$

subject to:

$$g_j(x), \qquad j = 1, 2, \dots, J$$

$$c_k(x), \qquad k = 1, 2, \dots, K,$$

where $\boldsymbol{X} = (x_1, x_2, \dots, x_d)\ X \in [x_{min}, x_{max}] \subset Rd.$

The objective functions are denoted by $f_i\ (x)$, and the problem's constraints are represented by $g_j(x)$ and $c_k(x)$. Each feasible solution, which comprises input values from the search space, is a set of design variables X that satisfy the requirements [6].

2.1 Constraints based classification

The optimization cases for this branch can be assigned into multiple categories which are being detailed in the following sections [7]. Unconstrained optimization and constrained optimization cases.

2.2 Nonlinear Programming Problem (NLPP)

A general optimization case is to adopt n decision variables $(\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n})$ from a provided feasible area in such a way for the optimization (minimization or maximization) of a nominated function $f\ (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n})^T$ which has the controlling variables. The case is named a nonlinear programming problem (NLPP) when the function is nonlinear and the feasible area is determined by constraints that are nonlinear [8].

2.3 Multi-Objective Optimization Programming Problems (MOOPP)

A case with a multi-objective optimization problem is of the from

$$\text{Optimize } \{f_1(x), f_2(x), \dots, f_k\ (x)\}$$

subject to: $x \in S,$

where for $k \geq 2$ objective functions $f_i: R_n \rightarrow R$, the decision (variables) vectors $\boldsymbol{X} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n})^T$ are allocated in the non-empty feasible area S which is considered a group of the decision variable area R^n. When all the functions and the constraints that are forming the feasible area are linear, then the multi-objective optimization case is called linear. If at least one of the objectives or the constraint functions is nonlinear, the problem is called a nonlinear multi-objective optimization problem. Multi-objective optimization problems are usually solved by scalarization approach. The scalarization means that the problem is converted into a single (scalar) or a family of single objective optimization problems [9].

2.4 Multi-Objective Quadratic Programming Problems (MOQPP)

The mathematical form of a multi-objective quadratic programming problem is [9]:

$$Max. [f_1(x), f_2(x), \ldots, f_r(x)]$$
$$Min. [f_{(r+1)}(x), f_{(r+2)}(x), \ldots, f_s(x)],$$
$$f_k(x) = x^T Q_k x + C_k^T x,$$
$$k = 1, 2, \ldots, r, r+1, \ldots, s.$$

subject to:

$$Ax [\geq, \leq, =] B,$$

where Q is an $(n \times n)$ symmetric matrix of coefficients, $\boldsymbol{x}$ is an n-dimensional vector of decision variables, C is the n-dimensional vector of constants, B is m-dimensional vector of constants, A is $(n \times m)$ matrix of coefficients, r is number of objective functions to be maximized, s is the number of objective functions to be maximized and minimized and $(s - r)$ is the number of objective functions that is minimized, all vectors are assumed to be column vectors unless transposed (T).

3 KARUSH-KUHN-TUCKER METHOD (KKT)

The optimality conditions for a constrained local optimum are called the Karush Kuhn Tucker (KKT) conditions and they play an important role in constrained optimization theory and algorithm development. The KKT conditions for optimality are a set of necessary conditions for a solution to be optimal in a mathematical optimization problem. They are necessary and adequate conditions for a local minimum in nonlinear programming problems. The KKT conditions consist of the following elements, Consider the following optimization problem [10].

$$Minimize\ f(\boldsymbol{x})$$
$$Subject\ to:$$
$$gi(x) \leq 0, \quad i = 1, 2, \ldots, m$$
$$x \geq 0,$$

where, $f(x)$ is the objective function, and $gi(x)$ are inequality function of the constraints. Defined the Lagrange function by $L(x, \lambda) = f(x) + \sum_i^m \lambda_i g_i(x)$. Stationarity of the gradient of the Lagrange multipliers must be zero at the optimal point, $\nabla f(\boldsymbol{x}^*) + \sum_i^m \lambda_i \nabla g_i(\boldsymbol{x}^*) = 0$, for all i.

4 RESPONSE SURFACE MODEL

A response surface model (RSM) is a tool which is utilized to manage the data in both mathematical and statistical approaches to build multi-objective functions for the process of optimization of a system. The RSM considers the global effect of many input variables with different mathematical terms such as linear, quadratic, and interactions between the variables. The RSM determines the type of the relation between the involving variables and detecting the extent of controlling the output result by the mentioned variables [2, 11].

