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INTRODUCTION TO COMPOSITE DECKS FIBROUS LAMINATES

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ABSTRACT: In materials science, a composite laminate used in decks industry, is an assembly of layers of fibrous materials. The individual layers consist of high-modulus, high-strength fibers impregnated in an appropriate polymeric, metallic, or ceramic matrix material. Layers of different materials may be used, resulting in a hybrid laminate. The individual layers generally are orthotropic or transversely isotropic with the laminate then exhibiting anisotropic, orthotropic, or quasi-isotropic properties. Quasi-isotropic laminates exhibit isotropic in plane response but are not restricted to isotropic out-of-plane response. Depending upon the stacking sequence of the individual layers, the laminate may exhibit coupling between in plane and out of plane response. An example of bending-stretching coupling is the presence of curvature developing as a result of in-plane loading. The properties of a composite laminate depend on the geometrical arrangement and the properties of its constituents. The exact analysis of such structure – property relationship is rather complex because of many variables involved. Therefore, a few simplifying assumptions regarding the structural details and the state of stress within the composite have been introduced. The deformation of a plate subjected to transverse loading is caused either by flexural deformation due to rotation of cross-sections, or shear deformation due to sliding of sections or layers. The resulting deformation depends on the thickness to length ratio and the ratio of elastic to shear moduli. When the thickness to length ratio is small, the plate is considered thin, and it deforms mainly by flexure or bending; whereas when the thickness to length and the modular ratios are both large, the plate deforms mainly through shear. Due to the high ratio of in-plane modulus to transverse shear modulus, the shear deformation effects are more pronounced in the composite laminates subjected to transverse loads than in the isotropic plates under similar loading conditions.

KEYWORDS: Microstructure and Macrostructure, Composites, Laminates, Modeling, Theories of Plates, Numerical methods

1 GENERAL INTRODUCTION

A composite material can be defined as a combination of two or more materials that results in better properties than those of the individual components used alone. In contrast to metallic alloys, each material retains its separate chemical, physical, and mechanical properties. The two constituents are a reinforcement and a matrix. The main advantages of composite materials are their high strength and stiffness, combined with low density, when compared with bulk materials, allowing for a weight reduction in the finished part. The reinforcing phase provides the strength and stiffness. In most cases, the reinforcement is harder, stronger, and stiffer than the matrix. The reinforcement is usually a fiber or a particulate. Particulate composites have dimensions that are approximately equal in all directions. They may be spherical, platelets, or any other regular or irregular geometry. Particulate composites tend to be much weaker and less stiff than continuous fiber composites, but they are usually much less expensive. Particulate reinforced composites usually contain less reinforcement (up to 40 to 50 volume percent) due to processing difficulties and brittleness. A fiber has a length that is much greater than its diameter. The length-to-diameter (l/d) ratio is known as the aspect ratio and can vary greatly. Continuous fibers have long aspect ratios, while discontinuous fibers have short aspect ratios. Continuous-fiber composites normally have a preferred orientation, while discontinuous fibers generally have a random orientation. Examples of continuous reinforcements include unidirectional, woven cloth, and helical winding, while examples of discontinuous reinforcements are chopped fibers and random mat. Continuous-fiber composites are often made into laminates by stacking single sheets of continuous fibers in different orientations to obtain the desired strength and stiffness properties with fiber volumes as high as 60 to 70 percent. Fibers produce high-strength composites because of their small diameter; they contain far fewer defects (normally surface defects) compared to the material produced in bulk. As a general rule, the smaller the diameter of the fiber, the higher its strength, but often the cost increases as the diameter becomes smaller.

