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STUDY OF THE EFFECTS OF SOME VARIABLES ON NON – DIMENSIONAL DEFLECTIONS OF RECTANGULAR LAMINATED DECKS PLATES

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ABSTRACT: Using dynamic relaxation numerical technique (DR), it was decided to undertake some study cases and generate new results for uniformly loaded laminated rectangular decks plates. The plates were assumed to be either simply supported or clamped on all edges. The effects of transverse shear deformation, material anisotropy, orientation, and coupling between stretching and bending on the deflections of laminated plates are investigated.

KEYWORDS: Boundary conditions, rectangular laminated decks plates, load, length to thickness ratio, number of layers, material anisotropy, fiber orientation, aspect ratio, reversing lamination scheme.

1 INTRODUCTION

In materials science engineering, a composite laminate is an assembly of layers of fibrous composite laminated materials one on top of another which can be joined to provide required engineering properties, including in-plane stiffness, out of plane stiffness, bending stiffness, strength, Poisson ratio and coefficient of thermal expansion.

The individual layers consist of high-modulus, high-strength fibers impregnated in an appropriate polymeric, metallic, or ceramic matrix material. Typical fibers used include cellulose, graphite, glass, boron, and silicon carbide, and some matrix materials are epoxies, polyimides, Aluminium, titanium, and alumina.

Layers of different materials may be used, resulting in a hybrid laminate. The individual layers generally are orthotropic (that is, with principal properties in orthogonal directions) or transversely isotropic (with isotropic properties in the transverse plane) with the laminate then exhibiting anisotropic (with variable direction of principal properties), orthotropic, or quasi-isotropic properties. Quasi-isotropic laminates exhibit isotropic (that is, independent of direction) in plane response but are not restricted to isotropic out-of-plane (bending) response. Depending upon the stacking sequence of the individual layers, the laminate may exhibit coupling between in plane and out of plane response. An example of bending-stretching coupling is the presence of curvature developing as a result of in-plane loading.

The properties of a composite laminate depend on the geometrical arrangement and the properties of its constituents. The exact analysis of such structure – property relationship is rather complex because of many variables involved. Therefore, a few simplifying assumptions regarding the structural details and the state of stress within the composite have been introduced.

The deformation of a plate subjected to transverse loading is caused either by flexural deformation due to rotation of cross-sections, or shear deformation due to sliding of sections or layers. The resulting deformation depends on the thickness to length ratio and the ratio of elastic to shear moduli. When the thickness to length ratio is small, the plate is considered thin, and it deforms mainly by flexure or bending; whereas when the thickness to length and the modular ratios are both large, the plate deforms mainly through shear. Due to the high ratio of in-plane modulus to transverse shear modulus, the shear deformation effects are more pronounced in the composite laminates subjected to transverse loads than in the isotropic plates under similar loading conditions. Refer to David Roylance [1], Osama Khayal [2] and [3], Turvey and Mahmoud Yassin Osman [4], [5] and [6].

Mathematical models for rectangular laminated decks plates in bending need to determine the real stress strain state in the laminated plate, which requires the application of more accurate theories. In addition, it is important to find a balance between the desired accuracy and calculation costs.

Different theories for rectangular plate analysis have been reviewed. These theories can be divided into two major categories, the individual layer theories (IL), and the equivalent single layer (ESL) theories. These categories are further divided into sub – theories by the introduction of different assumptions. Refer to Marina Rakočević [7], and Seloodeh A.R and Karami G. [8], Noor A.K. [9], Dorgruoglu A.N., Omurtag M.H. [10], and Huang M.H. [11], Pagano [12], Reddy [13] and Phan and Reddy [14], Reddy [15], and Reddy and Chao [16].

The method of dynamic relaxation in its early stages of development was perceived as a numerical finite difference technique. It was first used to analyze structures, then skeletal and cable structures, and plates. The method relies on a discretized continuum in which the mass of the structure is assumed to be concentrated at given points (i.e. nodes) on the surface. The system of concentrated masses oscillates about the equilibrium position under the influence of out of balance forces. With time, it comes to rest under the influence of damping. The iterative scheme reflects a process, in which static equilibrium of the system is achieved by simulating a pseudo dynamic process in time. In its original form, the method makes use of inertia term, damping term and time increment. Refer to Rushton K.R. [17], Cassell A.C. and Hobbs R.E. [18], Day A.S. [19], and Osama Khayal [20] and [21].

2 BOUNDARY CONDITIONS

The proper boundary conditions for the rectangular laminated decks plates in this study are those which are sufficient to guarantee a unique solution of the governing equations. To achieve that goal, one term of each of the following five pairs must be prescribed along the boundary.

$$N_n \text{ or } u_n \;;\; N_{ns} \text{ or } u_s \;;\; M_n \text{ or } \varphi_n \;;\; M_s \text{ or } \varphi_s \;;\; Q \text{ or } w$$

Where the subscripts n and s indicate the normal and tangential directions respectively. The boundary conditions used in this research article are given in Figures from 1 to 5 for simply supported boundary conditions and in Figures from 6 to 10 for clamped boundary conditions.

