

## **ENHANCED DYNAMIC RESPONSE OF BRIDGE SYSTEMS WITH CELLULAR SHEAR WALLS**

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**ABSTRACT:** Light-weight shear wall panels with deterministic cellular periodic architecture could provide the means for controlling vibrations in large scale structural systems. The behavior of shear walls comprising of cellular solids when excited by seismic loading is studied. An investigation to evaluate the effects of different cellular configurations is conducted, including the orientation angles and the shapes of the cells (honeycomb, reentrant, and chiral architecture). The cellular walls may be appropriately arranged between the columns of a pier of a concrete bridge system for seismic resistance enhancement in both directions. Appropriate simplified Finite Element Models are developed to predict the stiffness, strength, and energy dissipation capacity of shear wall panels when subjected to monotonic and cyclic shear loading. The total column-cellular wall-deck system behavior is studied with simple stick models after the calibrated wall properties have been taken into account. Aiming to enhance the stiffness and damping properties of the cellular walls, the idea of inserting viscoelastic fillers into the cells is developed in the final part of this study.

**KEYWORDS:** Cellular Materials, Shear Walls, Damping, Energy Dissipation, Viscoelastic Materials.

### **1 INTRODUCTION**

Cellular solids are assemblies that result from appropriate interconnection of solid struts or plates [1]. Cellular materials have attracted great attention due to their advantageous properties such as the ultra-low-density format, the light weight and the good thermal behavior. Honeycomb architectures have been commonly used as the filling layer of sandwich composite panels. They have also been implemented in other fields such as aerodynamics for controlling vibrations due to wind turbulence, in automotive crash test barriers for absorbing energy [2] and in thermal insulator materials due to their low thermal conductivity. Man-made cellular solids imitate the construction of many natural

materials as: wood, plant leaves, trabecular bones or fruits. A light-weight steel shear wall panel of deterministic-cellular-periodic-architecture could be a promising alternative for an efficient and cost-effective response modification system for controlling seismic vibration in large scale structural systems.

Vian and Bruneau (2005) [3] proposed the use of shear wall panels with perforations to mitigate the over-strength effects of solid steel plate shear walls. In the limit, when the number of perforations is maximized and the size of perforation is minimized, a steel shear wall panel with perforations turns to a cellular-solid shear wall panel. Interesting results of steel plate shear walls, both continuous and perforated, attached between the double and triple column piers of bridge models have been presented by Keller and Bruneau [4]. Tsopelas and Chen [5] studied the seismic behavior of cellular-solid-shear walls with various cell orientations. A perforated-aluminum-shear panel in combination with steel braces was also proposed by Foti et al. in 2009 as an energy dissipation device/mechanism in structural frames [6].

The main purpose of this study is to investigate the behavior of different cellular configurations, including the orientation angles and the shapes of the cells (honeycomb, reentrant and chiral architecture), under seismic loading. Simplified Finite Element Models are developed to predict the stiffness, strength, and energy dissipation characteristics of shear wall panels, utilizing ABAQUS software, when subjected to monotonic and cyclic shear loading. Moreover, the different cellular wall architectures are appropriately arranged between the double-column pier (Figure 1a) of a concrete bridge system for an improved transverse seismic response. The shear wall panels can be applied in both directions in case the pier consists of four columns (Figure 1b).

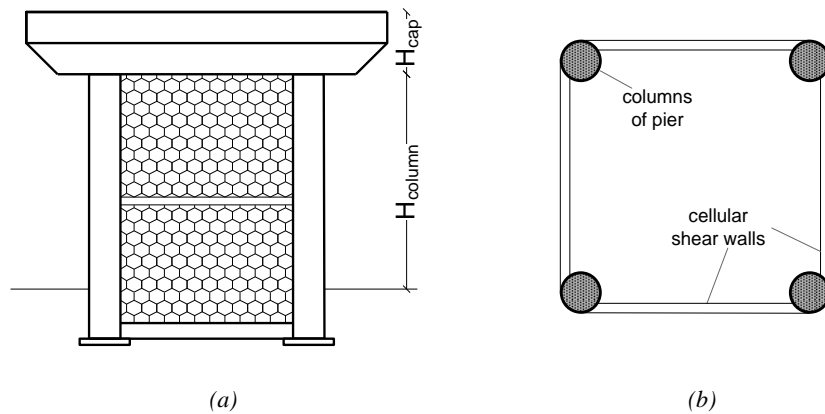


Figure 1. a) Cellular shear wall (honeycomb) arranged between double-column pier of a bridge, b) Plan view of a four-column pier with cellular walls applied in both directions (Keller and Bruneau [4]).

The combined column-cellular wall-deck system is studied with simple stick models after the calibrated properties of the different shear wall arrangements have been considered.

