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ROUGHNESS ON ROAD AND ON BRIDGE

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ABSTRACT: A new point of view of how the surface roughness operates on the cars is the subject of this paper. The paper has two parts. The first studies the roughness on road while the second studies the roughness on the bridge deck. The theoretical formulation is based on a continuous approach that has been used in the literature to analyze such bridges, the procedure is carried out by the modal superposition method, while the obtained equations are solved using Duhamel's integrals.

KEYWORDS: Roughness; Road surface; Bridge deck roughness.

1 INTRODUCTION

The influence of a rough deck-surface on the dynamic response of a bridge depends on various factors. The irregularities, on a deck-surface, may be due to the roughness of the deck-surface, but also due to random or on purpose existing anomalies for traffic reasons. The parameter of the road surface (or bridge deck) roughness, dealt with excessively in the recent literature.

In 1960, Carey and Irick [1] showed that surface roughness was the primary variable needed to explain the driver's opinion of the quality of the serviceability provided by a pavement surface. Most of the followed investigations focused on the study, characterization and classification of the pavements [2 to 6].

Numerous studies adopt the Power Spectral Density (PSD) functions for roughness, as modified by Wang and Huang [7], or the simpler harmonically varying surface irregularity presented in Cheng et al [8].

Among these one most quote the significant contribution by Fafard et al [9], Cheng and Lee [10], Huang and Wang [11], Kou and DeWolf [12] as well as by Yang et al [13]. Their findings have shown that the foregoing parameter is one of the most important factors affecting elastic dynamic response, especially applicable to steel highway bridges.

The research of today is focused on two main subjects. A number of researchers are studying the noise and inconvenience caused by the pavement roughness [14, 15], or the percentage influence of roughness on the vehicle-structure coupled interaction [16, 17]. Other researchers are trying to determine the bridges' frequencies by studying the influence of the road surface roughness on a moving vehicle or a pair of vehicles [18 to 20].

It is obvious that the wheel of a vehicle cannot follow the entire surface of the roughness because of the wheels' dimensions and the surface of the roughness. So the inconvenience caused should be sought for other reasons. Although some researchers have suspected and reported [21, 22] the effect of tire bounce, the study in this area did not continue further.

In this paper the problem is studied by another point of view. A possible reason could be the forces developing during the rolling of the wheels on the abnormal surface of the deck-road. The roughness is considered as a series of repeated small irregularities, which the wheel passes without changing its level of motion but with the development of impact forces from each small irregularity (fig. 2).

The paper has two parts. The first studies the roughness on road while the second studies the roughness on the bridge deck. The theoretical formulation is based on a continuous approach that has been used in the literature to analyze such bridges, the procedure is carried out by the modal superposition method, while the obtained equations are solved using Duhamel's integrals.

A variety of numerical examples allows one to draw important conclusions for structural design purpose.

2 INTRODUCTORY CONCEPTS

When the solid bodies shown in figure A1, with rotational velocities θ_1 *and* θ_2 and rotational inertias I_1 *and* I_2 are collided, after their bouncing, the new rotational velocity of the first body after impact is given by:

$$\theta_{1a} = \frac{(I_1 - \varepsilon I_2)\theta_{1f} + (1+\varepsilon) I_2 \theta_{2f}}{I_1 + I_2} \tag{1a}$$

where ε is the percentage of the energy remaining after impact (which for regular wheels is between 0.80 and 0.97, depending on the inner pressure of the wheel) and f and α express the rotational velocities before and after impact.

Setting into (1a) $I_1 = \infty$ and $\theta_{1f} = 0$, we obtain:

$$\theta_{2a} = -\varepsilon\, \theta_{2f} = -\varepsilon\, \theta \tag{1b}$$

A body under the action of a moment for a period of time Δt, acquires a torsional momentum (or impulse) equal to:

$$G = M \cdot \Delta t \tag{2a}$$

It is proved that:

$$G = I \cdot \omega \tag{2b}$$

where I the rotational inertia of the body and ω its rotational velocity.