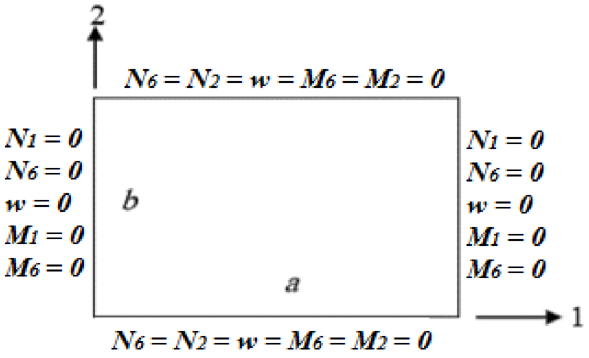

Figure 1. Simply supported (SS1) boundary condition

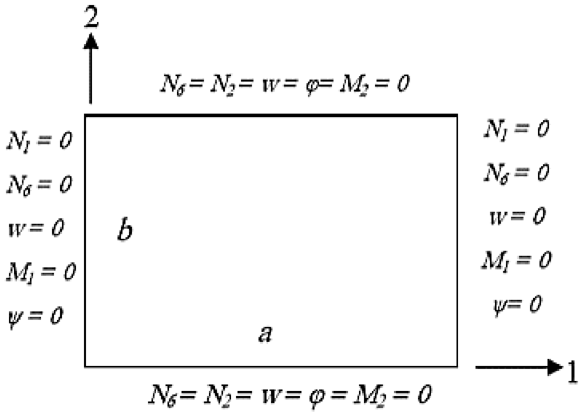

Figure 2. Simply supported (SS2) boundary condition

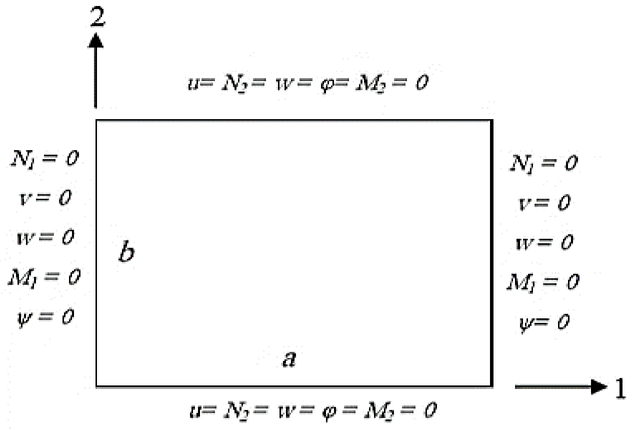

Figure 3. Simply supported (SS3) boundary condition

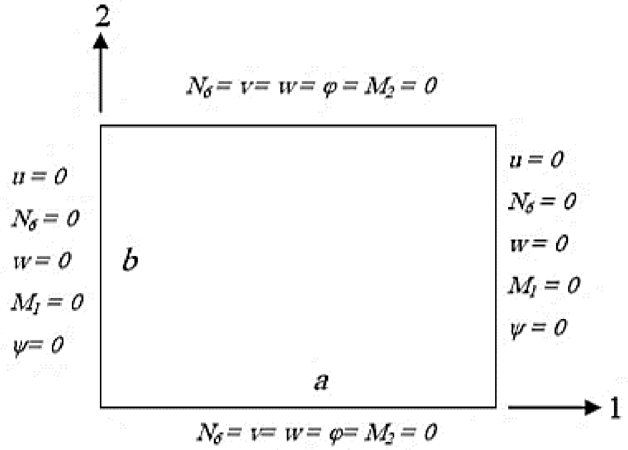

Figure 4. Simply supported (SS4) boundary condition

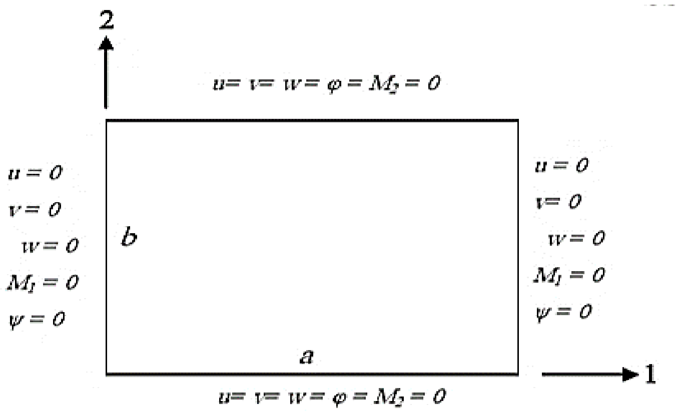

Figure 5. Simply supported in-plane fixed (SS5) boundary condition

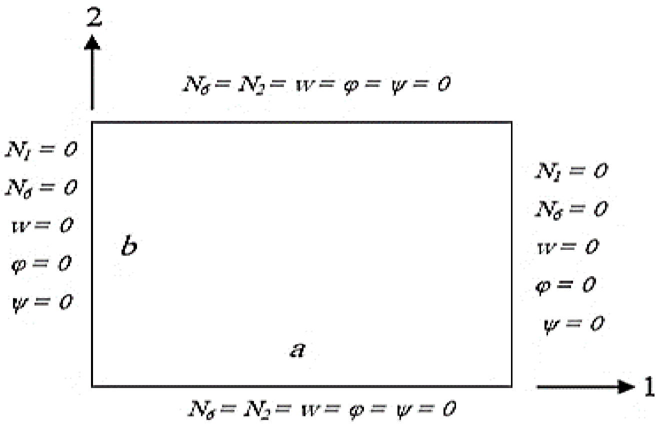

Figure 6. Clamped supported (CC1) boundary condition

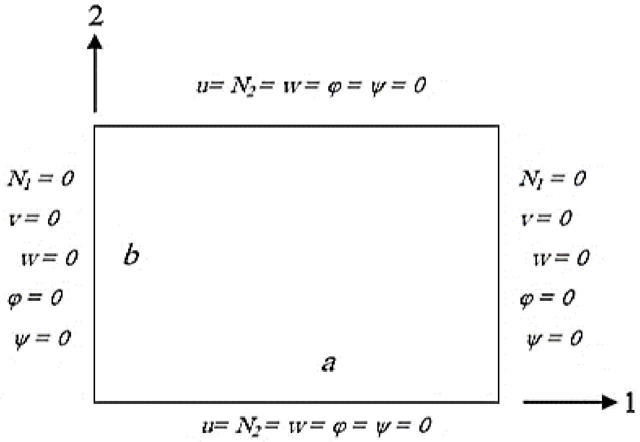

Figure 7. Clamped supported (CC2) boundary condition

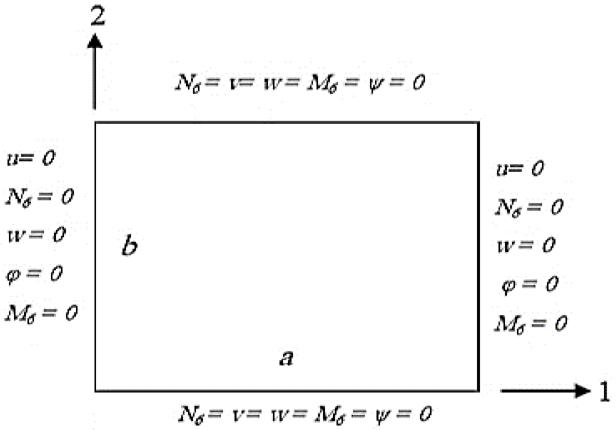

Figure 8. Clamped supported (CC3) boundary condition

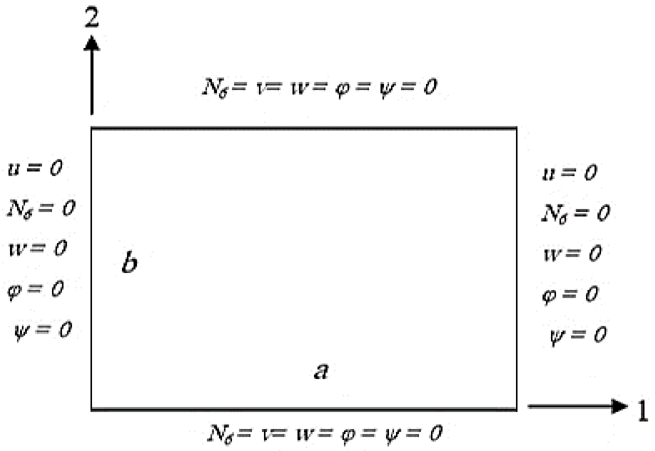

Figure 9. Clamped supported (CC4) boundary condition

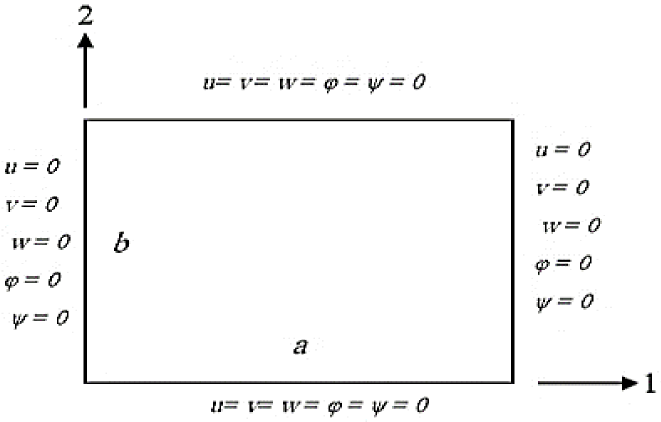

Figure 10. Clamped supported (CC5) boundary condition

3 THE EFFECTS OF SOME VARIABLES ON DECKS PLATES

3.1 Effect of load on decks plates

The variations of the center deflections, $\overline{w}_C$ with load, $\overline{q}$ for thin ($h/a = 0.02$) and thick ($h/a=0.2$) isotropic plates of simply supported in-plane fixed (SS5) condition shown in figure 5 are given in Table 1, and Figure 11. It is observed that, the center deflections of thin and thick plates increase with the applied load, and that the deflections of thick plates are greater than those of thin plates under the same loading conditions. The difference in linear deflection is due to shear deformation effects which are significant in thick plates. Whereas, the non-linear deflection of thin and thick plates, which are nearly coincident, implies that the shear deformation effect vanishes as the load is increased.

Table 1. Variation of central deflection $\overline{w_c}$ with Load, $\bar{q}$ of Thin (h/a = 0.02) and Thick (h/a =0.2) isotropic plates of simply supported (SS5) condition $(\nu = 0.3)$

$\bar{q}$	S	$\overline{w_c}$	
		h/a =0.02	h/a = 0.2
20	1	0.8856	1.0635
	2	0.5846	0.6159
40	1	1.7708	2.1271
	2	0.8432	0.8626
60	1	2.6562	3.1906
	2	1.0138	1.0262
80	1	3.5416	4.2542
	2	1.1447	1.1526
100	1	4.4270	5.3177
	2	1.2527	1.2573
120	1	5.3125	6.3812
	2	1.3455	1.3478
140	1	6.1979	7.4448
	2	1.4275	1.4279
160	1	7.0833	8.5083
	2	1.5012	1.5001
180	1	7.9685	9.578
	2	1.5685	1.5662
200	1	8.8541	10.6354
	2	1.6306	1.6274

S (1): *Linear* , S (2): *Nonlinear*

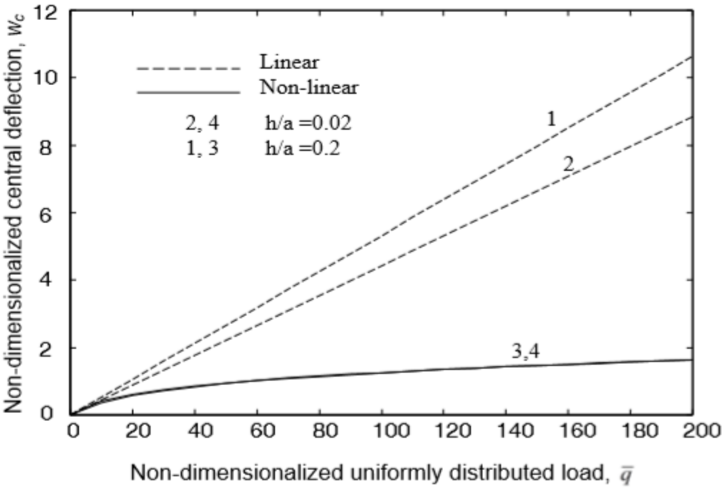

Figure 11. Variation of central deflection, $\overline{w}_c$ with load, $\overline{q}$ of thin (h/a =0.02) and thick (h/a =0.2) simply supported (SS5) square isotropic plate

3.2 Effect of length to thickness ratio on decks plates

Table 2 and Figure 12 contain numerical results and plots of center deflection versus length to thickness ratio of anti-symmetric cross-ply [0°/90°/0°/90°] and angle-ply [45°/-45°/45°/-45°] square plates under uniform lateral load ($\bar{\mathrm{q}} = 1.0$) for two boundary conditions (i.e. simply supported (SS1) and clamped (CC1) as shown in Figures 1 and 6 respectively). The maximum percentage difference in deflections for a range of length / thickness ratio between 10 and 100, fluctuates between 35% for simply supported (SS1) cross-ply laminate and 73.3% for angle-ply laminate as the length/thickness ratio increases to a value of a/h = 40.0, and then become fairly constant. It is evident that shear deformation effect is significant for a/h < 40.0. It is obvious that shear deformation reduces as the length/thickness ratio increases. The orientation effect is clearly noticeable when the plate is simply supported while it is not apparent when the plate is clamped.