In addition, smaller-diameter high-strength fibers have greater flexibility and are more amenable to fabrication processes such as weaving or forming over radii. Typical fibers include glass, aramid, and carbon, which may be continuous or discontinuous. The continuous phase is the matrix, which is a polymer, metal, or ceramic. Polymers have low strength and stiffness, metals have intermediate strength and stiffness but high ductility, and ceramics have high strength and stiffness but are brittle. The matrix (continuous phase) performs several critical functions, including maintaining the fibers in the proper orientation and spacing and protecting them from abrasion and the environment. In polymer and metal matrix composites that form a strong bond between the fiber and the matrix, the matrix transmits loads from the matrix to the fibers through shear loading at the interface. In ceramic matrix composites,

the objective is often to increase the toughness rather than the strength and stiffness; therefore, a low interfacial strength bond is desirable.
Composites were first considered as structural materials a little more than half a century ago. From that time to now, they have received increasing attention in all aspects of material science, manufacturing technology, and theoretical analysis.

The term composite could mean almost anything if taken at face value, since all materials are composites of dissimilar subunits if examined at close enough details. But in modern materials engineering, the term usually refers to a matrix material that is reinforced with fibers. For instance, the term "FRP" which refers to Fiber Reinforced Plastic usually indicates a thermosetting polyester matrix containing glass fibers, and this particular composite has the lion's share of today commercial market.

Many composites used today are at the leading edge of materials technology, with performance and costs appropriate to ultra-demanding applications such as space craft, naval industry and bridge engineering technology. But heterogeneous materials combining the best aspects of dissimilar constituents have been used by nature for millions of years. Ancient societies, imitating nature, used this approach as well: The book of Exodus speaks of using straw to reinforce mud in brick making, without which the bricks would have almost no strength. Here in Sudan, people from ancient times dated back to Merowe civilization, and up to now used *zibala* mixed with mud as a strong building material.

As seen in Table 1 below, which is cited by David Roylance [1], Osama Khayal [2] and [3], Turvey and Mahmoud Yassin Osman [4], [5] and [6] the fibers used in modern composites have strengths and stiffnesses far above those of traditional structural materials. The high strengths of the glass fibers are due to processing that avoids the internal or surface flaws which normally weaken glass, and the strength and stiffness of polymeric aramid fiber is a consequence of the nearly perfect alignment of the molecular chains with the fiber axis.

Table 1. Properties of composite reinforcing fibers

Material	E (GN/m^2)	σ_b (GN/m^2)	ε_b (%)	ρ (Mg/m^3)	E/ρ (MN.m/kg)	σ_b/ρ (MN.m/kg)
E-glass	72.4	2.4	2.6	2.54	28.5	0.95
S-glass	85.5	4.5	2.0	2.49	34.3	1.8
Aramid	124	3.6	2.3	1.45	86	2.5
Boron	400	3.5	1.0	2.45	163	1.43
H S graphite	253	4.5	1.1	1.80	140	2.5
H M graphite	520	2.4	0.6	1.85	281	1.3

Where E is Young's modulus, σ_b is the breaking stress, ε_b is the breaking strain, and ρ is the mass density.

Of course, these materials are not generally usable as fibers alone, and typically they are impregnated by a matrix material that acts to transfer loads to the fibers, and also to protect the fibers from abrasion and environmental attack. The matrix dilutes the properties to some degree, but even so very high specific (weight – adjusted) properties are available from these materials. Polymers are much more commonly used, with unsaturated Styrene – hardened polyesters having the majority of low – to – medium performance applications and Epoxy or more sophisticated thermosets having the higher end of the market. Thermoplastic matrix composites are increasingly attractive materials, with processing difficulties being perhaps their principal limitation.

Composites possess two desirable features: the first one is high strength to weight ratio, and the second is their properties that can be tailored through variation of the fiber orientation and stacking sequence which gives the designers a wide spectrum of flexibility. The incorporation of high strength, high modulus and low-density filaments in a low strength and a low modulus matrix material is known to result in a structural composite material with a high strength / weight ratio. Thus, the potential of a two-material composite for use in aerospace, under-water, and automotive structures has stimulated considerable research activities in the theoretical prediction of the behavior of these materials. One commonly used composite structure consists of many layers bonded one on top of another to form a high-strength laminated composite plate. Each lamina is fiber- reinforced along a single direction, with adjacent layers usually having different filament orientations. For these reasons, composites are continuing to replace other materials used in structures such as those mentioned earlier. In fact composites are the potential structural materials of the future as their cost continues to decrease due to the continuous improvements in production techniques and the expanding rate of sales.