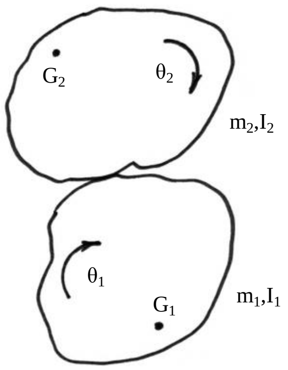


Figure 1. Collision of two bodies

A wheel, has rotational inertia $I = m_o i^2$ or finally

$$I = m_o R^2 / 2 \tag{3}$$

where m_o is the mass of the wheel, i its inertia radius and R the radius of the wheel.

According to appendix A, the impact of a wheel with radius R on a solid plate is expressed by

$$G = M \cdot \Delta t = N \cdot \Delta t \cdot R = I \cdot \omega = m_o \varepsilon R^2 \omega = m_o \varepsilon \cdot \frac{R^2}{2} \cdot \frac{\upsilon}{R}$$

and finally

$$N = G_{\Delta t \to 0} = \varepsilon \cdot m_o \upsilon / 2 \tag{4}$$

where N is the force arising from the impact and acting on the common tangent of two solids at the point of impact (see also Figure 2).

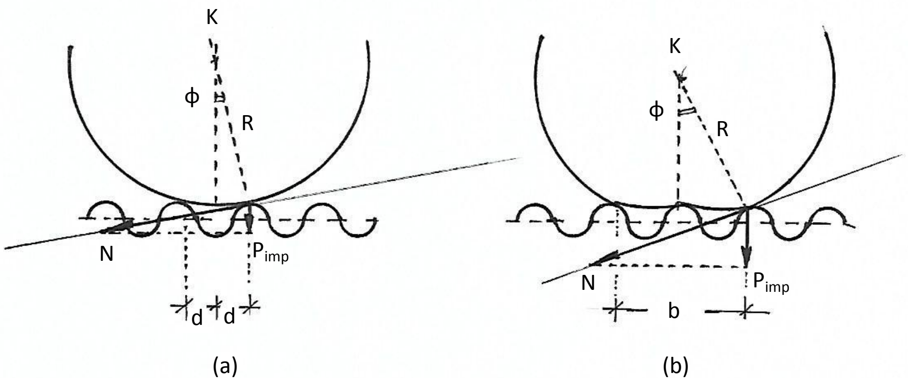


Figure 2. Impact because of irregularities

3 EQUATIONS OF THE PROBLEM

3.1 Roughness on road

The acting on the road force F is:

$$F = (M + m_o)g - M\ddot{z} + P_{imp} \tag{5a}$$

Cutting at a-a and taking the equilibrium of the cut off part we have:

$$F = kz + c\dot{z} \tag{5b}$$

From (5a) and (5b) we obtain the following equation of the problem:

$$M\ddot{z} + c\dot{z} + kz = (M + m_o)g + P_{imp} \tag{5c}$$

But
$$P_{imp} = N \cdot \sin\varphi = N \cdot \frac{d}{R} \tag{5d}$$

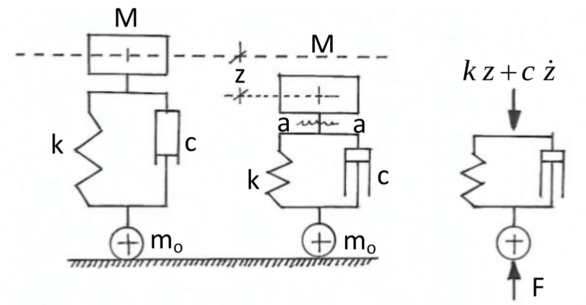


Figure 3. Mass load on road

Equation (5c), because of (4) and (5d) becomes:

$$\left.\begin{aligned} &M\ddot{z} + c\dot{z} + kz = (M + m_o)g + \frac{\varepsilon m_o \upsilon d}{2R} \cdot \delta(t - t_n) \\ &where: \quad t_n = (n-1)\frac{2d}{\upsilon} \end{aligned}\right\} \tag{5e}$$