As shown in Table 3 and Figure 13, the maximum percentage difference in deflection ($\bar{q} = 200.0$) for a range of length/ thickness ratio between 10 and 100 fluctuates between 6.36% for simply supported (SS1) cross-ply laminate and 38.7% for clamped (CC1) angle-ply laminate. This means that the center deflections become independent on the length/thickness ratio as the load gets larger.

Table 2. Comparison of the non-dimensional center deflections vs. side to thickness ratio of a four layered anti-symmetric cross-ply [0°/90°/0°/90°] and angle-ply [45°/-45°/45°/-45°] square laminates under uniform lateral load ($\bar{q}$ =1.0)

a/h	$\overline{w_c}$			
	SS1		CC1	
	[0°/90°/0°/90°]	[45°/-45°/45°/-45°]	[0°/90°/0°/90°]	[45°/-45°/45°/-45°]
10	0.0148	0.0115	0.0045	0.0048
15	0.0138	0.0102	0.0035	0.0037
20	0.0134	0.0097	0.0032	0.0032
25	0.0133	0.0095	0.0030	0.0030
30	0.0132	0.0094	0.0029	0.0029
35	0.0131	0.0093	0.0029	0.0029
40	0.0131	0.0092	0.0028	0.0028
50	0.0130	0.0092	0.0028	0.0028
80	0.0130	0.0091	0.0027	0.0027
100	0.0130	0.0091	0.0027	0.0027

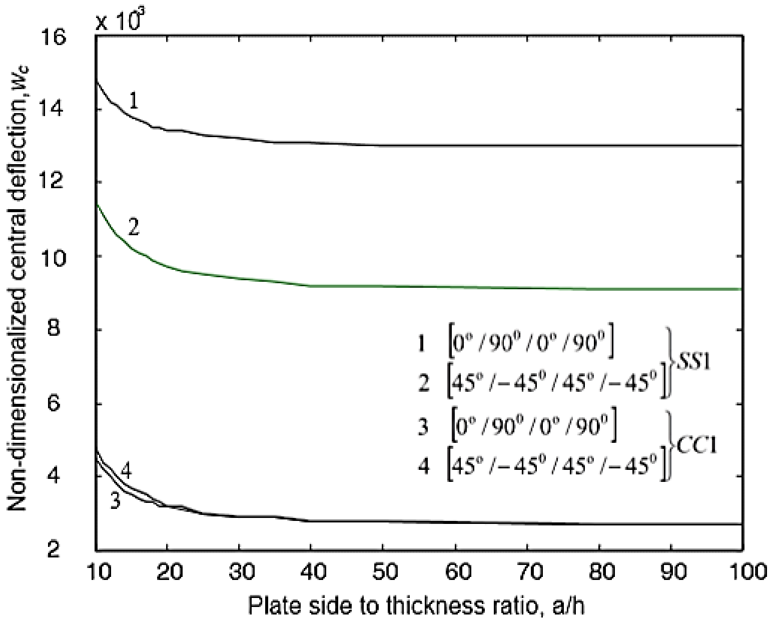

Figure 12. Comparison of center deflections versus side to thickness ratio of anti-symmetric cross-ply and angle-ply square laminates under uniform lateral load

Table 3. Comparison of the non-dimensional center deflections vs. side to thickness ratio of a four layered anti-symmetric cross-ply [0°/90°/0°/90°] and angle-ply [45°/-45°/45°/-45°] square laminates under uniform lateral load ($\bar{q}$ =200.0)

a/h	$\overline{w_c}$			
	SS1		CC1	
	[0°/90°/0°/90°]	[45°/-45°/45°/-45°]	[0°/90°/0°/90°]	[45°/-45°/45°/-45°]
10	1.8682	1.6788	0.8488	0.8874
15	1.8027	1.5700	0.6842	0.7152
20	1.7792	1.5144	0.6225	0.6447
25	1.7682	1.4860	0.5932	0.6092
30	1.7622	1.4697	0.5771	0.5889
35	1.7585	1.4595	05673	05763
40	1.7562	1.4528	0.5609	0.5679
50	1.7534	1.4446	0.5534	0.5578
80	1.7504	1.4356	0.5451	0.5467
100	1.7497	1.4335	05432	0.5440

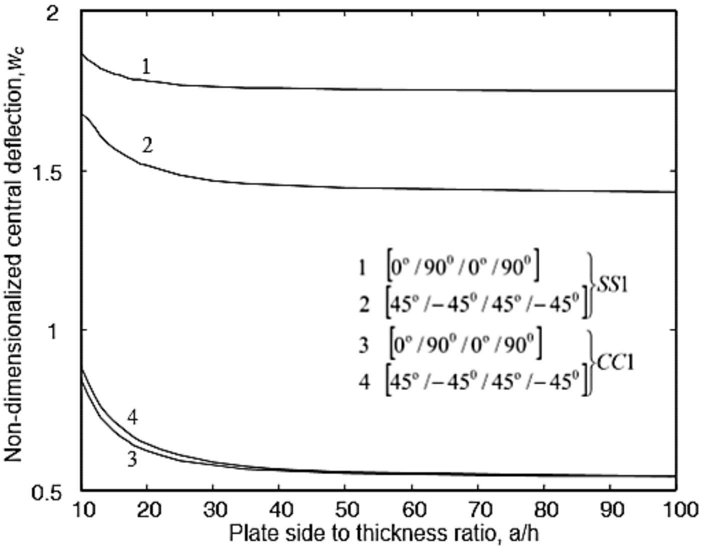

Figure 13. Comparison of center deflections versus side to thickness ratio of anti-symmetric cross-ply and angle-ply square laminates under uniform lateral load

3.3 Effect of number of layers on decks plates

Figure 14 shows a plot of the maximum deflection of a simply supported (SS5) anti-symmetric cross-ply [($0^o/90^o$) $_n$] (*n=1, 2, 3, 4, 8*) square plates under uniformly distributed load of a moderately thick plate (h/a = 0.1).

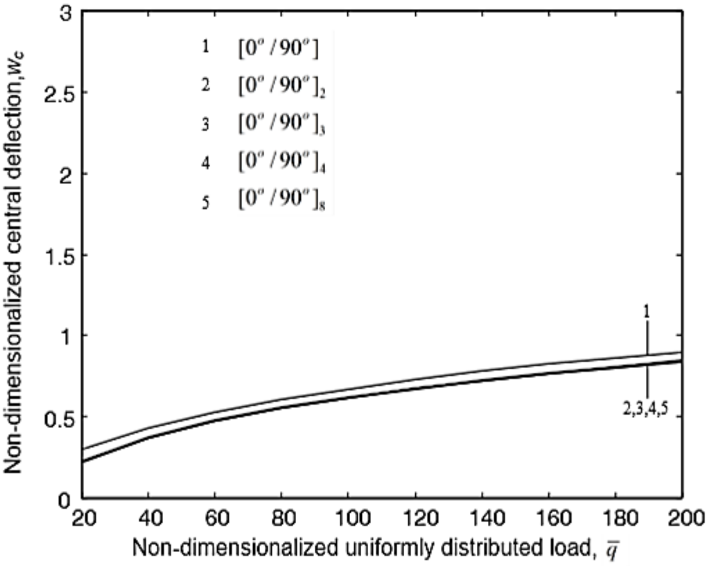

Figure 14. Number of layers effect on a simply supported (SS5) antisymmetric cross-ply [(0^o/ 90^o) n] square plate under uniformly distributed loads (h/a = 0.1)

The numerical results are given in Table 4. Two, four, six, eight, and sixteen-layer laminates are considered. The results show that as the number of layers increases, the plate becomes stiffer and the deflection becomes smaller. This is mainly due to the existence of coupling between bending and stretching which generally increases the stiffness of the plate as the number of layers is increased. When the number of layers exceeds 8, the deflection becomes independent on the number of layers. This is because the effect of coupling between bending and stretching does not change as the number of layers increases beyond 8 layers.

In Table 5 and Figure 15, the deflection of simply supported (SS5) angle-ply plates $[(45^o/-45^o)_n]$ is given. Similar features can be noted as in the case of cross-ply plates $[(0^o/90^o)_n]$ mentioned above.