The exact data for the construction of the elements of the RSM is collected through many ways such as equations, laboratory test, and numerical simulations. The gathered data would be further processed to determine the regression coefficients by least square approach, were large matrices are solved in the process. Finally, the objective functions are constructed. The equation is representing the relation between the variables in the system which is a function denoted by $\mathbf{y}$, and the variables are $(\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n})$. Generally, the function is being tested for the reliability to fully represent the system output. The equation is written as follows:

$$y = f(\boldsymbol{X})\beta + \varepsilon \quad (1)$$

Where the vector $\boldsymbol{X} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n})$, and the function $f\,(\boldsymbol{X})$ is a vector of n elements which contains many terms as mentioned above. The syntax β represents a vector of k number of regression coefficients. And ε is a random experimental error of experiments which is always random and considered to have a mean of zero value. The term $f(\boldsymbol{X})\beta$ is the mean output of the system. In our study, we will consider multi-objective function y which provides prediction for two outputs. The following equations (1) and (2) are used in linear cases and nonlinear cases.The linear problem is represented by Eq. (2):

$$y = \beta_0 + \sum_{i=1}^k \beta_i \boldsymbol{x_i} + \varepsilon \quad (2)$$

While the non-linear problem is represented by Eq. (3):

$$y = \beta_0 + \sum_{i=1}^k \beta_i \boldsymbol{x_i} + \sum_{i<j} \sum \beta_{ij} \boldsymbol{x_i} \boldsymbol{x_j} + \sum_{i=1}^k \beta_{ii} \boldsymbol{x_i^2} + \varepsilon \quad (3)$$

The vector β is calculated by the least square method. Generally, Eq. (3) is being updated to represent a matrix:

$$Y = \beta \boldsymbol{X} + \varepsilon \quad (4)$$

Eq. (4) is solved using the following formula:

$$\boldsymbol{\beta} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \mathrm{Y} \quad (5)$$

where $\boldsymbol{X}^T$ represents the transpose matrix $\boldsymbol{X}$ and the term $(\boldsymbol{X}^T \boldsymbol{X})^{-1}$ is the inverse of the resulting matrix ($\boldsymbol{X}^T\boldsymbol{X}$) [2].

4.1 Variable limit values

A surrogate models is created by dedicating five variables utilizing the Latin Hypercube design approach by creating 21 models of the pre-stressed concrete member in ABAQUS program. The adopted models are involved in the simulation process to determine the maximum principal strain and the maximum deflection data. Table 1 shows the adopted factors and their range values.

Table 1. Variable limit values

Variable Symbol	Variable	Limit Value
$\mathbf{x_1}$	Concrete Density ($\boldsymbol{\rho}$c) kg/m3	2200 – 2600
$\mathbf{x_2}$	Concrete Young's Modulus (Ec) GPa	24 – 35
$\mathbf{x_3}$	Steel Density ($\boldsymbol{\rho}$s) kg/m3	7800 – 8000
$\mathbf{x_4}$	Steel Young's Modulus (Es) GPa	190 – 230
$\mathbf{x_5}$	Pre-stressing (MPa)	1.2 – 2

4.2 Latin hypercube sampling method

The generated 21 models is organized in Table 2 (Appendix 1). The numerical models were constructed using the design sampling method of Latin Hypercube in parallel with MATLAB. Five factors were considered (concrete density, concrete Young's modulus, steel density, steel Young's modulus, and pre-stressing) [12].

4.3 Surrogate models

A surrogate model is that tool which is used to predict the behavior or the output of any system. The prediction process is being established through an equation which represents the relation between the output and the dependent variables. The reliability of the surrogate models is then guaranteed through the use of the coefficient of determination R^2. We construct the surrogate models for this purpose and will be further processed for optimization using the KKT Method. Concrete Density, Concrete Young's Modulus, Steel Density, Steel Young's Modulus, Pre-stressing are the five input factors taken into account. The factors are categorized by their minimum magnitudes for each surrogate models. The model consists of five components: linear, nonlinear (quadratic curve), and interaction terms, as shown in Eq. (6).