For enhancing the energy dissipation potential in cellular shear wall systems, it is proposed to utilize viscoelastic materials as cell fillers. A honeycomb cellular shear wall with viscoelastic inclusions is examined under dynamic shear loading with appropriate finite element model, in order to compare its performance with a similar shear wall without the inclusions. Analyses results show the effectiveness of the proposed concept.

## 2 CELLULAR-SOLID SHEAR WALL SYSTEMS

Cellular-solid shear wall systems demonstrate the following advantages over the conventional steel shear wall panels: a) they are less prone to out of plane buckling, b) they weigh less, c) they offer controlled stiffness (flexibility when compared to a solid wall) to the structure attached to, while dissipating energy through plastic strains, and d) they offer architectural elegance. The geometry of the basic cell, the so-called “unit cell”, affects the mechanical behavior of the cellular solid. In the ideal case when the cell is equiaxed, its behavior can be considered isotropic. In case the unit cell is slightly elongated or flattened, its properties appear to differ significantly between the two perpendicular directions  $x$  and  $y$ , showing anisotropic behavior. In reality the mechanical properties of unit cells are neither isotropic nor homogeneous on their microstructure. Only the macroscopic behavior of cellular solids is considered homogeneous, given that the cell size is relatively small compared to the size of the structural element.

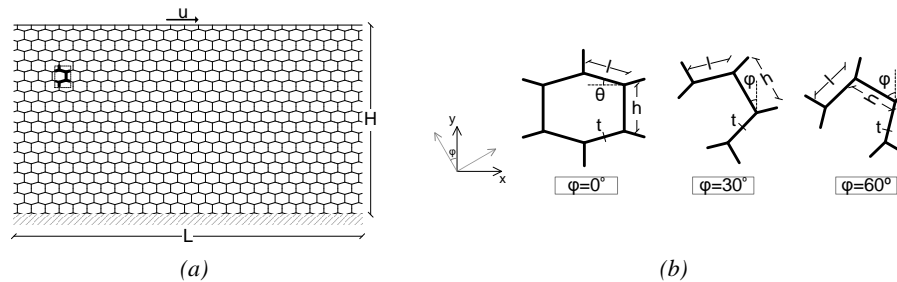


Figure.2. a). Honeycomb Cellular Shear Wall of dimensions  $L \times H$ , b) Different orientations ( $\varphi=0^\circ$ ,  $30^\circ$ ,  $60^\circ$ ) for honeycomb unit cells used in shear walls of this study.

Three different cellular configurations are chosen to be studied, in order to reveal the sensitivity of the overall shear behavior to the shapes and orientations of unit cells with respect to an initial frame. Figure (2a) depicts a honeycomb cellular shear wall, while Figure (2b) shows three different orientations for the

honeycomb unit cell to be compared. The effect on the strength and energy dissipation of the shear walls is quantified by analyzing detailed Finite Element models utilizing a commercial software, ABAQUS.

The honeycomb assembly chosen for the Finite Element Analyses is not the regular one ( $h=l=1$ ) but is slightly elongated, with the ratio of its wall lengths to be  $h/l \approx 1.1$ . Two more configurations were considered for the unit cell of shear walls, in compliance with honeycomb's geometry: the reentrant and the chiral architectures (Figure 3) with the same variation of orientation angles. Both honeycomb and reentrant geometries have the same ratio  $l/t \approx 4$ . Especially for the chiral unit cell the topology aspect ratio [7] was depicted so as to be  $L/R=0.87$ . For comparison purposes the thickness of the cell walls was chosen so as to correspond to the same relative density value  $\rho^*=0.3$  ( $=\rho_{\text{cell}}/\rho_{\text{solid}}$ ) for each unit cell.

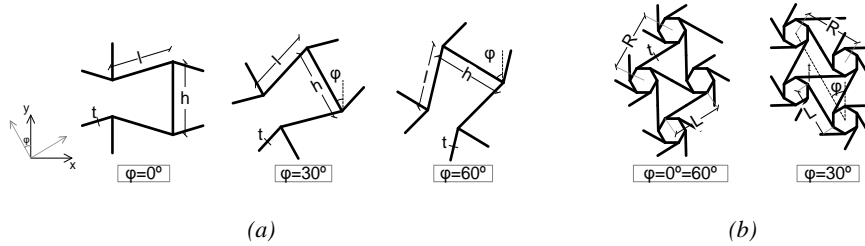


Figure 3. a) Different orientations ( $\varphi=0^\circ, 30^\circ, 60^\circ$ ) for reentrant unit cells and, b) chiral unit cells used in shear walls of this study.