Where δ is the Dirac Delta function and n the nth irregularity of the roughness. The above (5e) can be written as follows:

$$\left.\begin{aligned} &\ddot{z} + 2\beta\dot{z} + \omega^2 z = \frac{(M + m_o)g}{M} + \frac{\varepsilon m_o \upsilon d}{2RM} \cdot \delta(t - t_n) \\ &where: \quad 2\beta = \frac{c}{M}, \quad \omega^2 = \frac{k}{M} \end{aligned}\right\} \tag{6}$$

The first term of the right side member of equation (6) gives the time needed to tranquillize the system in its equilibrium state after the application of mass M. The second term expresses the roughness influence. Solving equation (6) without the term due to roughness we obtain:

$$\left.\begin{aligned} & z_s(t)=\frac{(M+m_o)g}{\omega_o M}\int_0^t e^{-\beta(t-\tau)}\sin\omega_o(t-\tau)d\tau=\frac{(M+m_o)g}{\omega_o M}\left[\omega_o-e^{-\beta t}(\beta\sin\omega_o t+\omega_o\cos\omega_o t)\right] \\ & where: \quad \omega_o=\sqrt{\omega^2-\beta^2} \end{aligned}\right\} \tag{7a}$$

Putting for example $M=300\,kg$, $m_o=8\,kg$, $k=60\,000\,dN/m$, $c=1000\,dN\,sec/m$ and applying equation (7), we get the plot of figure 4.

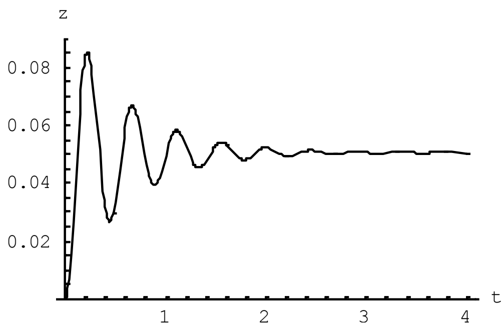


Figure 4. Tranquillization of system

We observe that after a needed time (in this example it is ~2.5 sec), the system tranquillizes after equilibrium at $z=5\,cm$. This verifies the equation $z=Mg/k=3000/60000=0.05m$, which gives the position of the static equilibrium.

We suppose that at $t=0$ the vehicle has already taken its static equilibrium position and starts to move with speed υ, on a road with roughness. The equation of motion is:

$$\left.\begin{aligned} & \ddot{z}+2\beta\dot{z}+\omega^2 z=\frac{\varepsilon m_o\upsilon d}{2RM}\cdot\delta(t-t_n) \\ & where: \quad t_n=(n-1)\frac{2d}{\upsilon} \end{aligned}\right\} \tag{7b}$$

When the load moves from irregularity (n-1) to irregularity (n), the solution of equation (7b) is given by the Duhamel's integral:

$$z_{n-1}(t)=\frac{\varepsilon m_o\upsilon d}{2\omega_o RM}\int_0^t e^{-\beta(t-\tau)}\sin\omega_o(t-\tau)\delta(\tau-t_{n-1})d\tau$$

or finally:

$$z_{n-1}(t)=\frac{\varepsilon m_o\upsilon d}{2\omega_o RM}\cdot e^{-\beta(t-t_{n-1})}\sin\omega_o(t-t_{n-1})\left[H(t-t_{n-1})-H(t-t_n)\right] \tag{7c}$$

where H is the Haeviside unit step function.
When the load moves from irregularity (n) to (n+1) one, the solution of equation (7d) is:

$$\left.\begin{aligned} z_n(t) &= e^{-\beta(t-t_n)}\left[A\sin\omega_o(t-t_n)+B\cos\omega_o(t-t_n)\right]\cdot\left[H(t-t_n)-H(t-t_{n+1})\right] \\ &+\frac{\varepsilon m_o\upsilon d}{2\omega_o RM}\cdot e^{-\beta(t-t_n)}\sin\omega_o(t-t_n)\cdot\left[H(t-t_n)-H(t-t_{n+1})\right] \end{aligned}\right\} \quad (7d)$$