$$y=\beta_0+\beta_1\mathbf{x_1}+\beta_2\mathbf{x_2}+\beta_3\mathbf{x_3}+\beta_4\mathbf{x_4}+\beta_5\mathbf{x_5}+\beta_6\mathbf{x_1}^2+\beta_7\mathbf{x_2}^2+\beta_8\mathbf{x_3}^2+\beta_9\mathbf{x_4}^2+\beta_{10}\mathbf{x_5}^2+\beta_{11}\mathbf{x_1}\mathbf{x_2}+\beta_{12}\mathbf{x_1}\mathbf{x_3}+\beta_{13}\mathbf{x_1}\mathbf{x_4}+\beta_{14}\mathbf{x_1}\mathbf{x_5}+\beta_{15}x_2x_3+\beta_{16}\mathbf{x_2}\mathbf{x_4}+\beta_{17}\mathbf{x_2}\mathbf{x_5}+\beta_{18}\mathbf{x_3}\mathbf{x_4}+\beta_{19}\mathbf{x_3}\mathbf{x_5}+\beta_{20}\mathbf{x_4}\mathbf{x_5} \quad (6)$$

where **y** is the prediction output, β_0 is a constant coefficient term, and $\mathbf{x_1}$, $\mathbf{x_2}$, $\mathbf{x_3}$, $\mathbf{x_4}$, and $\mathbf{x_5}$ are the adopted variables and β_1, β_2, β_3, β_4, and β_5 are the first degree coefficients; and β_6, β_7, β_8, β_9, and β_{10} are the quadratic coefficients; β_{11}, β_{12}, …,

β_{20} are the coefficients of interactions between the variables.

Surrogate modeling analysis is important to generate a tool for prediction of the outputs, where the results collected from the exact output and the results collected from the surrogate models are being verified. The coefficient of determination R^2 is being utilized to recognize the strength of the surrogate models. The limit value of the R^2 is bounded between (0 and 1). R^2 uses a verification between the exact outputs and the outputs of surrogate models. The magnitude near 1 means efficient surrogate models [2, 13, 14].

5 FINITE ELEMENT MODEL

The pre-stressed concrete beam is created in ABAQUS program with dimensions (874*15.24*30.48) cm. The beam is loaded by multiple loads in for locations (see Figure 1). The beam has two pre-stressed tendons of 1.12 cm diameter for each, and dour steel reinforcement bars located in top and bottom of the beam with a diameter of 2 cm for each steel bar. The numerical model is constrained at the supports locations. The constraint is different where there is a need of releasing the support in one location to move in horizontal direction to expel the stresses generated due to loading.

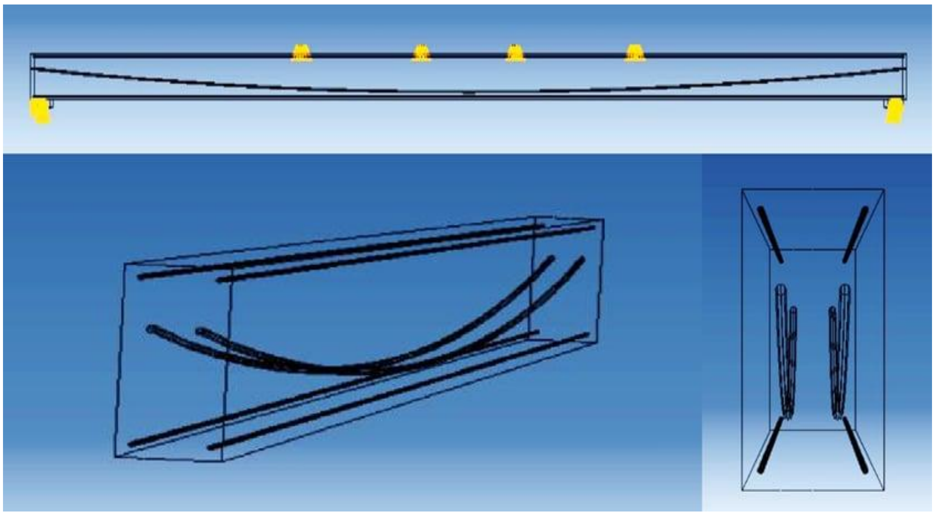

Figure 1. Finite element model

6 RESULTS AND DISCUSSION

6.1 Maximum principal strain

Table 3 (Appendix 2) shows the data of the regression coefficients, which were used to generate the surrogate model for the maximum principal strain output. The function of the surrogate model is denoted by f_1:

$$f_1 = 0.00386184346766488325\, \mathbf{x_1} - 0.875776821696116215\, \mathbf{x_2} - 0.00870462141076268974\, \mathbf{x_3} + 0.291492460782349511\, \mathbf{x_4} - 10.9396975656360713\, \mathbf{x_5} - 0.00000058177589641980639 1\, \mathbf{x_1}^2 + 0.00577282382463858444\, \mathbf{x_2}^2 + 0.0000008308790354519850 32\, \mathbf{x_3}^2 + 0.000509010212177061309\, \mathbf{x_4}^2 - 0.259775660658358809\, \mathbf{x_5}^2 + 0.0000618323357507319108\, \mathbf{x_1x_2} + 0.00000012974510634193745 7\, \mathbf{x_1x_3} - 0.0000261155807997875 65\, \mathbf{x_1x_4} + 0.000811829966181199093\, \mathbf{x_1x_5} + 0.000120219688441336623\, \mathbf{x_2x_3} - 0.00227804444100949204\, x_2x_4 - 0.0561297428276724856\, x_2x_5 - 0.000050096845455541665 9\, \mathbf{x_3x_4} + 0.00107367441029970281\, \mathbf{x_3x_5} + 0.0148404413293488307\, \mathbf{x_4x_5} + 22.8395039952906588 \tag{7}$$