## 2.1 Modelling of cellular solid

Simple beam elements have been successfully used by many researchers for modeling both the linear and nonlinear mechanical properties of cellular materials (Papka and Kyriakides 1994, Overaker et al. 1998) [8, 9]. In this work, the ABAQUS [10] software is used for the analyses. The element B21 is used for modeling the walls of the cells, while appropriate meshing was chosen so as both bending and shearing effects to be monitored. The overall dimensions of the walls are  $H \times L \times d$  ( $d$  is the depth of a cell and of the wall).

A parametric study is conducted in the beginning to quantify the mechanical properties of the cellular solid shear wall panels under monotonic shear loading as a function of the shape of unit cell used (honeycomb, reentrant, and chiral) and the vertical cell wall orientation angle ( $\varphi$ ), for a material with given yield strength. The shear loading is applied as lateral deformation ( $u$ ) of increasing amplitude on the top of the shear wall considered (see Figure 2a). The ratio of the applied deformation at the top of the wall,  $u$ , over its height,  $H$ , is defined as average shear strain ( $\gamma_{\text{aver}} = u/H$ ) of the cellular solid (shear wall “material”). Similarly, the reaction force at the bottom of the wall normalized by the cross-sectional area of the shear wall panel is defined as the average shear stress of

the “cellular solid” ( $\tau_{aver} = F/d/L$ ). This normalization of the lateral displacements and reaction forces makes the results scalable under the assumptions of the analysis and aids the development of “spring” like hysteretic models for full scale cellular shear walls. The material considered is steel with properties shown in Table 1. A multi-linear kinematic hardening material model was utilized in the analyses performed. The dimensions of the shear wall were chosen so as  $H/L=0.54$  ( $=4.2/8.8m$ ).

Table 1. Properties of steel material

Steel	Young's Modulus, E (GPa)	Yielding Strength, $\sigma_y$ (MPa)	Poisson's Ratio, $\nu$
	200	450	0.30

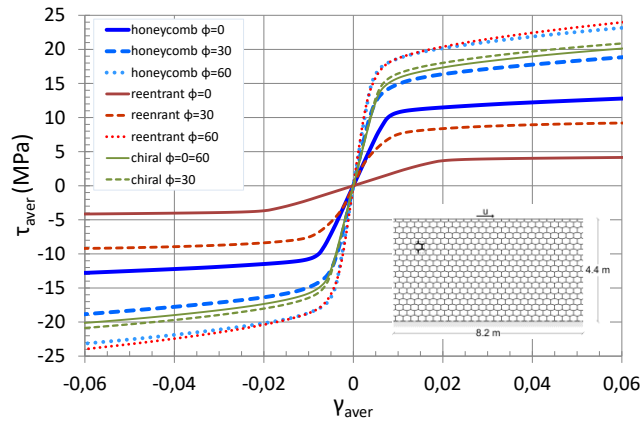


Figure 4. Effect of the shape and orientation of unit cell on the  $\tau_{aver}$  vs  $\gamma_{aver}$  response of a cellular solid wall in monotonic loading.

The effect of the cell shape and orientation on the mechanical properties of cellular solids is demonstrated by plots of  $\tau_{aver}$  vs  $\gamma_{aver}$  in Figure 4, for monotonic loading in both directions. Large discrepancies in stiffness, strength and plastic strains are observed for the different cellular assemblies of the same wall. The behavior of the same cellular solid wall ( $H/L=4.2m/8.8m=0.54$ ) consisting of three different unit cell shapes, oriented by  $\phi=0^\circ$  and  $\phi=60^\circ$  is studied under cyclic loading in Figure 5. The hysteretic responses are symmetric. This is due to absence of buckling of cells of the wall modeled. When the cell walls are under compression, they may suffer severe buckling and this may lead to an important load resistance reduction. The resulting symmetric hysteretic behavior is preferable to a designer when it is compared to the response of a brace in a braced-frame structure under cyclic loading, which is more likely to experience buckling.

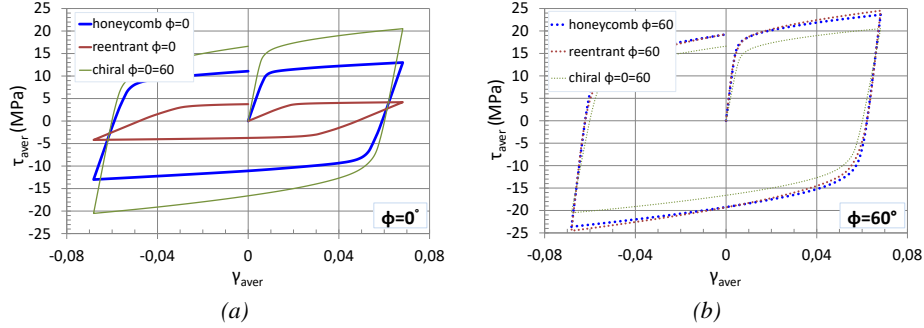


Figure 5. a) Effect of the shape of unit cell on the  $\tau_{aver}$  vs  $\gamma_{aver}$  response of a cellular solid wall in cyclic loading for orientation angle  $\phi=0^\circ$  and b)  $\phi=60^\circ$ .