With time conditions $z_{n-1}(t_n)=z_n(t_n)$, $\dot{z}_{n-1}(t_n)=\dot{z}_n(t_n)$ we determine the constants A and B as follows:

$$\left.\begin{aligned} A &= \frac{\varepsilon m_o\upsilon d}{2\omega_o RM}\cdot e^{-\beta(t_n-t_{n-1})}\cos\omega_o(t_n-t_{n-1}) \\ B &= \frac{\varepsilon m_o\upsilon d}{2\omega_o RM}\cdot e^{-\beta(t_n-t_{n-1})}\sin\omega_o(t_n-t_{n-1}) \end{aligned}\right\} \quad (8a)$$

In the above equations, the term $L=e^{-\beta(t_n-t_{n-1})}$ has limit ->1. This is clear in the plot of figure 5 (for $M=300\,kg, m_o=8\,kg, k=60000\,dN/m, c=1000\,dN\,sec/m$), different vehicle's speeds and roughness quality.

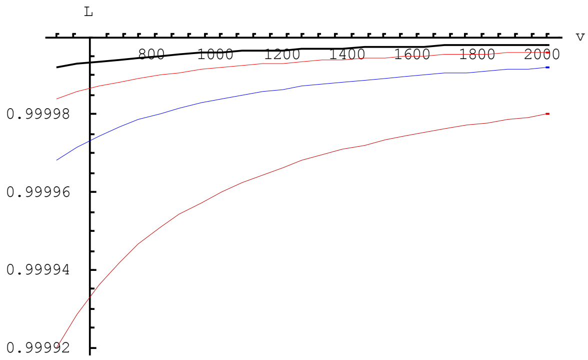


Figure 5. d=0.01(black), d=0.02(red, d=0.04(blue), d=0.10 (café)

Therefore one can write:

$$\left.\begin{aligned} A &= \frac{\varepsilon m_o\upsilon d}{2\omega_o RM}\cdot\cos\omega_o(t_n-t_{n-1}) \\ B &= \frac{\varepsilon m_o\upsilon d}{2\omega_o RM}\cdot\sin\omega_o(t_n-t_{n-1}) \end{aligned}\right\} \quad (8b)$$

3.2 Roughness on bridge

Easily one can find that:

$$P_{imp} = \frac{\varepsilon\, m_o \upsilon}{2} \cdot \sin\varphi = \frac{\varepsilon\, m_o \upsilon}{2} \cdot \frac{d}{R} \tag{9}$$

with d, determined according to ISO 8608 and 4287.

Figure 2a assumes that the wheel will be undeformed. Otherwise, figure 2b applies when $b > 3d$. In the first case it will be $\tan\varphi = d / R$ while in the second one $\tan\varphi = b / 2R$.

Assuming that two successive elevations of roughness are 2d apart, then when the wheel impacts on the a_i^{th} elevation, the mathematical formulation will be:

$$G = P_{imp} \cdot \delta(x - a_i) \cdot \delta(t - t_{a_i}) \tag{10}$$

where α_i is the distance if the ith elevation from the left end of the bridge.

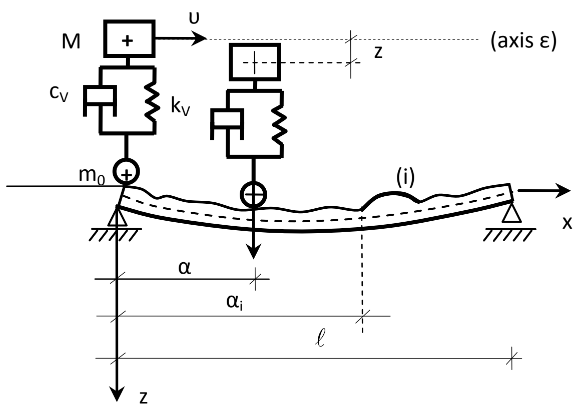


Figure 6. Mass load on bridge

Therefore the corresponding equations of bridge's motion are:

$$\left.\begin{aligned} & E I_y w'''' + c_b \dot{w} + m\ddot{w} = [(M + m_o)g - M\ddot{z} - m_o \ddot{w}] \cdot \delta(x - a) + P_{imp} \sum_{i=1}^{\zeta} \delta(x - a_i) \cdot \delta(t - t_{ai}) \\ & M\ddot{z} + c_p \dot{z} + k_p z = k_p w(x) + c_p \dot{w}(x) \quad ,with \quad \zeta = L / 2d,\ t_{ai} = i \cdot 2d / \upsilon \end{aligned}\right\}$$