The R^2 for the maximum principal strain outputs was R^2=1 that indicates an excellent efficiency of the surrogate model for the prediction of the maximum principal strain in the pre-stressed concrete beam models.

6.2 Maximum deflection

Table 4 (Appendix 3) lists the information of the regression coefficients that were used to construct the surrogate model for the output of maximum deflection.
The function of the surrogate model that calculates the maximum deflection output in the pre-stressed concrete beam is represented by f_2\;

$$f_2 = 0.00469252383145175736 2\, \mathbf{x_1} - 1.37024823990113387\, \mathbf{x_2} - 0.0960969224374603614\, \mathbf{x_3} + 0.300262428518838854\, \mathbf{x_4} + 14.0472661880508154\, \mathbf{x_5} + 0.00000003179716800247886 88\, \mathbf{x_1}^2 + 0.00290688831528073307\, \mathbf{x_2}^2 + 0.0000066065489266386712 3\, \mathbf{x_3}^2 + 0.000206101165782022201\, \mathbf{x_4}^2 - 0.109276724414906985\, \mathbf{x_5}^2 + 0.0000297561656730545783\, \mathbf{x_1x_2} - 0.00000006076543529637628 75\, \mathbf{x_1x_3} - 0.0000073295634864362 54\, \mathbf{x_1x_4} + 0.000340052233457333142\, \mathbf{x_1x_5} + 0.000181719312768606514\, \mathbf{x_2x_3} - 0.00128278265218584585\, \mathbf{x_2x_4} - 0.0266285784473415859\, \mathbf{x_2x_5} - 0.000043467552581557602 2\, \mathbf{x_3x_4} - 0.00194497334072859 75\, \mathbf{x_3x_5} + 0.00795040656168949732\, \mathbf{x_4x_5} + 351.676280216378406 \tag{8}$$

The R^2 for the results of the exact outputs and the surrogate model outputs was 1, which is an excellent efficiency of the surrogate model in calculating the maximum deflection in the pre-stressed concrete beam models.

7 COEFFICIENT OF DETERMINATION RESULTS

7.1 Maximum principal strain

The R^2 for the maximum principal strain for the exact outputs and the surrogate models' outputs was 1, which indicates an excellent efficiency of the surrogate model to calculate the maximum principal strain output in the pre-stressed concrete beam model (Figure2), which is actually very satisfactory.

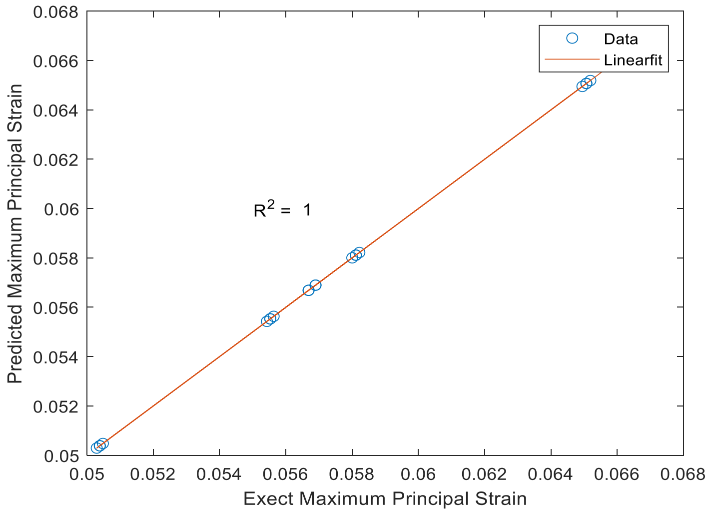

Figure 2. Coefficient of determination - Maximum principal strain

7.2 Maximum deflection

The R^2 value for the maximum deflection output was 1, which is an excellent value indicating the efficiency of the surrogate model to calculate the maximum deflection in the pre-stressed concrete beam model (Figure 3), which is very satisfactory.