Table 4. Number of layers effect on a simply supported (SS5) anti-symmetric cross-ply $[(0^o/90^o)_n]$ square plate under uniformly distributed loads (h/a = 0.1)

$\bar{q}$	$\overline{w_c}$				
	$[0^o/90^o]$	$[0^o/90^o]_2$	$[0^o/90^o]_3$	$[0^o/90^o]_4$	$[0^o/90^o]_8$
20	0.2953	0.2278	0.2250	0.2241	0.2232
40	0.4323	0.3769	0.3728	0.3714	0.3702
60	0.5287	0.4807	0.4758	0.4742	0.4727
80	0.6057	0.5605	0.5551	0.5533	0.5517
100	0.6725	0.6258	0.6201	0.6182	0.6165
120	0.7294	0.6815	0.6756	0.6736	0.6718
140	0.7791	0.7304	0.7242	0.7221	0.7202
160	0.8236	0.7740	0.7676	0.7655	0.7636
180	0.8639	0.8136	0.8071	0.8049	0.8029
200	0.9009	0.8500	0.8433	0.8411	0.8390

Subscripted values 2, 3, 4, and 8: No. of the arrangements of a two layered laminate.

Table 5. Number of layers effect on a simply supported (SS5) anti-symmetric angle-ply $[(45^o/-45^{o)\ n}]$ square plate under uniformly distributed loads. (h/a = 0.1)

$\bar{q}$	$\overline{w_c}$				
	$[0^o/90^o]$	$[0^o/90^o]_2$	$[0^o/90^o]_3$	$[0^o/90^o]_4$	$[0^o/90^o]_8$
20	0.2160	0.1637	0.1583	0.1565	0.1549
40	0.3715	0.3009	0.2926	0.2899	0.2875
60	0.4841	0.4103	0.4010	0.3979	0.3951
80	0.5721	0.4993	0.4897	0.4865	0.4835
100	0.6446	0.5740	0.5644	0.5611	0.5582
120	0.7067	0.6384	0.6289	0.6257	0.6228
140	0.7612	0.6953	0.6859	0.6827	0.6798
160	0.8101	0.7462	0.7370	0.7339	0.7310
180	0.8544	0.7924	0.7834	0.7804	0.7775
200	0.8952	0.8349	0.8260	0.8231	0.8203

Subscripted values 2, 3, 4, and 8: No. of the arrangements of a two layered laminate.

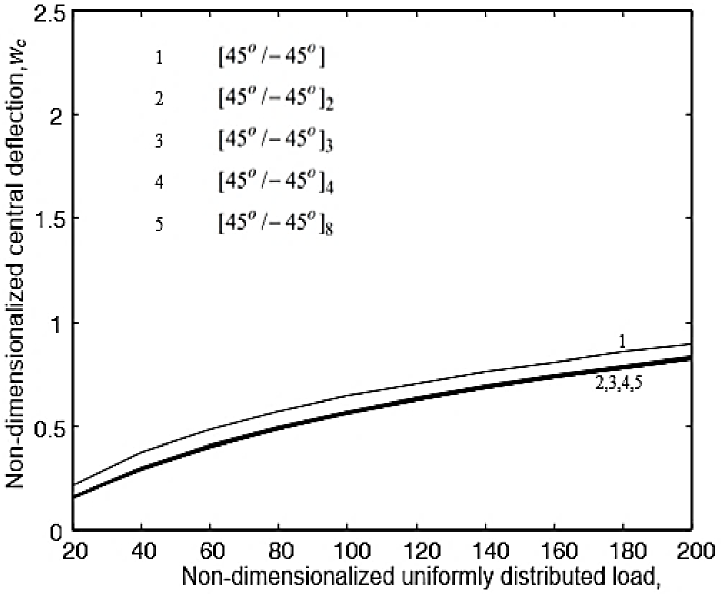

Figure 15. Number of layers effect on a simply supported (SS5) antisymmetric angle ply [(45° / -45°) n] square plate under uniformly distributed loads (h/a = 0.1)

3.4 Effect of material anisotropy on decks plates

According to Whitney and Pagano [22], the severity of shear deformation effects depends on the material anisotropy, E_1/E_2 of the layers.

The exact maximum deflections of clamped supported plate (CC5) is illustrated in Figure 10. Four-layer symmetric cross-ply [0°/90°/90°/0°] and angle-ply [45°/-45°/-45°/45°] laminates are compared in Table 6 and Figure 16 for various degrees of anisotropy. It is observed that, when the degree of anisotropy is small the deflection is large. As the degree of the anisotropy increases, the plate becomes stiffer. This may be attributed to the shear deformation effects which increase as the material anisotropy is decreased. When the degree of anisotropy becomes greater than 40.0, the deflection becomes approximately independent on the degree of anisotropy. This is due to the diminishing of the shear deformation effects and the dominance of bending effects.

The results in Table 7 and the plot in Figure 17 is for simply supported (SS5) laminates which follow a similar behavior but the deflections are relatively smaller. The apparent difference between the non-linear deflections of both clamped (CC5) and simply supported (SS5) symmetric laminates, as shown in Figures 16 and 17 may be attributed to the different boundary conditions used in each case which either permits edge rotation or prohibits it.

Table 6. Effect of material anisotropy on the non-dimensional center deflections of a four layered symmetric cross-ply and angle-ply clamped laminates (CC5) under uniform lateral load ($\overline{q}$ =100.0, h/a = 0.1)

E_1/E_2	$\overline{w_c}$	
	[0°/90°/90°/0°]	[45°/-45°/-45°/45°]
2	0.8211	0.8318
4	0.6574	0.6882
6	0.5631	0.6006
8	0.5015	0.5408
10	0.4580	0.4970
12	0.4254	0.4633
14	0.4000	0.4364
20	0.3485	0.3804
25	0.3210	0.3498
30	0.3010	0.3273
35	0.2876	0.3099
40	0.2732	0.2959
45	0.2631	0.2845
50	0.2545	0.2748

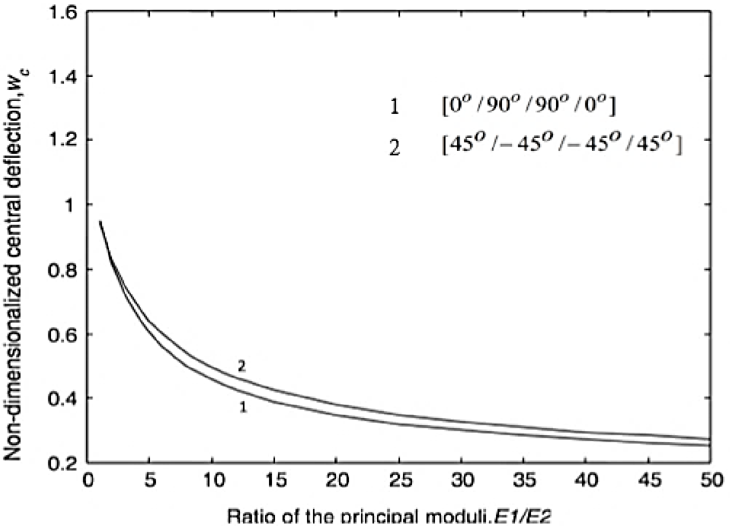

Figure 16. Effect of material anisotropy on the non-dimensional center deflections of a four layered symmetric cross-ply and angle-ply clamped laminates (CC5) under uniform lateral load ($\overline{q}$ = 100.0, h/a = 0.1)

Table 7. Effect of material anisotropy on the non-dimensional center deflections of a four layered symmetric cross-ply and angle-ply simply supported laminates (SS5) under uniform lateral load ($\overline{q}$ =100.0, h/a = 0.1)

E_1/E_2	$\overline{w_c}$	
	[0°/90°/90°/0°]	[45°/-45°/-45°/45°]
2	1.1114	1.1128
4	0.9424	0.9397
6	0.8362	0.8272
8	0.7610	0.7466
10	0.7041	0.6851
12	0.6589	0.6362
14	0.6218	0.5962
20	0.5410	0.5098
25	0.4944	0.4609
30	0.4589	0.4242
35	0.4306	0.3955
40	0.4076	0.3724
45	0.3883	0.3534
50	0.3718	0.3374

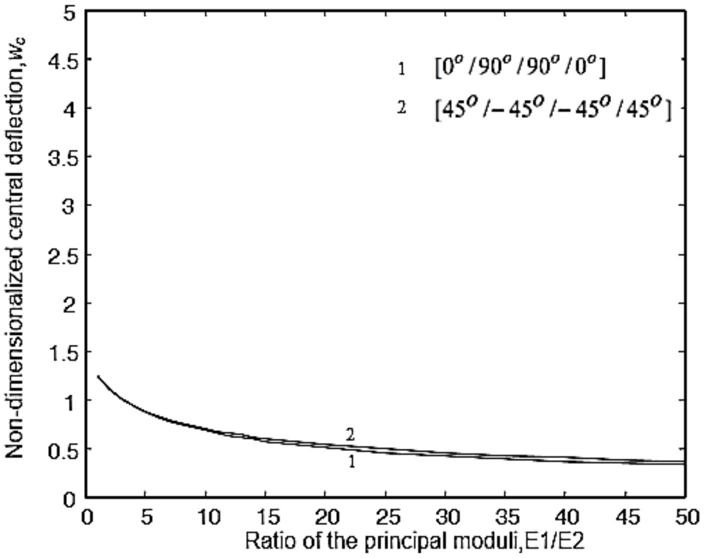

Figure 17. Effect of material anisotropy on the non-dimensional center deflections of a four layered symmetric cross-ply and angle-ply simply supported laminates (SS5) under uniform lateral load ($\overline{q}$ = 100.0, h/a = 0.1)