2 STRUCTURE OF COMPOSITES

There are many situations in engineering where no single material will be suitable to meet a particular design requirement. However, two materials in combination may possess the desired properties and provide a feasible solution to the materials selection problem. A composite can be defined as a material that is composed of two or more distinct phases, usually a reinforced material supported in a compatible matrix, assembled in prescribed amounts to achieve specific physical and chemical properties.

In order to classify and characterize composite materials, distinction between the following two types is commonly accepted; see Vernon [7], Jan Stegmann and Erik Lund [8], and David Roylance [1].

1. Fibrous composite materials: Which consist of high strength fibers embedded

in a matrix. The functions of the matrix are to bond the fibers together to protect them from damage, and to transmit the load from one fiber to another. See Figure 1.

2. Particulate composite materials: This composed of particles encased within a tough matrix, e.g. powders or particles in a matrix like ceramics.

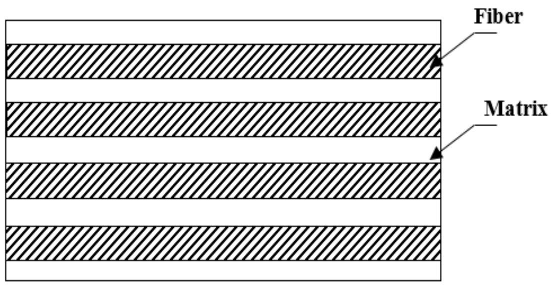

Figure 1. Structure of a fibrous composite laminates

In this research article the focus will be on fiber-reinforced composite materials, as they are the basic building element of a rectangular laminated plate structure. Typically, such a material consists of stacks of bonded-together layers (i.e. laminas or plies) made from fiber-reinforced material. The layers will often be oriented in different directions to provide specific and directed strengths and stiffnesses of the laminate. Thus, the strengths and stiffnesses of the laminated fiber-reinforced composite material can be tailored to the specific design requirements of the structural element being built.

2.1 Mechanical properties of a fiber-reinforced lamina

Composite materials have many mechanical characteristics, which are different from those of conventional engineering materials such as metals. More precisely, composite materials are often both inhomogeneous and non-isotropic.

Therefore, and due to the inherent heterogeneous nature of composite materials, they can be studied from a micromechanical or a macro-mechanical point of view. In micromechanics, the behavior of the inhomogeneous lamina is defined in terms of the constituent materials; whereas in macro-mechanics the material is presumed homogeneous and the effects of the constituent materials are detected only as averaged apparent macroscopic properties of the composite material. This approach is generally accepted when modeling gross response of composite structures. The micromechanics approach is more convenient for the analysis of the composite material because it studies the volumetric percentages of the constituent materials for the desired lamina stiffnesses and strengths, i.e. the aim of micromechanics is to determine the moduli of elasticity and strength of a lamina in terms of the moduli of elasticity, and volumetric percentage of the fibers and the matrix. To explain further, both the fibers and the matrix are assumed homogeneous, isotropic and linearly elastic.

The fibers may be oriented randomly within the material, but it is also possible to arrange for them to be oriented preferentially in the direction expected to have the highest stresses. Such a material is said to be anisotropic (i.e. different properties in different directions), and control of the anisotropy is an important means of optimizing the material for specific applications. At a microscopic level, the properties of these composites are determined by the orientation and distribution of the fibers, as well as by the properties of the fiber and matrix materials.