According to Figure 5 the cell walls appear to yield almost uniformly resulting in stable hysteretic response with significant yielding strength and post yielding stiffness for the case of chiral geometry ( $\phi=0^\circ=60^\circ$ , see Fig 3b.) On the other hand, the same wall consisting of reentrant cells of same relative density and same orientation ( $\phi=0^\circ$ ) appear to develop much smaller elastic stiffness, almost zero post yielding stiffness and overall dissipate much lesser energy. The honeycomb geometry lies between the two aforementioned behaviors. For the case that  $\phi=60^\circ$ , all three geometries appear to have almost identical properties and dissipate similar amount of seismic energy.

### 3 RESPONSE OF A BRIDGE SYSTEM WITH DOUBLE-COLUMN FITTED WITH CELLULAR-SOLID SHEAR WALLS UNDER SEISMIC EXCITATION

To determine the effect of a cellular shear wall on the overall seismic behavior of a bridge pier, we consider a double-column pier model of a concrete bridge inspired by the one proposed by Keller and Bruneau [4], referred as Prototype Bridge #2. The original bridge is a three span one, with two piers of 4 columns each, two at each direction. For simplicity we consider the equivalent double column pier (transverse direction of the original bridge) that consists of one cellular wall along the total height that is made of four identical cellular shear panels. A brief sketch of the bridge with the four shear panels of the wall is given in Figure 6. Each panel is connected with the bridge only at its top and bottom faces and that is the reason why horizontal steel strips are considered between them. The properties of the bridge and the cellular shear panels and wall are given in Table 2. The mass of the wall panels are neglected. The values of the properties agree with the proposed ones by Keller and Bruneau [4] after being appropriately modified for the case only two columns are considered. The stiffness of the system in the transverse direction and the percentage of participation of each part to the overall stiffness [shear wall: 32%, concrete frame: 68%] are identical to the one presented by the authors after pushover

analysis was performed in the same report [4].

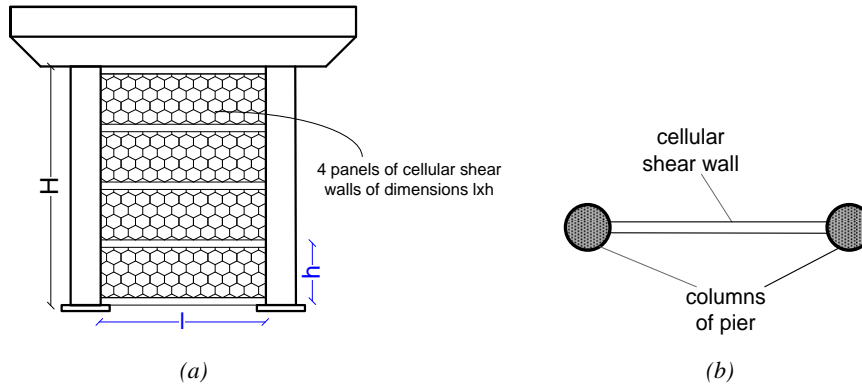


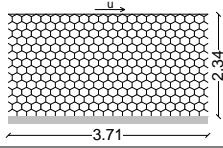
Figure 6. a) The two-column pier considered with 4 cellular panels attached at their top and bottom faces. b) Plan view of the double-column pier with the wall applied in transverse direction.

The two concrete columns are considered to have elastic behavior. In order to use the results of Figure 4 that correspond to the shear behavior of a wall of dimensions  $8.2 \times 4.4$ , the ratio of the total dimensions  $l/h$  is examined, based on the fact that the effective shear modulus of one wall will be identical to any other with the same ratio  $l/h$ , made of the same material. For simplicity we consider that the ratios  $l/h$  of the wall already analyzed and each of the four panels applied between the pier columns are almost the same ( $8.2/4.4 \approx 3.71/2.34$ ). The depth of the cellular wall was picked  $d=0.06\text{m}$  so as the honeycomb configuration of orientation  $\varphi=0^\circ$  to correspond to the desired stiffness compared to the overall one ( $K_{\text{wall}}=0.32K_{\text{total}}$ ), as described before. The same depth was considered for all the other configurations as well. The shear modulus  $G$  and the corresponding stiffness of each cellular panel configuration are presented in Table 3.