(11a)

Solving the second of equations (11a) we conclude to the following expressions:

$$\left.\begin{aligned} &\ddot{z}(t)=\Phi(t)+\frac{1}{\overline{\omega}_p}\int_0^t \Xi(\tau)\cdot\Phi(\tau)\,d\tau \\ &where: \quad \Phi(t)=\omega_p^2 w(a,t)+2\beta_p\dot{w}(a,t) \\ &\qquad \Xi(t)=e^{-\beta_p(t-\tau)}[(\beta_p^2-\overline{\omega}_p^2)\sin\overline{\omega}_p(t-\tau)+2\beta_p\overline{\omega}_p\cos\overline{\omega}_p(t-\tau) \\ &\qquad \beta_p=c_p/2M\,, \quad \omega_p^2=k_p/M\,, \quad \overline{\omega}_p=\sqrt{\omega_p^2-\beta_p^2} \end{aligned}\right\} \qquad (11b)$$

In order to solve the first of equations (11a), we are searching for a solution of the form:

$$w(x,t)=\sum_n X_n(x)\cdot T_n(t) \qquad (11c)$$

where X_n are the shape functions of the beam and T_n are the time functions to be determined. Following the well known procedure and using approaching methods to solve the equations governing the motion of the bridge [23], we finally find:

$$\left.\begin{aligned} T_k(t)=&\frac{2}{mL\overline{\omega}_k}\int_0^t[(M+m_o)g-M\ddot{z}(\tau)-m_o\sum_n \sin\Omega_n\tau\ddot{\overline{T}}_n(\tau)]\sin\Omega_k\tau\cdot e^{-\beta(t-\tau)}\sin\overline{\omega}_k(t-\tau)d\tau \\ &+\frac{\varepsilon m_o\upsilon d}{mLR\overline{\omega}_k}\sum_{i=1}^{\zeta}\sin\Omega_k t_a\cdot e^{-\beta(t-t_a)}\sin\overline{\omega}_k(t-t_a)H(t-t_a) \end{aligned}\right\}$$

(12a)

where $\zeta=L/2d\,,\ \ t_a=i\cdot 2d/\upsilon$.

After the impact of the wheel on the ith irregularity of the roughness, its influence lasts until its impact on the (i+1)th one. Then at $t_{i+1}=t_i+2d/\upsilon$, the beam vibrates under the action of P_{imp} on (i+1)th irregularity and under the appeared free motion due to the previous action of P_{imp} on the ith irregularity. This last term for the k mode has the following time function:

$$T_k^{free}(t)=\frac{2P_{imp}}{mL\omega_k}\cdot e^{-\beta(t-t_i)}[A_k\sin\omega_k(t-t_i)+B_k\cos\omega_k(t-t_i)] \qquad (12b)$$

Using the time conditions

$$\left.T_k(t_i)=T_k^{free}(t_i)\,, \quad and \quad \dot{T}_k(t_i)=\dot{T}_k^{free}(t_i)\right\} \qquad (12c)$$

We finally find:

$$\left.\begin{aligned} A_k&=e^{-\beta(t_i-t_{i-1})}\cdot\sin\Omega_k t_{i-1}\cdot\cos\omega_k(t_i-t_{i-1}) \\ B_k&=e^{-\beta(t_i-t_{i-1})}\cdot\sin\Omega_k t_{i-1}\cdot\sin\omega_k(t_i-t_{i-1}) \end{aligned}\right\} \qquad (12d)$$

Let us now consider the vehicle of figure 7.