Consider a typical region of material of unit dimensions, containing a volume fraction, V_f of fibers all oriented in a single direction. The matrix volume fraction is then, $V_m = 1 - V_f$. This region can be idealized by gathering all the fibers together, leaving the matrix to occupy the remaining volume. If a stress σ_l is applied along the fiber direction, the fiber and matrix phases act in parallel to support the load. In these parallel connections the strains in each phase must be the same, so the strain ε_l in the fiber direction can be written as:

$$\varepsilon_l = \varepsilon_f = \varepsilon_m \tag{1}$$

Where the subscripts *l, f* and *m* denote the lamina, fibers and matrix respectively.

The forces in each phase must add to balance the total load on the material. Since the forces in each phase are the phase stresses times the area (here numerically equal to the volume fraction), we have

$$\sigma_l = \sigma_f V_f + \sigma_m V_m = E_f \varepsilon_l V_f + E_m \varepsilon_l V_m \tag{2}$$

The stiffness in the fiber direction is found by dividing the strain:

$$E_l = \frac{\sigma_l}{\varepsilon_l} = E_f V_f + E_m V_m \tag{3}$$

(Where E is the longitudinal Young's modulus)

This relation is known as a rule of mixtures prediction of the overall modulus in terms of the moduli of the constituent phases and their volume fractions.

Rule of mixtures estimates for strength proceed along lines similar to those for stiffness. For instance, consider a unidirectional reinforced composite that is strained up to the value at which the fiber begins to fracture. If the matrix is more ductile than the fibers, then the ultimate tensile strength of the lamina in equation (2) will be transformed to:

$$\sigma_l^u = \sigma_f^u V_f + \sigma_m^f (1 - V_f) \tag{4}$$

Where the superscript *u* denotes an ultimate value, and σ_m^f is the matrix stress when the fibers fracture as shown in Figure 2.

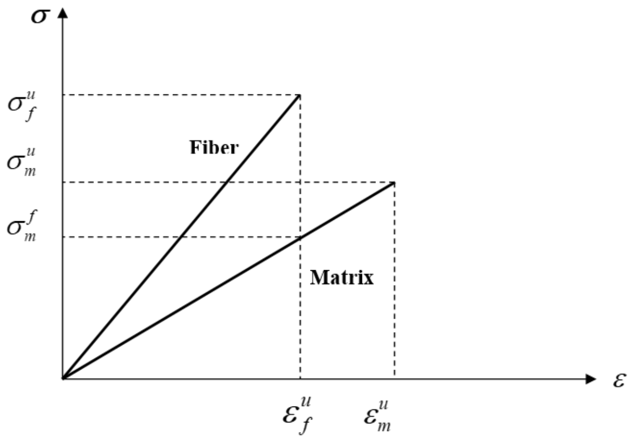

Figure 2. Stress-strain relationships for fiber and matrix

It is clear that if the fiber volume fraction is very small, the behavior of the lamina is controlled by the matrix.
This can be expressed mathematically as follows:

$$\sigma_l^u = \sigma_m^u\left(1 - V_f\right) \tag{5}$$

If the lamina is assumed to be useful in practical applications, then there is a minimum fiber volume fraction that must be added to the matrix. This value is obtained by equating equations (4) and (5) i.e.

$$V_{min} = \frac{\sigma_m^u - \sigma_m^f}{\sigma_f^u + \sigma_m^u - \sigma_m^f} \tag{6}$$

The variation of the strength of the lamina with the fiber volume fraction is illustrated in Figure 3.