3.5 Effect of fiber orientation on decks plates

The variation of the maximum deflection, $\overline{w}_C$ with fiber orientation of a square laminated plate is shown in Table 8 and Figure 18 for $\overline{q} = 120.0$, and *h/a = 0.1*. Four simply supported boundary conditions SS2, SS3, SS4 which are shown in Figures 2, 3 and 4 and SS5 are considered in this case. The non-linear curves SS2 and SS3 conditions show minimum deflection at $\theta = 45^o$. However, this trend is different for a plate under SS4 and SS5 conditions in which the non-linear deflection increases with θ. This is due to the in-plane fixed edges in the latter case. Also, the non-linear curves for clamped boundary conditions CC1, and (CC3, CC4 which are shown in Figures 8 and 9) and CC5 as shown in Table 9 and Figure 19 indicate the same trend as in the simply supported SS4 and SS5. These differences indicate that the type of end support is a determinant factor in the deflections for different orientations.

Table 8. Effects of fiber orientation θ on the deflection of a simply supported square plate ($\overline{q}$ = 120.0, h/a = 0.1)

θ	$\overline{w_c}$			
	SS2	SS3	SS4	SS5
0	1.3706	1.2346	0.6511	0.6513
5	1.3671	1.2274	0.6655	0.6537
10	1.3560	1.2074	0.7011	0.6606
15	1.3359	1.1769	0.7434	0.6713
20	1.3070	1.1366	0.7805	0.6843
25	1.2752	1.0876	0.8060	0.6979
30	1.2438	1.0321	0.8173	0.7101
35	1.2129	0.9745	0.8161	0.7194
40	1.1898	0.9259	0.8089	0.7249
45	1.1815	0.9056	0.8049	0.7267
50	1.1898	0.9259	0.8089	0.7249
55	1.2129	0.9745	0.8161	0.7194
60	1.2438	1.0321	0.8173	0.7101
65	1.2752	1.0876	0.8060	0.6979
70	1.3070	1.1366	0.7805	0.6843
75	1.3359	1.1769	0.7434	0.6713
80	1.3560	1.2074	0.7011	0.6606
85	1.3671	1.2274	0.6655	0.6537
90	1.3706	1.2346	0.6511	0.6513

Another set of results showing the variation of center deflections, $\overline{w}_C$ with Load, $\overline{q}$ for a range of orientations is given in Tables 10 and 11, and Figures 20 and 21. Table 10 and Figure 20 show the variations in the center deflection of thick laminates (*h/a=0.2*) with load ranges between $\overline{q} = 20.0$ and $\overline{q} = 200.0$

for a simply supported (SS4), 4–layer anti-symmetric square plate of orientation $[\theta^o / -\theta^o / \theta^o / -\theta^o]$.

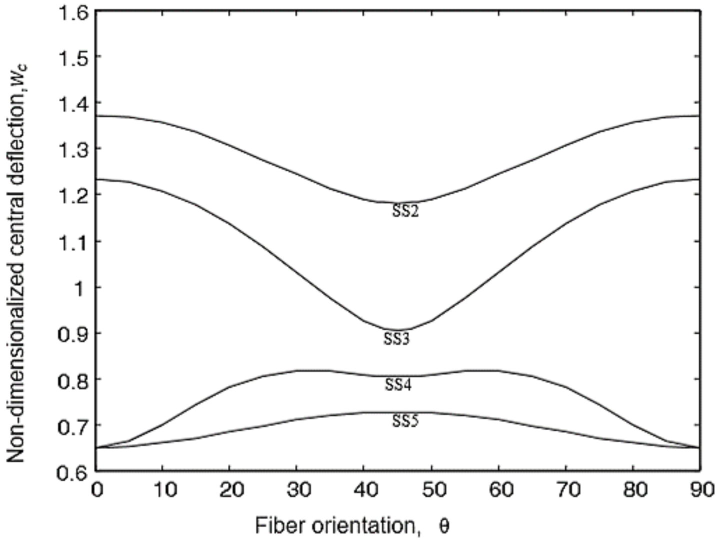

Figure 18. Effects of fiber orientation, θ on the deflection of a simply supported square plate ($\bar{q}$ = 120.0, h/a = 0.1)

Table 9. Effects of fiber orientation θ on the deflection of a clamped square plate *($\bar{q}$ = 120.0, h/a = 0.1)*

θ	$\overline{w_c}$				
	CC1	CC2	CC3	CC4	CC5
0	0.5895	0.5815	0.4713	0.4709	0.4708
5	0.5920	0.5835	0.4789	0.4778	0.4730
10	0.5995	0.5896	0.4985	0.4952	0.4793
15	0.6110	0.5992	0.5242	0.5173	0.4895
20	0.6254	0.6106	0.5514	0.5400	0.5027
25	0.6419	0.6212	0.5764	0.5606	0.5178
30	0.6584	0.6279	0.5960	0.5769	0.5331
35	0.6712	0.6280	0.6078	0.5871	0.5467
40	0.6788	0.6223	0.6118	0.5908	0.5561
45	0.6813	0.6183	0.6122	0.5913	0.5594
50	0.6788	0.6223	0.6118	0.5908	0.5561
55	0.6712	0.6280	0.6078	0.5871	0.5467
60	0.6584	0.6279	0.5960	0.5769	0.5331
65	0.6419	0.6212	0.5764	0.5606	0.5178
70	0.6254	0.6106	0.5514	0.5400	0.5027
75	0.6110	0.5992	0.5242	0.5173	0.4895
80	0.5995	0.5896	0.4985	0.4952	0.4793
85	0.5920	0.5835	0.4789	0.4778	0.4730
90	0.5895	0.5815	0.4713	0.4709	0.4708

It is noticed from Figure 20 that the deflection of thick laminates increases with the applied load as the angle of orientation is decreased (i.e. from 45° to 0°) to a point where $60 < \bar{q} \le 70$ and then increases as the angle of orientation is increased beyond that point. This results in the inflection of the deflection curves at a point where $60 < \bar{q} \le 70$. This behavior is caused by coupling between bending and stretching which arises as the angle of orientation increases.

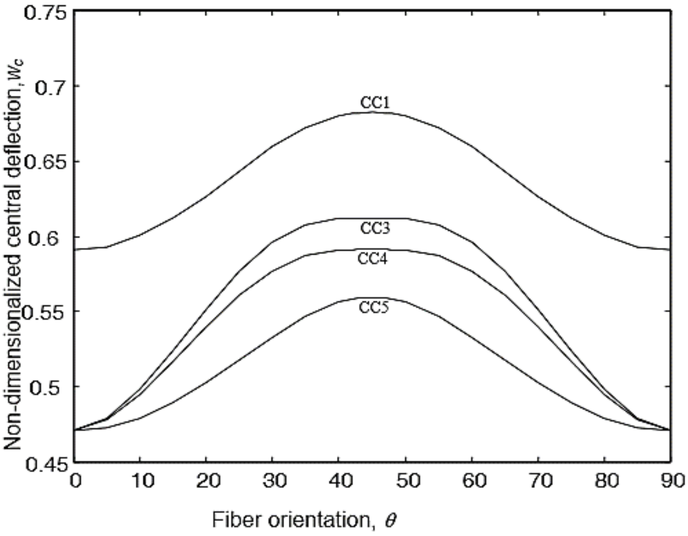

Figure 19. Effects of fiber orientation, θ on the deflection of a clamped square plate ($\bar{q}$ =120.0, h/a = 0.1)

Table 10. Variation of central deflection $\overline{w_c}$ *with a high pressure range* $\bar{q}$ *of a simply supported (SS4) four-layered anti-symmetric square plate of the arrangement* $[\theta^\circ / -\theta^\circ / \theta^\circ / -\theta^\circ]$ *with different orientations* (h/a =0.2)

$\bar{q}$	$\overline{w_c}$			
	θ= 0° or 90°	θ= 15° or 75°	θ = 30° or 60°	θ= 45°
20	0.2922	0.2799	0.2568	0.2466
40	0.4268	0.4209	0.4098	0.4039
60	0.5150	0.5141	0.5141	0.5135
80	0.5826	0.5853	0.5943	0.5984
100	0.6382	0.6438	0.6603	0.6685
120	0.6859	0.6940	0.7169	0.7286
140	0.7281	0.7382	0.7667	0.7816
160	0.7660	0.7779	0.8114	0.8292
180	0.8007	0.8141	0.8521	0.8725
190	0.8170	0.8311	0.8712	0.8929
200	0.8326	0.8475	0.8896	0.9124

Similar behavior is exhibited by thick anti-symmetric clamped (CC3) laminates as shown in Table 11 and Figure 21 but with a low response due to the different boundary conditions used in each case.