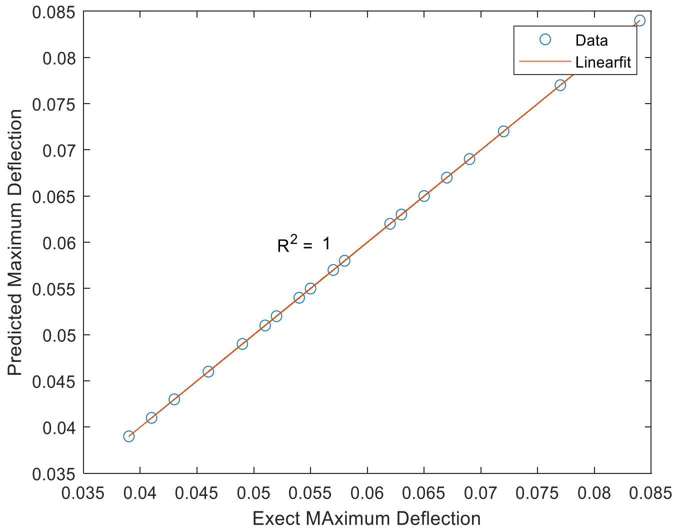

Figure 3. Coefficient of determination - Maximum deflection

8 SIMULATION RESULTS

The output of the tensile damage generated in the reinforced concrete beam when dedicating a 1.8 MP for the pre-stressing variable leads to a certain appearance of the tension stresses in the tension zones in both sides (see Figure 4). It is considered a logical behavior of the structural member under the external load which can be controlled through the optimization process by studying the other results. While the decrease of the pre-stressing in the tendons to 1.3 MP results in an increase in the generation and propagation of the tension stresses in the same locations and further deflection in the reinforced concrete beam (see Figure 5). Finally, when the pre-stressing is further decreased to 0.8 MP it would increase the tension stresses highly with a great propagation to the top region of the beam (see Figure 6).

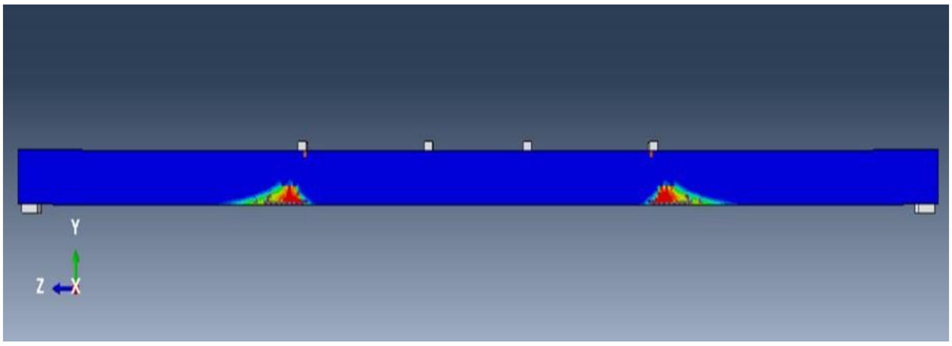

Figure 4. Tensile damage - Prestressing = 1.8 MPa

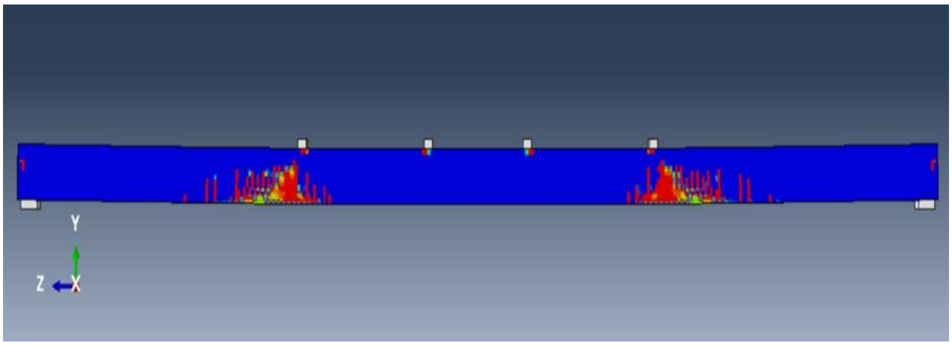

Figure 5. Tensile damage - Prestressing = 1.3 MPa

The increase of the pre-stressing variable is obviously resulting in the decrease of the generated tension stresses in the tension zones. It is known that concrete is weak in tension compared to compression when externally loaded where it becomes the ongoing step for cracking and the failure of the entire structural system under loading.