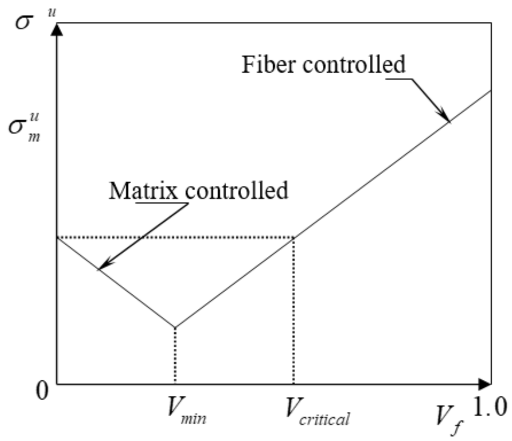

Figure 3. Variation of unidirectional lamina strength with the fiber volume fraction

It is obvious that when $0 < V_f < V_{min}$ the strength of the lamina is dominated by the matrix deformation which is less than the matrix strength. But when the fiber volume fraction exceeds a critical value (i.e.$V_f > V_{critical}$), Then the lamina gains some strength due to the fiber reinforcement.

The micromechanical approach is not responsible for the many defects which may arise in fibers, matrix, or lamina due to their manufacturing. These defects, if they exist include misalignment of fibers, cracks in matrix, non-uniform distribution of the fibers in the matrix, voids in fibers and matrix, delaminated regions, and initial stresses in the lamina as a result of its manufacture and further treatment.

The above mentioned defects tend to propagate as the lamina is loaded causing an accelerated rate of failure. The experimental and theoretical results in this case tend to differ. Hence, due to the limitations necessary in the idealization of the lamina components, the properties estimated on the basis of micromechanics should be proved experimentally. The proof includes a very simple physical test in which the lamina is considered homogeneous and orthotropic. In this test, the ultimate strength and modulus of elasticity in a direction parallel to the fiber direction can be determined experimentally by loading the lamina longitudinally. When the test results are plotted, as in Figure 4 below, the required properties may be evaluated as follows:

$$E_1 = \sigma_1 / \varepsilon_1 \quad ; \quad \sigma^u = P^u / A \quad ; \quad \nu_{12} = -\varepsilon_2 / \varepsilon_1$$

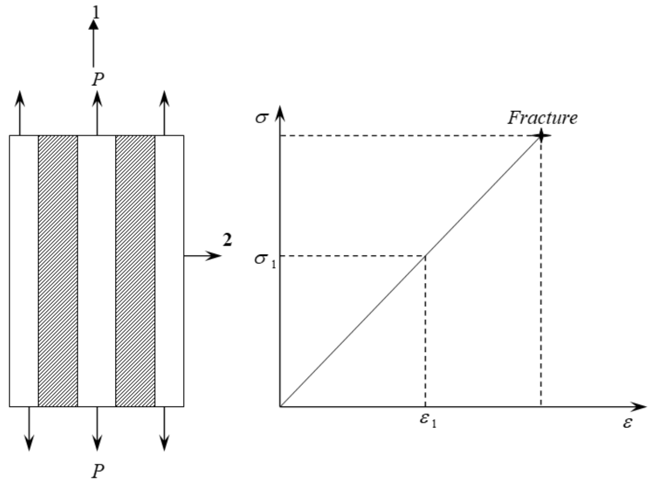

Figure 4. Unidirectional lamina loaded in the fiber-direction

Similarly, the properties of the lamina in a direction perpendicular to the fiber-direction can be evaluated in the same procedure.

2.2 Analytical modeling of composite laminates

The properties of a composite laminate depend on the geometrical arrangement and the properties of its constituents. The exact analysis of such structure – property relationship is rather complex because of many variables involved. Therefore, a few simplifying assumptions regarding the structural details and the state of stress within the composite have been introduced.

It has been observed, that the concept of representative volume element and the selection of appropriate boundary conditions are very important in the discussion of micromechanics. The composite stress and strain are defined as the volume averages of the stress and strain fields, respectively, within the representative volume element. By finding relations between the composite stresses and the composite strains in terms of the constituent properties expressions for the composite moduli could be derived. In addition, it has been shown that, the results of advanced methods can be put in a form similar to the rule of mixtures equations.