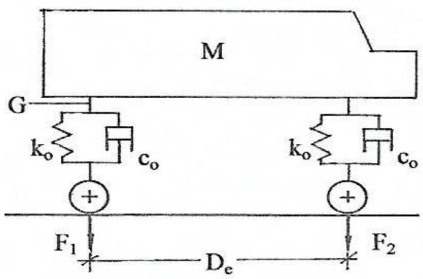


Figure 7. Vehicle's model

Assuming that the weight of the vehicle is equally distributed between the front and back wheels, the forces applied on the bridge will be:

$$\left.\begin{aligned}F_1 &= [\frac{M}{2}(g-\ddot{z})+m_o(g-\ddot{w})]\delta(x-a)+\frac{\varepsilon m_o\upsilon}{2}\sin\varphi\sum_{i=1}^{\zeta}\delta(x-a)\delta(t-t_a)\\ F_2 &= [\frac{M}{2}(g-\ddot{z})+m_o(g-\ddot{w})]\delta(x-a+D_e)+\frac{\varepsilon m_o\upsilon}{2}\sin\varphi\sum_{i=1}^{\zeta}\delta(x-a+D_e)\delta(t-t_a)\end{aligned}\right\} \quad (13a)$$

where $\ddot{z}$ from equation (11d), and D_e the wheelbase of the vehicle.
Searching for a solution under the form:

$$w(x,t)=\sum_n X_n(x)\cdot T_n(t) \quad (13b)$$

we determine the time function $T_k(t)$ as follows:

$$\left.\begin{aligned}T_k &= \frac{2}{mL\overline{\omega}_k}\int_0^t G_1(\tau)e^{-\beta(t-\tau)}\sin\overline{\omega}_k(t-\tau)d\tau+\frac{\varepsilon m_o\upsilon}{2m\overline{\omega}_k}\sin\varphi\sum_{i=1}^{\zeta}\sin\Omega_k t_{a1}e^{-\beta(t-t_{a1})}\sin\overline{\omega}_k(t-t_{a1})H(t-t_{a1})\\ &+\frac{2}{mL\overline{\omega}_k}\int_0^t G_2(\tau)e^{-\beta(t-\tau)}\sin\overline{\omega}_k(t-\tau)d\tau+\frac{\varepsilon m_o\upsilon}{2m\overline{\omega}_k}\sin\varphi\sum_{i=1}^{\zeta}\sin\Omega_k t_{a2}e^{-\beta(t-t_{a2})}\sin\overline{\omega}_k(t-t_{a2})H(t-t_{a2})\end{aligned}\right\} \quad (13c)$$

where:

$$\left.\begin{aligned}G_1(t) &= \left[(\frac{M}{2}+m_o)g-\frac{M}{2}\ddot{z}(t)-m_o\sum_n \sin\Omega_n t_1\ddot{T}_n(t)\right]\sin\Omega_k t_1\\ G_2(t) &= \left[(\frac{M}{2}+m_o)g-\frac{M}{2}\ddot{z}(t)-m_o\sum_n \sin\Omega_n t_2\ddot{T}_n(t)\right]\sin\Omega_k t_2\\ t_{a1} &= \frac{\delta_i+\xi_o}{\upsilon},\quad t_{a2}=\frac{\delta_i+\xi_o-D_e}{\upsilon},\quad t_1=\frac{\delta_i}{\upsilon},\quad t_2=\frac{\delta_i-D_e}{\upsilon}\end{aligned}\right\} \quad (13d)$$

4 NUMERICAL EXAMPLES AND DISCUSSION

The purpose of this paper is to study the effect of the impact of the vehicle's tires on the roughness' irregularities for a vehicle moving either on the road or on a bridge.

The paper focuses on the following parameters:

- The quality of the road surface and its effect on the so called passengers' trouble.
- The effect of the roughness on the bridge motion

For the study of the road surface we use the vehicle of figure 3, while for the study of the bridges we use the vehicle of figure 7.

The used vehicle, has the following characteristics:

$M = 300\,kg$, $m_o = 8\,kg$, $R = 0.30\,m$, $k_o = 60000\ dN / m$, $c_o = 1000\ dN \cdot sec/\ m$.

The vehicles' speed is $\upsilon = 10\,m / sec$.

As for the roughness study three kinds of pavement are considered. The soft with $d = 0.01m$, the middle with $d = 0.02m$, and the bad with $d = 0.035m$. Finally the case of a cobbled road is studied with $d = 0.10\,m$.