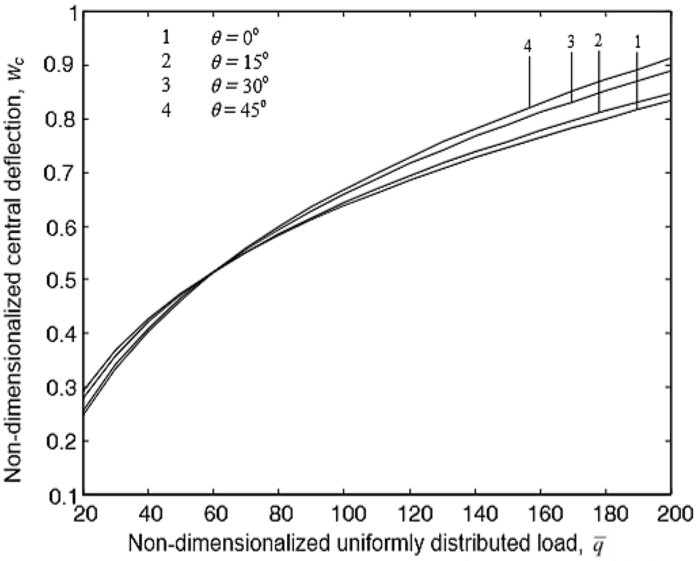

Figure 20. Variation of central deflection, with pressure, of simply supported (SS4) antisymmetric square plate with different orientations (h/a =0.2)

Table 11. Variation of central deflection $\overline{w_c}$ with a high pressure range $\bar{q}$ of clamped (CC3) four-layered anti-symmetric square plate of the arrangement $[\theta^\circ/-\theta^\circ/\theta^\circ/-\theta^\circ]$ with different orientations (h/a =0.2)

$\bar{q}$	$\overline{w_c}$			
	θ= 0° or 90°	θ= 15° or 75°	θ= 30° or 60°	θ= 45°
20	0.2136	0.2125	0.2064	0.2003
40	0.3521	0.3553	0.3572	0.3531
60	0.4478	0.4550	0.4667	0.4668
80	0.5211	0.5317	0.5521	0.5564
100	0.5812	0.5946	0.6224	0.6307
120	0.6324	0.6483	0.6826	0.6944
140	0.6774	0.6954	0.7355	0.7505
160	0.7177	0.7376	0.7828	0.8007
180	0.7543	0.7759	0.8257	0.8464
200	0.7880	0.8111	0.8652	0.8883

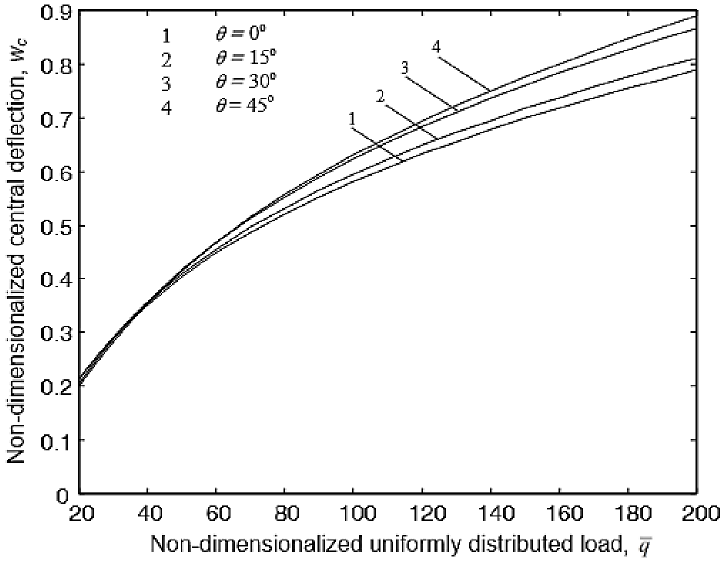

Figure 21. Variation of central deflection, with pressure, of clamped (CC3) antisymmetric square plate with different orientations (h/a = 0.2)

3.6 Effect of reversing lamination order on decks plates

The DR deflections of two-layer anti-symmetric cross-ply [0°/90°] simply supported in-plane fixed (SS5) rectangular laminates are given in Table 12 and plotted in Figure 22. The deflection of the plate with coupling stiffness $(B_{ij}=0)$ is also shown for the sake of comparison. The percentage difference between the center deflections $\overline{w}_1[0^o/90^o]$ and $\overline{w}_2[90^o/0^o]$ at $\overline{\mathrm{q}}=20.0$ is 146.5%.whilst when $\overline{q}=200.0$, it is 54.1%. It is obvious that the deflection depends on the direction of the applied load or the arrangement of the layers.

The coupling stiffness $(B_{ij}=0)$ serves as the limit between positive and negative coupling. For a positive coupling, the deflection increases as the magnitude of coupling increases. In other words, the apparent laminate bending stiffness decreases as the bending – extension coupling increases. Whereas, negative coupling is seen to stiffen the laminate. This contradicts the common notion that the bending – extension coupling lowers the laminate bending stiffness.

In a similar analysis, the deflection of an anti-symmetric angle-ply $\overline{w}_1[45^o/-45^o]$ and $\overline{w}_2[-45^o/45^o]$ simply supported in-plane fixed (SS5) laminates are shown in Table 13 and Figure 23. There is no difference in deflection between $[45^o/-45^o]$ and $[-45^o/45^o]$ as in the case of $[0^o/90^o]$ and $[90^o/0^o]$. This comparison with laminate $(B_{ij}=0)$ indicates that coupling between bending and twisting always lowers the laminate bending stiffness of angle-ply laminates.

Table 12. Central deflection of a two layer anti-symmetric cross-ply simply supported in-plane fixed (SS5) rectangular plate under uniform pressure (b/a = 5.0, h/a = 0.1)

$\overline{q}$	$\overline{w}_1$ [$0^o/90^0$]	$\overline{w}_2$ [$90^o/0^0$]	$\overline{w}_0$ (B_{ij} =0)	%S(1)	%S(2)	%S(3)
20	0.7051	0.2860	0.3387	108.2	-15.6	146.5
25	0.7599	0.3260	0.3879	95.9	-16.0	133.1
30	0.8052	0.3616	0.4303	87.1	-16.0	122.9
35	0.8442	0.3931	0.4677	80.5	-16.0	114.8
40	0.8787	0.4221	0.5013	75.3	-15.8	108.2
50	0.9380	0.4738	0.5599	67.5	-15.4	98.0
60	0.9884	0.5191	0.6103	62.0	-14.9	90.4
70	1.0325	0.5597	0.6546	57.7	-14.5	84.5
80	1.0721	0.5966	0.6945	54.4	-14.1	79.7
100	1.1412	0.6620	0.7641	49.4	-13.4	72.4
120	1.2007	0.7192	0.8241	45.7	-12.7	66.9
140	1.2534	0.7702	0.8772	42.9	-12.2	62.7
160	1.3009	0.8166	0.9250	40.6	-11.7	59.5
180	1.3444	0.8592	0.9686	39.8	-11.3	65.5
200	1.3846	0.8988	1.0089	37.2	-10.9	54.1

S (1):100 × ($\overline{w}_1$ - $\overline{w}_0$)/ $\overline{w}_0$, S (2):100 × ($\overline{w}_2$ - $\overline{w}_0$)/ $\overline{w}_0$, S (3):100 × ($\overline{w}_1$ - $\overline{w}_2$)/ $\overline{w}_2$

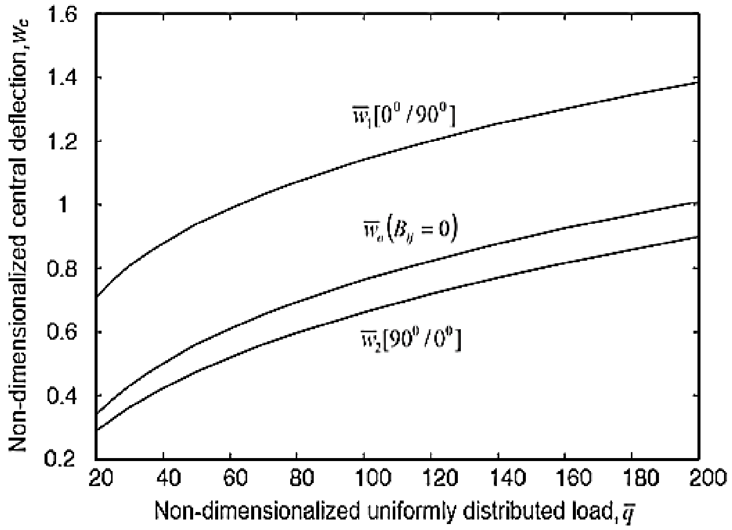

Figure 22. Central deflection of a two layer antisymmetric cross-ply simply supported (SS5) Rectangular Plate under Uniform Pressure (b/a = 5.0, h/a = 0.1)