Prediction of composite strengths is rather difficult because there are many unknown variables and also because failure critically depends on defects. However, the effects of constituents including fiber – matrix interface on composite strengths can be qualitatively explained. Certainly, failure modes can change depending on the material combinations. Thus, an analytical model developed for one material combination cannot be expected to work for a different one. Ideally a truly analytical model will be applicable to material combination. However, such an analytical model is not available at present.

Therefore, it has been chosen to provide models each of which is applicable only to a known failure mode. Yet, they can explain many of the effects of the constituents. (Refer to Ref. [9]).

3 DEVELOPMENTS IN THE THEORIES OF LAMINATED PLATES

From the point of view of solid mechanics, the deformation of a plate subjected to transverse loading consists of two components: flexural deformation due to rotation of cross-sections, and shear deformation due to sliding of sections or layers. The resulting deformation depends on two parameters: the thickness to length ratio and the ratio of elastic to shear moduli. When the thickness to length ratio is small, the plate is considered thin, and it deforms mainly by flexure or bending; whereas when the thickness to length and the modular ratios are both large, the plate deforms mainly through shear. Due to the high ratio of in-plane modulus to transverse shear modulus, the shear deformation effects are more pronounced in the composite laminates subjected to transverse loads than in the isotropic plates under similar loading conditions.

The three-dimensional theories of laminates in which each layer is treated as homogeneous anisotropic medium (see Reddy [10]) are intractable as the number of layers becomes moderately large. Thus, a simple two-dimensional theory of plates that accurately describes the global behavior of laminated plates seems to be a compromise between accuracy and ease of analysis.

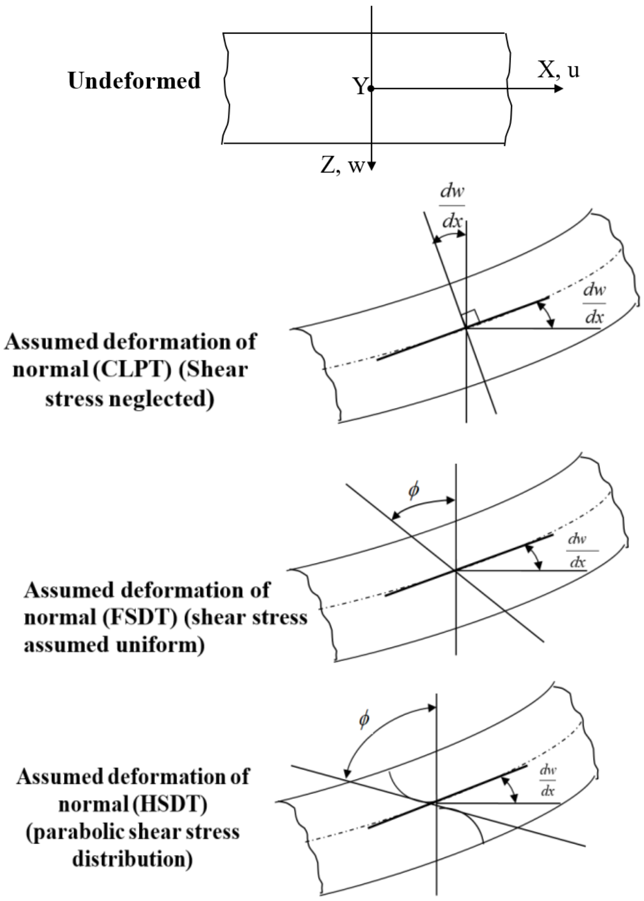

Figure 5. Assumed deformation of the transverse normal in various displacement base plate theories

Putcha and Reddy [11] classified the two-dimensional analyses of laminated composite plates into two categories: (1) the classical lamination theory, and (2) shear deformation theories. In both theories it is assumed that the laminate is in a state of plane stress, the individual lamina is linearly elastic, and there is perfect bonding between layers. The classical laminate theory (CLPT), which is an extension of the classical plate theory (CPT) applied to laminated plates was the first theory formulated for the analysis of laminated plates by Reissner and Stavsky [12] in 1961, in which the Kirchhoff-Love assumption that normal to the mid-surface before deformation remain straight and normal to the mid-surface after deformation is used (see Figure 5), but it is not adequate for the flexural analysis of moderately thick laminates. However, it gives reasonably accurate results for many engineering problems i.e. thin composite plates, as stated by Srinivas and Rao [13] and Reissner and Stavsky [12].