The bridges are made from homogeneous and isotropic material, having modulus of elasticity $E = 2.1 \cdot 10^{10}\ dN / m^2$.

Let us consider two kinds of bridges.

- One, relatively short bridge of length $L = 20\,m$, mass per unit length $m = 250\,kg / m$, and moment of inertia $I_y = 0.01\,m^4$.

Note should be taken of the following:

- The vehicles are supposed to move along the center line of the bridge. Thus no rotational motion is developed.
- They are studied the displacements of the bridge middle of the span.
- Only the first six flexural modes are taken into account.

4.1 Roughness on road

Applying the formulae of §3.1 and considering that the system of figure 3 is tranquilized in its equilibrium position, we obtain the following plots.

In the plot of figure 8 we see the motion of the vehicle for a smooth road surface (d=0.01m). The so produced trouble is the result of the vehicle motion from 0.000016 m to 0.000024 m or for 0.000008 m which can be considered as practically non-existent.

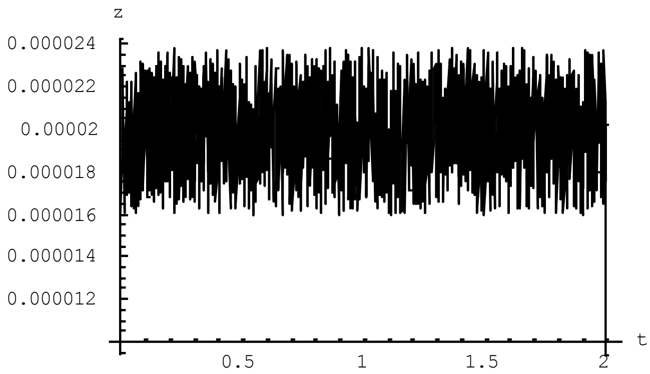


Figure 8. Roughness on road with d=0.01m

In the plot of figure 9 we see the motion of the vehicle for a middle road surface (d=0.02m). The so produced trouble is the result of the vehicle motion from 0.000063 m to 0.000097 m or for 0.000034 m, which causes a rather sufferable trouble.

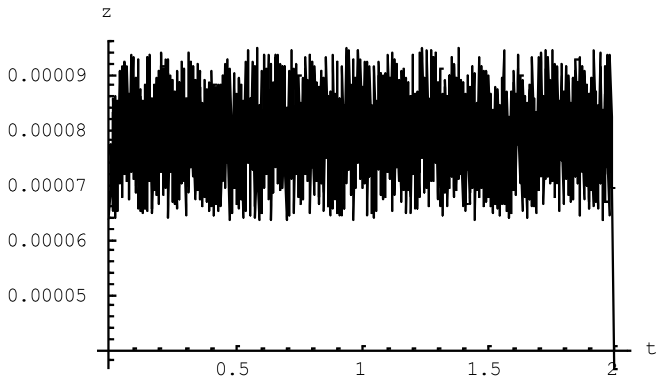


Figure 9. Roughness on road with d=0.02m

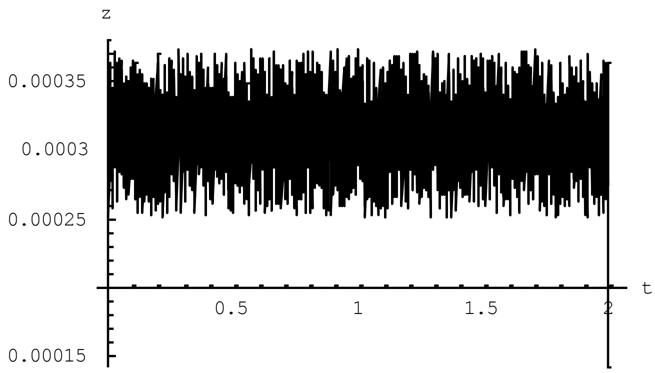


Figure 10. Roughness on road with d=0.04m

In the plot of figure 10 we see the motion of the vehicle for a bad road surface (d=0.04m). The so produced trouble is the result of the vehicle motion from 0.00043 m to 0.00024 m or for 0.00019 m, which causes a notable trouble.