Table 2. Bridge and cellular walls properties

Total height of two columns, $H$ (m)	9.37
Total length between two columns $L=l$ (m)	3.71
Dimensions of cellular panels of the wall, $l \times h$ (m)	$3.71 \times 2.34$
Mass of superstructure, pier cap and columns, $m_{\text{tot}}$ (Mg or tons)	1231.35
Stiffness of each elastic concrete column, $k_{\text{col}}$ (MN/m)	34.00
Poisson's ratio of both concrete and steel, $\nu$	0.30

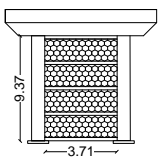
Table 3. Properties of cellular panels for the different configurations (Figure 4)



Cellular Pattern of panel	Shear Modulus, $G = t_{aver}/\gamma_{aver}$ (MN/m <sup>2</sup> )		Stiffness for $t=0.06m$ , $K=GA/h$ (MN/m)		Yield Force (MN)
	Elastic, $G_{el}$	Plastic, $G_{pl}$	Elastic, $K_{el}$	Plastic, $K_{pl}$	
Honeycomb $\phi=0^\circ$	1342.00	48.80	127.66	4.64	2.30
Honeycomb $\phi=30^\circ$	2180.00	101.80	207.38	9.68	3.05
Honeycomb $\phi=60^\circ$	3488.00	109.25	331.81	10.39	3.89
Reentrant $\phi=0^\circ$	200.00	15.69	19.025	1.49	0.79
Reentrant $\phi=30^\circ$	1268.00	34.45	120.62	3.28	1.66
Reentrant $\phi=60^\circ$	3725.00	136.88	354.35	13.02	3.72
Chiral $\phi=0^\circ$ and $60^\circ$	2285.00	130.70	217.37	12.43	3.29
Chiral $\phi=30^\circ$	2670.00	108.72	254.00	10.34	3.39

The total stiffness of the wall according to the stiffnesses of each of the four panels (Table 3) can be estimated if the panels are considered as springs connected in series. The total wall's and pier's properties are presented in Table 4. The pier is analyzed with a simple stick model, considering that the total mass of the pier is concentrated at the top of the column's height and that the overall stiffness is elastic-plastic according to the findings of Table 4. The pier's damping ratio was considered  $\xi=2\%$ .

Table 4. Properties of cellular wall for the different configurations



Cellular pattern of panel	Stiffness of total wall for $t=0.06m$ , $K=GA/h$ (MN/m)		Stiffness of pier= stiffness of total wall + 2*stiffness of column (MN/m)		Yield Force (MN)
	Elastic, $K_{wall, el}$	Plastic, $K_{wall, pl}$	Elastic, $K_{pier, el}$	Plastic, $K_{pier, pl}$	
Honeycomb $\phi=0^\circ$	31.92	1.16	99.92	69.16	7.20
Honeycomb $\phi=30^\circ$	51.84	2.42	119.85	70.42	7.06
Honeycomb $\phi=60^\circ$	82.95	2.60	150.95	70.60	7.09
Reentrant $\phi=0^\circ$	4.76	0.37	72.76	68.37	12.09
Reentrant $\phi=30^\circ$	30.16	0.82	98.16	68.82	5.42
Reentrant $\phi=60^\circ$	88.59	3.26	156.59	71.26	6.57
Chiral $\phi=0^\circ$ and $60^\circ$	54.34	3.11	122.34	71.11	7.40
Chiral $\phi=30^\circ$	63.50	2.59	131.50	70.58	7.03

One set of twenty ground motion time histories was used as excitation of the



stick bridge model. This set is identical to the far field motions used by Tsopeles et al. [11]. The motions are appropriately scaled so as the average spectrum to match the target design spectrum as presented in AASHTO for soil type II,  $A=0.4$ . The results of time history "Eq01" (Landers seismic event, 1992) are presented in Figure 7 in terms of total pier force vs deck displacement (relative to the ground) for the three cellular geometries compared before in Figure (5a): regular honeycomb, reentrant honeycomb and chiral for  $\phi=0^\circ$ .

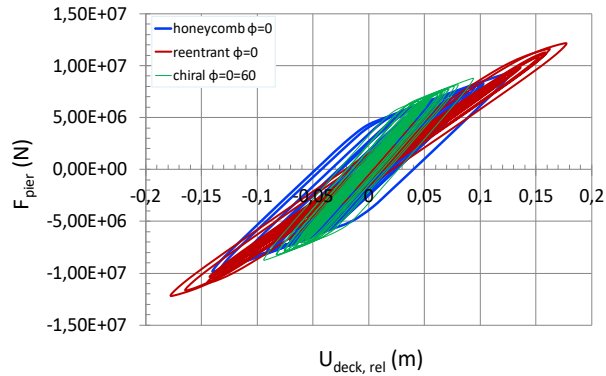


Figure 7. Total force of the pier vs relative displacement of the deck for three different cellular wall geometries with orientation  $\phi=0^\circ$ : regular honeycomb, reentrant honeycomb and chiral.