Table 13. Central deflection of a two layer anti-symmetric angle-ply simply supported in-plane fixed (SS5) rectangular plate under uniform pressure *(b/a = 5.0, h/a = 0.1)*

$\bar{q}$	$\overline{w_1}$ [45°/-45°] = $\overline{w_2}$ [-45°/45°]	$\overline{w_0}$ (B_{ij} =0)	%S(1)= %S(2)	%S(3)
20	0.4788	0.4503	6.3	0.0
25	0.5348	0.5082	5.2	0.0
30	0.5827	0.5578	4.5	0.0
35	0.6250	0.6013	3.9	0.0
40	0.6629	0.6404	3.5	0.0
50	0.7292	0.7084	2.9	0.0
60	0.7863	0.7669	2.5	0.0
70	0.8367	0.8184	2.2	0.0
80	0.8821	0.8648	2.0	0.0
100	0.9618	0.9459	1.7	0.0
120	1.0308	1.0160	1.5	0.0
140	1.0919	1.0780	1.3	0.0
160	1.1472	1.1340	1.2	0.0
180	1.1978	1.1852	1.1	0.0
200	1.2445	1.2324	1.0	0.0

S (1):100 × ($\overline{w_1}$ - $\overline{w_0}$)/ $\overline{w_0}$, S (2):100 × ($\overline{w_2}$ - $\overline{w_0}$)/ $\overline{w_0}$, S (3):100 × ($\overline{w_1}$ - $\overline{w_2}$)/ $\overline{w_2}$

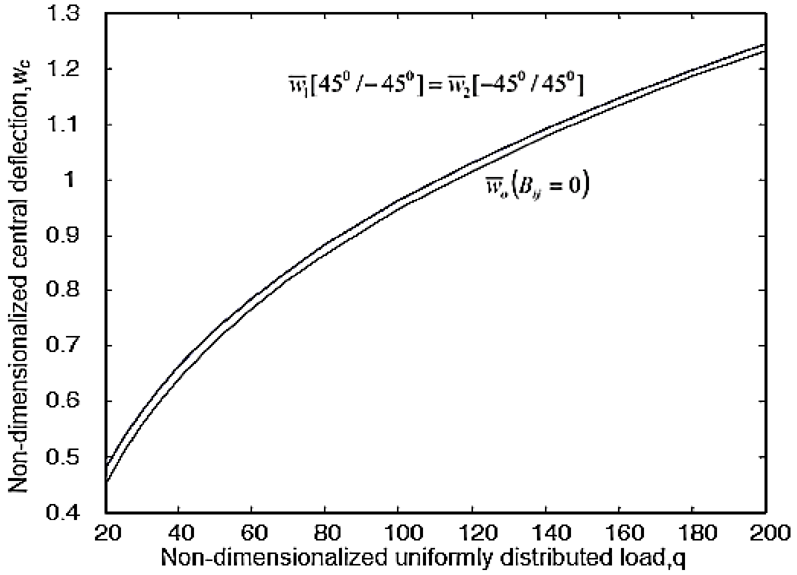

Figure 23. Central deflection of a two layer antisymmetric angle-ply simply supported (SS5) rectangular plate under uniform pressure (b/a = 5.0, h/a = 0.1)

3.7 Effect of aspect ratio on decks plates

Table 14. Central deflection of a two layer anti-symmetric cross-ply and angle-ply simply supported in-plane fixed (SS5) rectangular plate under uniform pressure and with different aspect ratios (h/a = 0.1, $\overline{q}$ =200.0)

b/a	$\overline{w}$	
	[0^o/90^0]	[45^o/-45^0]
5.00	1.3846	1.2445
4.00	1.3848	1.2448
3.00	1.3854	1.2431
2.50	1.3838	1.2370
2.00	1.3679	1.2145
1.90	1.3594	1.2055
1.80	1.3473	1.1940
1.75	1.3395	1.1871
1.70	1.3303	1.1793
1.60	1.3067	1.1606
1.55	1.2919	1.1494
1.50	1.2745	1.1369
1.45	1.2544	1.1227
1.40	1.2311	1.1069
1.35	1.2044	1.0891
1.30	1.1740	1.0693
1.25	1.1394	1.0471
1.20	1.1006	1.0225
1.00	0.9009	0.8952

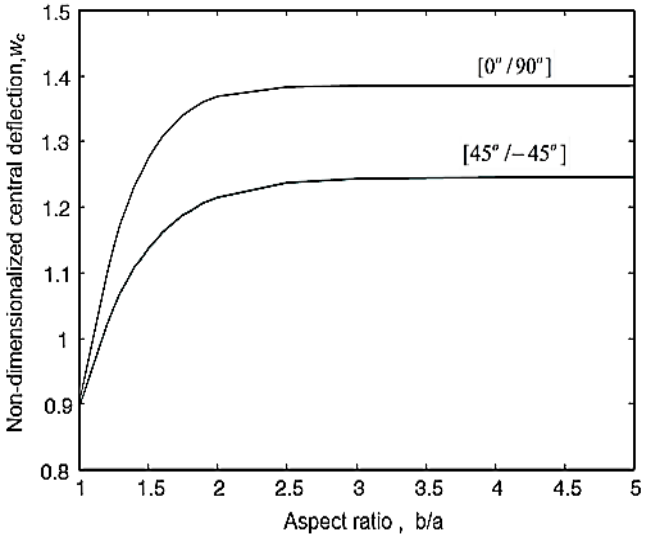

Figure 24. Central deflection of a two layer antisymmetric cross-ply and angle-ply simply supported (SS5) rectangular plate under uniform pressure and with different aspect ratios (h/a = 0.1, = 200.0).

Table 14 and correspondingly Figure 24 show the variations in the maximum deflection of a two-layer anti-symmetric cross-ply and angle-ply $\left[45^o/-45^o\right]$ simply supported in-plane fixed (SS5) rectangular laminate under uniform load and with different aspect ratios ($\overline{q} = 200.0$, and h/a =0.1).

It is noticeable that, when the aspect ratio is small the deflection is small, and as the aspect ratio increases further beyond 2.0, the deflection becomes independent on the aspect ratio. This is due to coupling between bending and stretching which becomes fairly constant beyond b/a=2.0 and therefore the plate behaves as a beam.

3.8 Effect of boundary conditions on decks plates

The type of boundary support is an important factor in determining the deflections of a plate along with other factors such as the applied load, the length / thickness ratio, the fiber orientation, etc.

Three sets of boundary conditions ranging between extreme in-plane fixed to in-plane free of an isotropic plate were considered and the results are given in Table 15 and shown graphically in Figure 25. The variations of center deflection, $\overline{w}_C$ with load**,** $\overline{q}$ for thin (h/a =0.02) isotropic simply supported (SS1) and (SS5) and clamped (CC5) plates are given. It is observed that, for all cases the deflections increase with the load but at different rates depending on whether the plate is simply supported in-plane free or clamped. The deflection is a maximum when the plate is simply supported in-plane free and a minimum when the plate is clamped.

Table 15. Variations of center deflection $\overline{w_c}$ with load, $\overline{q}$ of simply supported (SS1) and (SS5), and clamped (CC5) thin isotropic plates (h/a = 0.02, ν = 0.3)

$\overline{q}$	$\overline{w_c}$		
	SS1	SS5	CC5
10	0.4763	0.3688	0.1301
20	0.8582	0.5846	0.2576
30	1.1647	0.7310	0.3754
40	1.4192	0.8200	0.4803
50	1.6382	0.9351	0.5728
60	1.8318	1.0138	0.6548
70	2.0065	1.0828	0.7281
80	2.1662	1.1447	0.7943
90	2.3210	1.2010	0.8546
100	2.4692	1.2527	0.9101

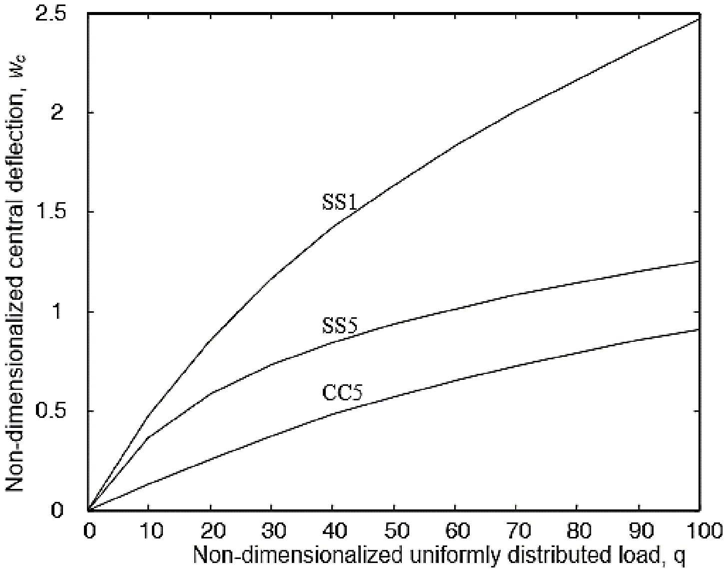

Figure 25. Variations of central deflection, with load, of thin (h/a =0.02) isotropic simply supported (SS1) and (SS5), and clamped (CC5) conditions (ν = 0.3)

3.9 Effect of lamination scheme on decks plates

In the present analysis the lamination scheme of plates is either symmetric or anti-symmetric. The anti-symmetric arrangement involves coupling between bending and stretching which affects greatly the deflections of both cross-ply and angle-ply laminates.