This theory ignores the transverse shear stress components and models a laminate as an equivalent single layer. The classical laminated plate theory (CLPT) under-predicts deflections as proved by Turvey and Mahmoud Yassin Osman [4], [5], and [6] and Reddy [9] due to the neglect of transverse shear strain. The errors in deflections are even higher for plates made of advanced filamentary composite materials like graphite -epoxy and boron-epoxy, whose elastic modulus to shear modulus ratios are very large (i.e. of the order of 25 to 40, instead of 2.6 for typical isotropic materials). However, these composites are susceptible to thickness effects because their effective transverse shear moduli are significantly smaller than the effective elastic modulus along the fiber direction. This effect has been confirmed by Pagano [14] who obtained analytical solutions of laminated plates in bending based on the three-dimensional theory of elasticity. He proved that classical laminated plate theory (CLPT) becomes of less accuracy as the side to thickness ratio decreases. In particular, the deflection of a plate predicted by CLPT is considerably smaller than the analytical value for side to thickness ratio less than 10. These high ratios of elastic modulus to shear modulus render classical laminate theory as inadequate for the analysis of composite plates.

Many theories which account for the transverse shear and normal stresses are available in the literature (see, for example Mindlin [15]). These are too numerous to review here. Only some classical papers and those which constitute a background for the present research article will be considered. These theories are classified according to Phan and Reddy [16] into two major classes on the basis of the assumed fields as: (1) stress based theories, and (2) displacement based theories. The stress-based theories are derived from stress fields, which are assumed to vary linearly over the thickness of the plate:

$$\sigma_i = \frac{M_i}{(h^2/6)} \times \frac{z}{(h/2)} \qquad (i = 1,2,6) \tag{7}$$

(Where M_i is the stress couples, h is the plate thickness, and z is the distance of the lamina from the plate mid-plane)

The displacement-based theories are derived from an assumed displacement field as:

$$\begin{aligned} u &= u_0 + zu_1 + z^2u_2 + z^3u_3 + \cdots \\ v &= v_0 + zv_1 + z^2v_2 + z^3v_3 + \cdots \\ w &= w_0 + zw_1 + z^2w_2 + z^3w_3 + \cdots \end{aligned} \tag{8}$$

Where, u_0, v_0 and w_0 are the displacements of the middle plane of the plate.

The governing equations are derived using the principle of minimum total potential energy. The theory used in the present work comes under the class of displacement-based theories. Extensions of these theories which include the linear terms in z in u and v and only the constant term in w, to account for higher -order variations and to laminated plates, can be found in the work of Yang, Norris and Stavsky [17], Whitney and Pagano [18] and Phan and Reddy [16]. In this theory which is called first-order shear deformation theory (FSDT), the transverse planes, which are originally normal and straight to the mid-plane of the plate, are assumed to remain straight but not necessarily normal after deformation, and consequently shear correction factor are employed in this theory to adjust the transverse shear stress, which is constant through thickness (see Figure 5). Recently Reddy [10] and Phan and Reddy [16] presented refined plate theories that use the idea of expanding displacements in the powers of thickness co-ordinate. The main novelty of these works is to expand the in-plane displacements as cubic functions of the thickness co-ordinate, treat the transverse deflection as a function of the x and y co-ordinates, and eliminate the functions u_2 ,u_3 ,v_2 and v_3 from equation (8) by requiring that the transverse shear stresses be zero on the bounding planes of the plate. Numerous studies involving the application of the first-order theory to bending analysis can be found in the works of Reddy [19], and Reddy and Chao [20].