For the specific seismic event, the pier with the regular honeycomb pattern of the cellular wall appears to dissipate the larger amount of incoming energy compared to the others. The pier with the chiral cellular wall corresponds to smaller amount of dissipated energy but to greater both elastic and post yielding stiffness.

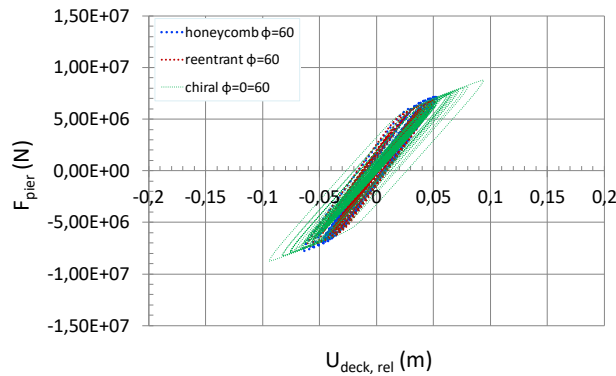


Figure 8. Total force of the pier vs relative displacement of the deck for three different cellular wall geometries with orientation  $\phi=60^\circ$ : regular honeycomb, reentrant honeycomb and chiral.

Finally, the pier with the reentrant wall exhibits elastic behavior due to the

maximum displacement imposed by "Eq01" seismic event being smaller than its yield displacement.

According to the results in Figure (8), in case that the three wall geometries (honeycomb-reentrant-chiral) are oriented at  $\varphi=60^\circ$  the overall pier behavior changes. As shown in Figure (4) for angle  $\varphi=60^\circ$  the wall panels show the largest elastic and post-yielding stiffnesses of all the other orientations. In addition, all three wall configurations dissipate more energy compared to the orientation of  $\varphi=0^\circ$ . Obviously, this is translated to much smaller lateral displacements but not necessarily significantly different pier forces.

#### 4 ADVANCED CELLULAR SHEAR WALLS

Researchers have suggested the placement of fillers into the gaps created in cellular architectures, in view of improving the overall dynamic behavior of a structural system [12-17]. In most cases, infills consist of viscoelastic materials, whose vibration absorption properties have been proven in the past, since they were mainly used in damping devices for the mitigation of structural response in high-rise buildings and bridges under wind or dynamic loading [18-22].

In this study, a shear wall panel with hexagonal honeycomb cells is examined under a harmonic excitation, considering two shear wall configurations: the unfilled steel honeycomb shear wall panel and the advanced fully filled with viscoelastic material shear wall panel. The results of the computational analysis of these two cases validate the hypothesis that the viscoelastic infills lead to a structural system with enhanced dynamic performance.

##### 4.1 Shear wall of cellular architecture with viscoelastic fillers

In addition to the investigation of the dynamic behavior of shear walls of cellular architecture with various types of cells, arranged in different orientation angles with respect to the dimensions of the shear wall, the idea of exploiting the gaps of a cellular shear wall as containers for viscoelastic infills is, also, explored in this study. In particular, a shear wall with hexagonal honeycomb cells is taken into consideration. The geometrical dimensions of the shear wall system are given in Table 5.

*Table 5.* Geometric dimensions of the considered shear wall system

Length, L (m)	Height, H (m)
8.20	4.40

Two cases of shear wall panel configurations are examined as shown in Figure 9. The first case is the plain (unfilled) steel honeycomb matrix (Figure 9a). The size and the geometry of each honeycomb cell are presented in Figure 10a. The same honeycomb matrix is also maintained in the second case where the

viscoelastic fillers are introduced. The configuration, shown in Figure 1b, is the fully filled shear wall, where all the honeycomb cells are filled with viscoelastic material.

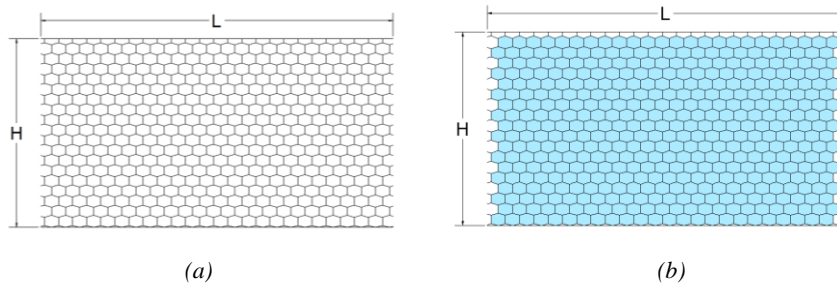


Figure 9. Shear wall configurations: (a) unfilled steel honeycomb shear wall panel, (b) fully filled shear wall panel

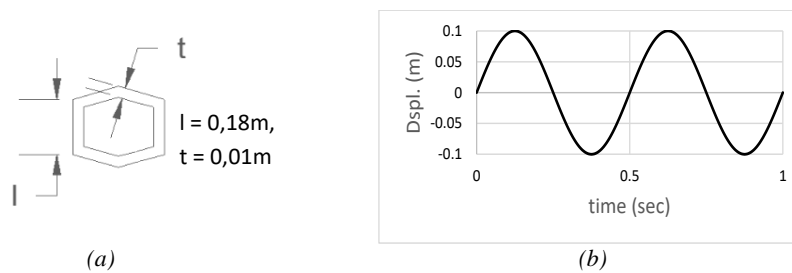


Figure 10. (a) Size and geometry of the honeycomb cells. (b) Harmonic excitation.