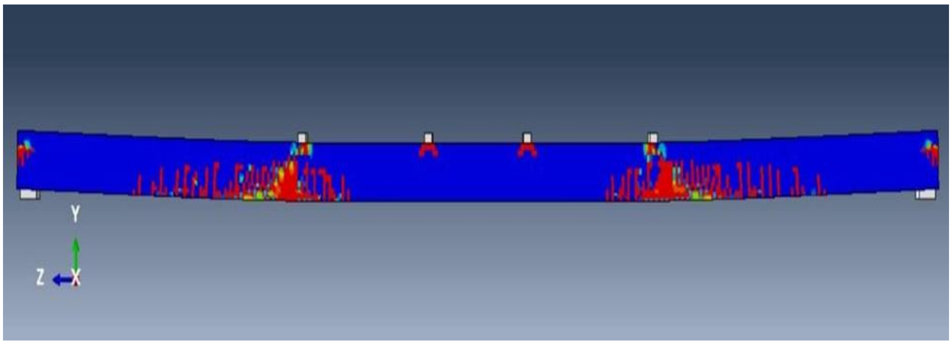

Figure 6. Tensile damage - prestressing = 0.8 MPa

The control of the failure and the propagation of such tension stresses is to optimize the design of the reinforced concrete beam with an optimized design of pre-stressing and optimized mechanical properties of the structure. The optimized control of the tension stresses is directly is a control of the deflection and increase of the factor of safety as a stiff structural member.

9 OPTIMIZATION RESULTS

9.1 KKT conditions with two objective functions

To build the constraints, we have 5 variables on the range in Table (1). There are different ways to construct the constraints. For the first constraint we combine the variables which is greater than the combined of the lower ranges:

$$900\mathbf{x_1}+\mathbf{x_2}+0.5\mathbf{x_3}-20\mathbf{x_4}-\mathbf{x_5} \geq 1980123$$

In second constraint we combine the variables which is less than the combine of the upper ranges

$$2\mathbf{x_1}-10\mathbf{x_2}+0.02\mathbf{x_3}+\mathbf{x_4}+0.00001\mathbf{x_5} \leq 5240$$

9.2 Multi-objective optimization problems

We use the method of KKT to solve them, because the two objectives are non-linear quadratic. Sen [15], presented a method convert a multi-objective to a single one, them solve with the same constraints. Many techniques can be used for this purpose [16]. Among these techniques, we used the harmonic mean.
We use the harmonic mean to convert them into single objective as follows:
The harmonic means that HM of asset of data is defined as the reciprocal of the arithmetic average of the reciprocal of the given values as in Eq (9). If ($\mathbf{x_1}$, $\mathbf{x_2}$, …, $\mathbf{x_n}$) are n observations, then:

$$HM = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_n}} \tag{9}$$

To combine the objective functions, we determine the common set of the variables from the following combined objective function. Let $Max.\ f_i = m_i$, $i =$

$1, \ldots, r$ and $Min.\ f_i = m_i,\ i = r+1, \ldots, s$. To formulate the problem to single objective and by using harmonic mean [17], we have Eq. (10)

$$Max. g = \sum_{k=1}^{r} \frac{Max. f_k}{HM_1} - \sum_{k=r+1}^{s} \frac{Min. f_k}{HM_2} \quad (10)$$

where HM_1 and HM_2 are the harmonic mean for maximized and minimized objectives, respectively. In this research, the objective functions of f_1 and f_2 are minimized, the objective function Max. g becomes:

$$Max. g = -\sum_{k=r+1}^{s} \frac{Min. f_k}{HM_2}. \quad (11)$$

So, thus, the multi-objective Quadratic programming problem (MOQPP) can be defined as in (Appendix 4). The algorithm is constructed as follows: Solving the objective function Min. f_k by the KKT-method. Go to the next step where m_i is the optimum value for Min. f_i, HM_2 the harmonic mean for Min. f_i. Optimize the Eq. (11) under the same constraints. Substitute the optimal value to the individual objective to get optimal solution for each one. Finally, stop.

$$Min. f_1 = 0.002592178874560303,$$

and $Min. f_2 = 0.02346141114221331.$

So, the harmonic mean is 0.004668544664367934. Now, divide the coefficients of each objective functions by 0.004668544664367934, and then sum them. The objective function of $Max. g$ with same constraints in (Appendix 5). Solving $Max. g$ by KKT method, $Max. g = -18.96644833081395$ and the optimal point of Max. g is:

$\mathbf{x}^*$= (2200.256613152524, 28.94127028963871, 7893.72803972978, 204.1186849607102, 1.383428211629868), which is in the range of the feasible solution.