In order to include the curvature of the normal after deformation, a number of theories known as Higher-order Shear Deformation Theories (HSDT) have been devised in which the displacements are assumed quadratic or cubic through the thickness of the plate. In this aspect, a variationally consistent higher-order theory which not only accounts for the transverse shear deformation but also satisfies the zero transverse shear stress conditions on the top and bottom faces of the plate and does not require shear correction factors was suggested by Reddy [10]. Reddy's modifications consist of a more systematic derivation of displacement field and variationally consistent derivation of the equilibrium equations. The refined laminate plate theory predicts a parabolic distribution of the transverse shear stresses through the thickness, and requires no shear correction coefficients.

In the non-linear analysis of plates considering higher-order theory (HSDT), shear deformation has received considerably less attention compared with linear analysis. This is due to the geometric non-linearity which arises from finite deformations of an elastic body and which causes more complications in the analysis of composite plates. Therefore fiber-reinforced material properties and lamination geometry have to be taken into account. In the case of anti-symmetric and unsymmetrical laminates, the existence of coupling between bending and stretching complicates the problem further.

Non-linear solutions of laminated plates using higher-order theories have been obtained through several techniques, i.e. perturbation method as in Ref. [21], finite element method as in Putcha and Reddy [10], the increment of lateral displacement method as in Ref. [22], and the small parameter method as in Ref. [23].

In the present work, a numerical method known as Dynamic Relaxation (DR) coupled with finite differences is used. The DR method was first proposed in 1960s; see Rushton [24], Cassell and Hobbs [25], Day [26]. In this method, the equations of equilibrium are converted to dynamic equation by adding damping and inertia terms. These are then expressed in finite difference form and the solution is obtained through iterations. The optimum damping coefficient and time increment used to stabilize the solution depend on a number of factors including the stiffness matrix of the structure, the applied load, the boundary conditions and the size of the mesh used, etc…

Numerical techniques other than the DR include finite element method, which is widely used in the literature. In a comparison between the DR and the finite element method, Aalami [27] found that the computer time required for finite element method is eight times greater than for DR analysis, whereas the storage capacity for finite element analysis is ten times or more than that for DR analysis. This fact is supported by Putcha and Reddy [11] who noted that some of the finite element formulations require large storage capacity and computer time. Hence, due to less computations and computer time involved in the present study, the DR method is considered more efficient than the finite element method. In another comparison Aalami [27] found that the difference in accuracy between one version of finite element and another may reach a value of 10% or more, whereas a comparison between one version of finite element method and DR showed a difference of more than 15%. Therefore, the DR method can be considered of acceptable accuracy. The only apparent limitation of DR method is that it can only be applied to limited geometries. However, this limitation is irrelevant to rectangular plates which are widely used in engineering applications.

4 CONCLUSIONS

Composites possess two desirable features, the first one is their high strength to weight ratio, and the second is their properties that can be tailored through variation of the fiber orientation and stacking sequence which gives the designers a wide spectrum of flexibility. For these reasons, composites are continuing to replace other materials used in structures such as classical materials.

Composite materials have many mechanical characteristics, which are different from those of conventional engineering materials such as metals. More precisely, composite materials are often both inhomogeneous and non-isotropic. Therefore, and due to the inherent heterogeneous nature of composite materials, they can be studied from a micromechanical or a macro-mechanical point of view. The micromechanics approach is more convenient for the analysis of the composite material because it studies the volumetric percentages of the constituent materials for the desired lamina stiffnesses and strengths.

The deformation of a deck plate subjected to transverse loading is caused either by flexural deformation due to rotation of cross-sections, or shear deformation due to sliding of sections or layers.

Solutions of laminated deck plates using different-order theories could be obtained through several techniques, i.e. dynamic relaxation method, finite differences method, perturbation method, finite element method, the increment of lateral displacement method, and the small parameter method.

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