Each one of the aforementioned configurations is subjected to the harmonic excitation of Figure 10b. The excitation frequency is equal to 2Hz, whereas the amplitude is of 0.10m (average shear strain or structural drift  $0.1/H=0.023 \approx 2\%$ ).

All computational analysis mentioned in the previous is carried out using ABAQUS software [10]. The material properties introduced in the computational model are given in Table 2 for the structural steel, used to create the cellular matrix of the shear wall panel, and in Table 6 for the viscoelastic material (Shen and Soong, 1995) [23].

Table 6. Elastic and plastic properties of structural steel

Young's modulus, E (GPa)	Poisson's ratio, $\nu$	Yield stress, $\sigma_y$ (MPa)	Yield strain, $\epsilon_y$ (%)	Ultimate stress, $\sigma_u$ (MPa)	Ultimate strain, $\epsilon_u$ (%)
200	0.30	275	0.1375	414	25

Table 7. Frequency-dependent properties of viscoelastic material at a

temperature of 21oC (Shen and Soong, 1995) [23]

Frequency, f (Hz)	Shear Storage Modulus, G' (MPa)	Shear Loss Modulus, G'' (MPa)
1	1.372	1.786
1.5	1.827	2.248
2	2.069	2.724
2.5	2.517	3.192
3	2.661	3.358

The stress-strain curves of the computational analysis of the two aforementioned shear wall configurations are presented in Figure 11. It is reminded that the structural steel of the honeycomb matrix is considered to behave bilinearly. As expected, the addition of the viscoelastic fillers improved the dynamic performance of the cellular shear wall system in both terms of stiffness and damping.

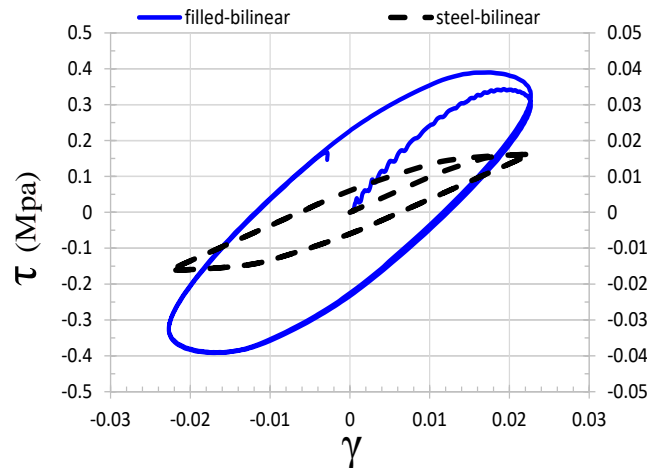


Figure 11. Stress-strain curves of the unfilled and the fully filled shear wall configurations.

## 5 CONCLUSIONS

According to the present study, cellular shear walls can be used as an effective response modification assembly for seismic vibration mitigation in large scale structural systems. The most important conclusions are: a) the cellular shear walls are lightweight structural members which may easily fit between the columns of a pier and are not prone to severe buckling effects and b) depending on the geometry and orientation of unit cells chosen, the walls can be designed to exhibit very different behavior: they may dissipate large amounts of energy through plastic displacements or they may contribute additional stiffness to the overall pier system and make it experience very small displacements. The analyses showed that the same pier exposed to a specific seismic event appears

to have a range of different responses, from elastic to fully plastic, depending on the unit cell shape and its orientation only. For additional energy dissipation the insertion of viscoelastic fillers in the gaps of the cellular shear walls is examined and its efficiency is validated through comparative results.