10 CONCLUSIONS

The flexural crack control in reinforced concrete beams are a major task for design engineers, whereas a pre-stressing technology is a must. The adopted nonlinear multi-objective optimization approach is critical need to fully control the design. The concluded findings of our research study are listed as follows;

1- The surrogate models of the prediction manifested 100% accuracy in predicting the outputs of maximum principal strain and maximum deflection for the pre-stressed concrete beam. The criteria of the reliability were the coefficient of determination R^2 which was 1 for both surrogate models.

2- The nonlinear multi-objective optimization approach displayed a strong efficiency in optimizing the values of the five factors in addition to the optimized surrogate models.

3- The process of controlling the flexural cracks is highly reliable through the application of the optimized values of the five factor as numerical simulation by ABAQUS program and through the application by the surrogate models.

4- The validation of the results for the optimization process were attained which is an evidence of a successful and smooth optimization approach, because any lateral or uncertain error will result in wide errors and failure in determining the targeted objectives of the study.

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APPENDICES

Appendix (1):

Table 2. Arrangement of 21 models

Mode l	x_1	x_2	x_3	x_4	x_5
1	2475.30506428678	32.43692234751	7802.72002227796	211.33365441304	1.65709254171
2	2360.93661406054	26.06344789230	7893.54412370220	194.45599015772	1.88979019537
3	2548.67549027058	32.28888157126	7809.64905952394	221.39059696123	1.48748580925
4	2334.71439824658	26.92477855004	7856.98096066760	210.32050207467	1.25415539027
5	2372.49123221203	33.56487094044	7904.38289076600	199.64919483272	1.98797693185
6	2449.46165086999	25.10369034679	7859.00637588633	204.76861533686	1.42879935714
7	2226.58845168778	34.46264826798	7829.90674013737	206.16913072163	1.73394693777
8	2315.69124272412	29.16603841371	7927.94045211430	222.87558497814	1.27754103272
9	2312.05816650644	30.06357324958	7945.19892684802	193.32539327053	1.86033275280
10	2269.69901889722	34.59951263806	7823.62366889154	218.86555284079	1.93424159328
11	2580.25275345555	31.05862813980	7984.89614788142	227.07018205335	1.83489361697
12	2208.79295522872	26.22384309769	7941.04063784599	201.59974069702	1.78960865652
13	2528.81468950047	24.95867700585	7909.49103368115	217.90341636163	1.41815792893
14	2444.16719042563	24.17142871119	7999.51509873660	197.74223831864	1.66787127511
15	2287.11911120844	31.53308665856	7923.27527863640	214.89958198260	1.55131361690
16	2426.70156272325	28.71069966743	7844.42669142934	196.49961836115	1.61595945830
17	2498.98625587278	27.27892948736	7965.41995469759	229.80616736308	1.38203504227
18	2242.28337881971	29.24242427345	7972.45735414273	214.06309344815	1.72556344785
19	2589.77218253868	33.36142881288	7869.77717498634	208.52219898101	1.32860574073
20	2509.16054413381	28.00159950652	7879.18793294762	191.39790952125	1.21046837655
21	2397.35345436898	30.70943781338	7955.27862101716	225.75897400109	1.51107184047

Appendix (2):

Table 3. Regression coefficients for maximum principal strain

Coefficient	Value
β_0	22.8395039952906588
β_1	0.00386184346766488325
β_2	-0.875776821696116215
β_3	-0.00870462141076268974
β_4	0.291492460782349511
β_5	-10.9396975656360713
β_6	-0.000000581775896419806391
β_7	0.00577282382463858444
β_8	0.000000830879035451985032
β_9	0.000509010212177061309
β_{10}	-0.259775660658358809
β_{11}	0.0000618323357507319108
β_{12}	0.000000129745106341937457
β_{13}	-0.0000261155807997787565
β_{14}	0.000811829966181199093

Coefficient	Value
β_{15}	0.000120219688441336623
β_{16}	-0.00227804444100949204
β_{17}	-0.0561297428276724856
β_{18}	-0.0000500968454555416659
β_{19}	0.00107367441029970281
β_{20}	0.0148404413293488307

Appendix (3):

Table 4. Regression coefficients for maximum deflection

Coefficient	Value
β_0	351.676280216378406
β_1	0.00469252383145175736
β_2	-1.37024823990113387
β_3	-0.0960969224374603614
β_4	0.300262428518838854
β_5	14.0472661880508154
β_6	0.000000031797168002478868
β_7	0.00290688831528073307
β_8	0.00000660654892663867123
β_9	0.000206101165782022201
β_{10}	-0.109276724414906985
β_{11}	0.0000297561656730545783
β_{12}	-0.000000607654352963762875
β_{13}	-0.0000073295634864362 54
β_{14}	0.000340052233457333142
β_{15}	0.000181719312768606514
β_{16}	-0.00128278265218584585
β_{17}	-0.0266285784473415859
β_{18}	-0.0000434675525815576022
β_{19}	-0.0019449733407285975
β_{20}	0.00795040656168949732