The variations of center deflection, $\overline{w}_C$ with load, $\overline{q}$ varying between 0 and 100 are given in Tables 16 and 17 and shown graphically in Figures 26 and 27. The transverse central deflection of 4- layered square laminated plates with simply supported (SS2) boundary condition subjected to uniformly distributed load is shown in Table 16 and Figure 26. The thickness of all layers is assumed equal. The results indicate that the anti-symmetric angle-ply [45°/-45°/45°/-45°] laminate is stiffer than the symmetric one, and that the symmetric cross-ply laminate is stiffer than the anti-symmetric one. This phenomenon is caused by coupling between bending and stretching which lowers the deflections of anti-symmetric angle-ply laminates, and raises the deflections of anti-symmetric cross-ply plates.

Similar behavior is shown by angle-ply laminates for clamped (CC2) condition which is shown in Figure 7. In the case of cross-ply laminates as given in Table 17 and shown in Figure 27 the anti-symmetric cross-ply is stiffer than the symmetric one. This is due to the restrained edge rotation in this case. Also, the results indicate that the anti-symmetric angle-ply laminate is stiffer than the symmetric one, and that the symmetric cross-ply laminate is stiffer than the anti-symmetric one.

Table 16. Variation of central deflection $\overline{w_c}$ with pressure $\overline{q}$ of a Simply supported (SS2) four-layered anti-symmetric and symmetric cross-ply and angle-ply square plate (h/a = 0.1)

$\overline{q}$	$\overline{w_c}$			
	[0°/90°/0°/90°]	[0°/90°/90°/0°]	[45°/-45°/45°/-45°]	[45°/-45°/-45°/ 45°]
10	0.1410	0.1299	0.0900	0.0934
20	0.2792	0.2577	0.1794	0.1862
30	0.4142	0.3814	0.26278	0.2777
40	0.5382	0.4999	0.3548	0.3674
50	0.6570	0.6126	0.4399	0.4548
60	0.7685	0.7192	0.5229	0.5398
70	0.8730	0.8200	0.6038	0.6221
80	0.9713	0.9154	0.6824	0.7016
90	1.0637	1.0057	0.7587	0.7785
100	1.1511	1.0915	0.8327	0.8528

Table 17. Variation of central deflection $\overline{w_c}$ with pressure $\overline{q}$ of Clamped (CC2) four-layered anti-symmetric and symmetric cross-ply and angle-ply square plate (h/a = 0.1)

$\overline{q}$	$\overline{w_c}$			
	[0°/90°/0°/90°]	[0°/ 90°/90°/0°]	[45°/-45°/45°/ -45°]	[45°/ -45°/-45°/ 45°]
10	0.0450	0.0457	0.0478	0.0499
20	0.0900	0.0913	0.0954	0.0997
30	0.1349	0.1368	0.1426	0.1489
40	0.1797	0.1822	0.1891	0.1971
50	0.2243	0.2274	0.2346	0.2441
60	0.2686	0.2724	0.2787	0.2895
70	0.3126	0.3169	0.3214	0.3331
80	0.3563	0.3611	0.3625	0.3749
90	0.3995	0.4048	0.4021	0.4149
100	0.4422	0.4479	0.4400	0.4532

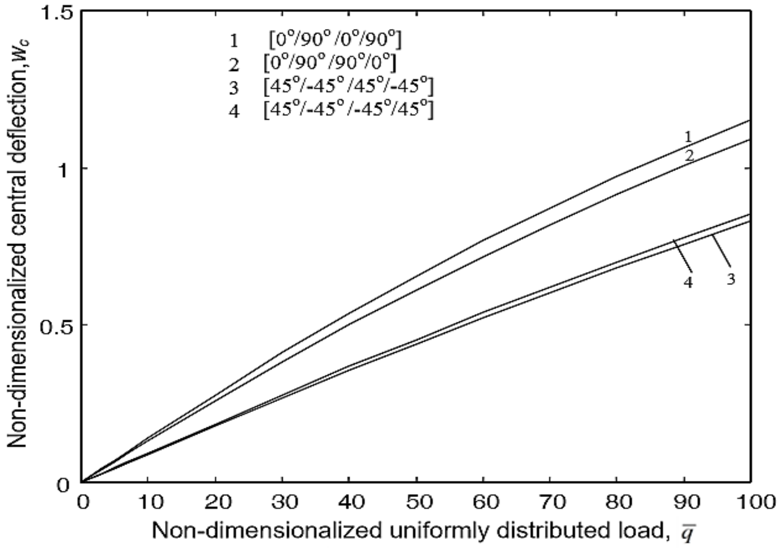

Figure 26. Variation of central deflection, $\overline{w_c}$ with pressure, of simply supported (SS2) 4-layered antisymmetric and symmetric cross-ply and angle-ply square laminate (h/a = 0.1)

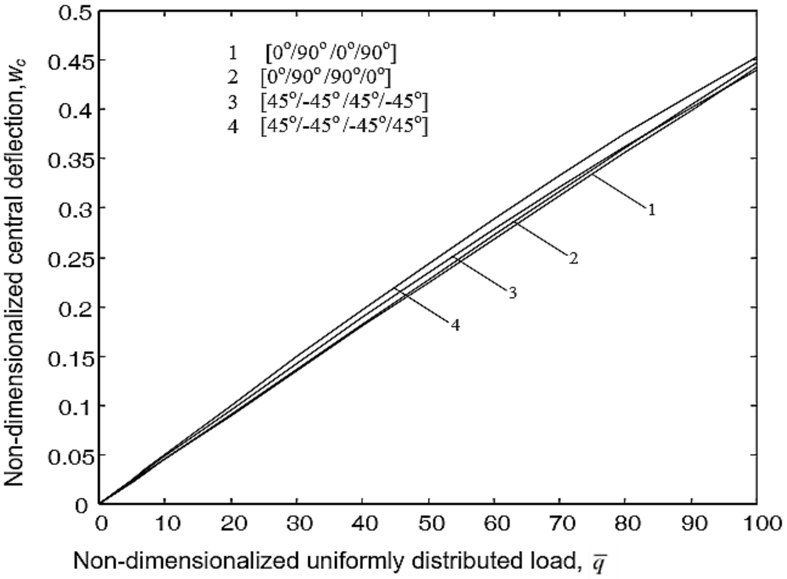

Figure 27. Variation of central deflection, $\overline{w_c}$ with pressure, of clamped (CC2) 4-layered anti-symmetric and symmetric cross-ply and angle-ply square laminate (h/a = 0.1)

4 CONCLUSIONS

A Dynamic Relaxation (DR) program based on finite differences has been developed for small and large deflection analysis of rectangular laminated decks plates using first order shear deformation theory (FSDT). The plate, which is assumed to consist of a number of orthotropic layers, is replaced by a single anisotropic layer and the displacements are assumed linear through the thickness of the plate. A series of new results for uniformly loaded thin, moderately thick, and thick plates with simply supported and clamped edges have been presented. These results show the following: The linear theory seriously over-predicts the deflection of plates; the deformations of a plate are dependent on bending and extension in the nonlinear theory, whereas they are dependent on bending alone in the linear theory; the convergence of the DR solution depends on several factors including boundary conditions, mesh size, the fictitious densities, and load; deflection is greatly dependent on plate length/ thickness ratio at small loads, and it becomes almost independent on that when the load is large; as the number of layers in a plate increases, the plate becomes increasingly stiffer; also, as the degree of anisotropy increases, the plate becomes stiffer and when it is greater than 40.0, the deflection becomes virtually independent on the degree of anisotropy; deflection of plates depends on the angle of orientation of individual plies. An increase of angle of orientation results in a decrease in the deflection at small loads and an increase in deflection at large loads; coupling between bending and stretching increases the deflection of [0°/90°] and decreases the deflection of [90°/0°] plates depending on whether it is positive or negative. Whereas, it always decreases the deflection of [45°/-45°] and [-45°/45°] plates. It also lowers the deflection of anti-symmetric angle-ply laminate [45°/-45°/45°/-45°] and increases that of anti-symmetric cross-ply laminate [0°/90° /0°/90°]; deflection depends on the aspect ratio of plate. When the aspect ratio becomes greater than 2.0, the plate behaves as a beam, and therefore the deflection becomes independent on the aspect ratio; and as the edges of a plate are more restrained, the deflection decreases.

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