## REFERENCES

- [1] Gibson L. J., Ashby M. F. (1997), *Cellular Solids: Structure and Properties*, Second Edition, Cambridge University Press, United Kingdom.
- [2] Zhou Q. (2001), *Applications of Cellular Materials and Structures in Vehicle Crashworthiness and Occupant Protection*, Proceedings of the International Symposium on Plasticity and Impact, World Scientific Publishing, Zhuhai China.
- [3] Vian D., Bruneau M. (2005), *Steel Plate Shear Walls for Seismic Design and Retrofit of Building Structures*, Multidisciplinary Center for Earthquake Engineering Research, University at Buffalo, State University of New York, Technical Report MCEER-05-0010.
- [4] Keller D., Bruneau M. (2008), *Development of a Steel Plate Shear Wall Bridge Pier System Conceived from a Multi-Hazard Perspective*, Multidisciplinary Center for Earthquake Engineering Research, University at Buffalo, State University of New York, Technical Report MCEER-08-0030.
- [5] Chen L., Tsopelas P. (2010), *Cellular-Solid Shear Walls under Seismic Excitations*, Proceedings of 9th US National Conference on Earthquake Engineering, Toronto, July 25-29.
- [6] Foti D., Diaferio M., Nobile R. (2010), *Optimal design of a new seismic passive protection device made in aluminium and steel*, *Journal of Structural Engineering and Mechanics*, Vol. 35(1): 119-122.
- [7] Spadoni A. (2008), *Application of chiral cellular materials for the design of innovative components*, Thesis for the for the Degree of Doctor of Philosophy, Georgia Institute of Technology.
- [8] Papka S.D. & Kyriakides S. (1994), *In-plane Compressive Response and Crushing of Honeycomb*. *J. Mech. Phys. Solids* 42(10): 1499-1532.
- [9] Overaker D.W., Cuitino A.M., & Langrana N.A. (1998), *Effect of morphology and orientation on the behavior of two-dimensional hexagonal*, *Mech. Mater.* 29: 43-52.
- [10] ABAQUS/Standard User's Manual, Version 6.14.
- [11] Tsopelas P., Constantinou M.C., Kircher C.A. et al. (1997), *Evaluation of Simplified Methods of Analysis for Yielding Structures*, National Center for Earthquake Engineering Research, University at Buffalo, Technical Report NCEER-97-0012.
- [12] Murray, G., Gandhi, F., & Hayden, E. (2012). *Polymer-filled honeycombs to achieve a structural material with appreciable damping*. *Journal of Intelligent Material Systems and Structures*, 23(6), 703–718.
- [13] Boucher, M. A., Smith, C. W., Scarpa, F., Rajasekaran, R., & Evans, K. E. (2013). *Effective topologies for vibration damping inserts in honeycomb structures*. *Composite Structures*, 106, 1–14. <https://doi.org/10.1016/j.compstruct.2013.05.036>
- [14] Aumjaud, P., Smith, C. W., & Evans, K. E. (2015). *A novel viscoelastic damping treatment for honeycomb sandwich structures*. *Composite Structures*, 119, 322–332. <https://doi.org/10.1016/j.compstruct.2014.09.005>
- [15] Ongaro, F., de Falco, P., Barbieri, E. & Pugno, N. M. (2016). *Mechanics of filled cellular materials*. *Mechanics of Materials*, 97, 26–47. <https://doi.org/10.1016/j.mechmat.2016.01.013>
- [16] Wang, Y. C., Lai, H. W., & Ren, X. J. (2020). *Enhanced Auxetic and Viscoelastic Properties of Filled Reentrant Honeycomb*. *Physica Status Solidi (B) Basic Research*, 257(10). <https://doi.org/10.1002/pssb.201900184>
- [17] Zhang, Ri-Hui, & Soong, T. T. (1992). *Seismic design of viscoelastic dampers for structural*

- applications. *J. Struct. Eng.*, 118, 1375-1392.
- [18] Kasai, K., & Munshi, J. A. (1994). Applications of viscoelastic dampers to high-rise buildings. *J. Struct. Eng.*, 120, 3680-3683.
- [19] Samali, B., & Kwok, K.C.S. (1995). Use of viscoelastic dampers in reducing wind-and earthquake-induced motion of building structures. In *Engineering Structures* (Vol. 17, Issue 9).
- [20] Shimizu, N., Yazaki, T., Nasuno, H., Student, G., & Sunakoda, K. (2002). DESIGN AND ANALYSIS OF VISCOELASTIC SEISMIC DAMPERS. <http://www.asme.org/ab>
- [21] Jung, W. Y., & Aref, A. J. (2003). A combined honeycomb and solid viscoelastic material for structural damping applications. *Mechanics of Materials*, 35(8), 831–844. [https://doi.org/10.1016/S0167-6636\(02\)00210-7](https://doi.org/10.1016/S0167-6636(02)00210-7)
- [22] Symans, M. D., Asce, A. M., Charney, F. A., Asce, F., Whittaker, ; A S, Asce, M., Constantinou, ; M C, Kircher, ; C A, Johnson, ; M W, & Mcnamara, R. J. (2008). Energy Dissipation Systems for Seismic Applications: Current Practice and Recent Developments. <https://doi.org/10.1061/ASCE0733-94452008134:13>
- [23] Shen, K.L. & Soong, T.T (1995). Modeling of viscoelastic dampers for structural applications. *J. Eng. Mech.*, 121, 694-701.