Appendix (4):

Min. f_1 = 0.00386184346766488325 $\mathbf{x_1}$ - 0.875776821696116215 $\mathbf{x_2}$ - 0.00870462141076268974 $\mathbf{x_3}$ + 0.291492460782349511 $\mathbf{x_4}$ - 10.9396975656360713 $\mathbf{x_5}$ - 0.0000005817758964198063 91 $\mathbf{x_1}^2$ + 0.00577282382463858444 $\mathbf{x_2}^2$ + 0.0000008308790354519850 32 $\mathbf{x_3}^2$ + 0.000509010212177061309 $\mathbf{x_4}^2$ - 0.259775660658358809 $\mathbf{x_5}^2$ + 0.0000618323357507319108 $\mathbf{x_1x_2}$ + 0.000000129745106341937457 $\mathbf{x_1x_3}$ - 0.0000261155807997875 65 $\mathbf{x_1x_4}$ + 0.000811829966181199093 $\mathbf{x_1x_5}$ + 0.000120219688441336623 $\mathbf{x_2x_3}$ - 0.00227804444100949204 $\mathbf{x_2x_4}$ - 0.0561297428276724856 $\mathbf{x_2x_5}$ - 0.0000500968454555416659 $\mathbf{x_3x_4}$ + 0.00107367441029970281 $\mathbf{x_3x_5}$ + 0.0148404413293488307 $\mathbf{x_4x_5}$ +

22.8395039952906588

$$Min.\ f_2 = 0.00469252383145175 7362\, x_1 - 1.37024823990113387\, x_2 - 0.0960969224374603614\, x_3 + 0.300262428518838854\, x_4 + 14.0472661880508154\, x_5 + 0.00000003179716800247886 88\, x_1^2 + 0.00290688831528073307\, x_2^2 + 0.00000660654892663867123\, x_3^2 + 0.000206101165782022201\, x_4^2 - 0.109276724414906985\, x_5^2 + 0.0000297561656730545783\, x_1x_2 - 0.00000060765435296376287 5\, x_1x_3 - 0.0000073295634864362 54\, x_1x_4 + 0.000340052233457333142\, x_1x_5 + 0.000181719312768606514\, x_2x_3 - 0.00128278265218584585\, x_2x_4 - 0.0266285784473415859\, x_2x_5 - 0.000043467552581557602 2\, x_3x_4 - 0.0019449733407285975\, x_3x_5 + 0.00795040656168949732\, x_4x_5 + 351.676280216378406$$

Subject to:

$900x_1+x_2+0.5x_3-20x_4-x_5 \geq 1980123$

$2x_1-10x_2+0.02x_3+x_4+0.00001x_5 \leq 5240$

$2200 \leq x_1 \leq 2600$

$24 \leq x_2 \leq 35$

$7800 \leq x_3 \leq 8000$

$190 \leq x_4 \leq 230$

$1.2 \leq x_5 \leq 2.$

Appendix (5):

$$Max.\ g = -1.832341321355828x_1+481.0974774943779x_2 + 22.44843979926064x_3 - 126.7536099242407x_4 - 665.6396898444325x_5 + 0.0001178051765499791\, x_1^2 - 1.859190125386633x_2^2 - 0.001593093457765515x_3^2 - 0.1531765098912042x_4^2 + 79.05084166592869x_5^2 - 0.01961821252837612x_1x_2 + 0.000102367928547285x_1x_3 + 0.007163933664700604x_1x_4 - 0.2467326077932005x_1x_5 - 0.06467518743356011x_2x_3 + 0.7627274341772699x_2x_4 + 17.72679222856243x_2x_5 + 0.02004144862342595x_3x_4 + 0.1866318077834773x_3x_5 - 4.881788550720429x_4x_5 - 80221.09910827513$$

Subject to:

$900x_1+x_2+0.5x_3-20x_4-x_5 \geq 1980123$

$2x_1-10x_2+0.02x_3+x_4+0.00001x_5 \leq 5240$

$2200 \leq x_1 \leq 2600$

$24 \leq x_2 \leq 35$

$7800 \leq x_3 \leq 8000$

$190 \leq x_4 \leq 230$

$1.2 \leq x_5 \leq 2.$