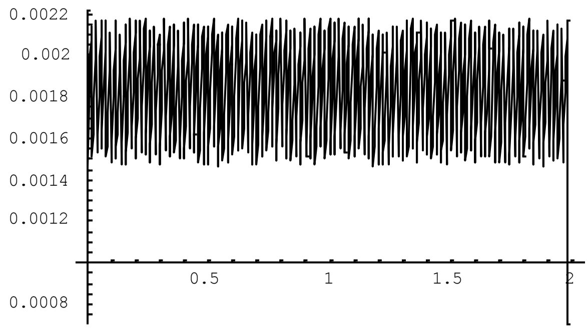


Figure 11. Roughness on road with d=0.10m

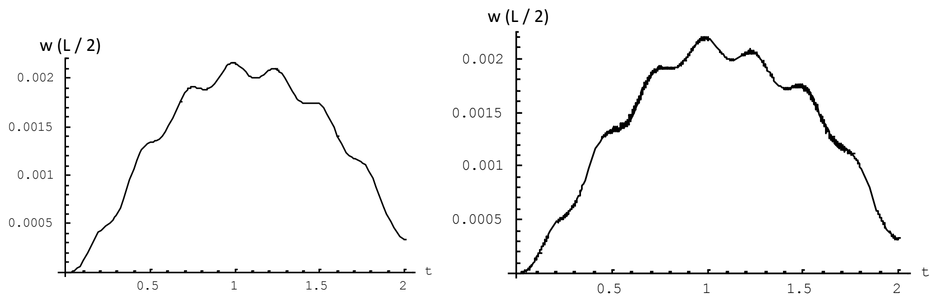
Finally in the plot of figure 11 we see the motion of the vehicle for a very bad road surface (d=0.10m – cobbled road). The so produced trouble is the result of the vehicle motion from 0.00132 m to 0.00233 m or for 0.00101 m, which causes an annoying trouble.

4.2 Roughness on bridge

Let us consider now two road surface qualities, the bad road surface $(d \leq 0.04\,m)$, and a surface with cobbled road $(d = 0.10\,m)$.

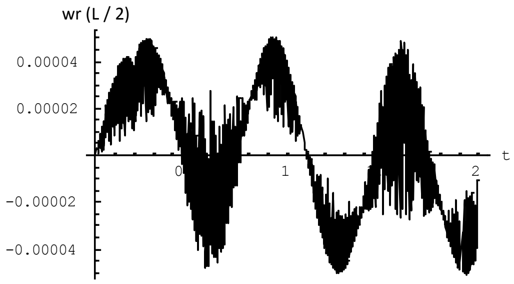
Applying the formulae of paragraph 3.2, we obtain the following diagrams related to the motion of the middle of the bridge for a vehicle moving with speed $\upsilon = 10\,m/sec$, and different kind of roughness.

In the following figure 12 we see the plots showing the motion of the middle of the bridge due to the passage of a light vehicle moving with speed 10 m/sec on a road surface with bad roughness $(d = 0.04\,m)$.



(a) (b)

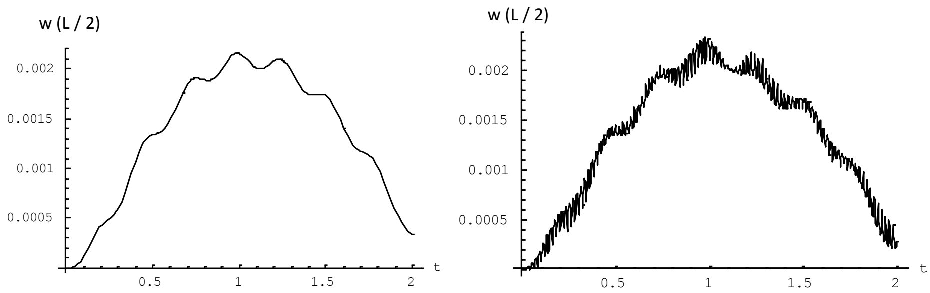
Figure 12. a) Bridge without roughness b) Bridge with roughness



(c)

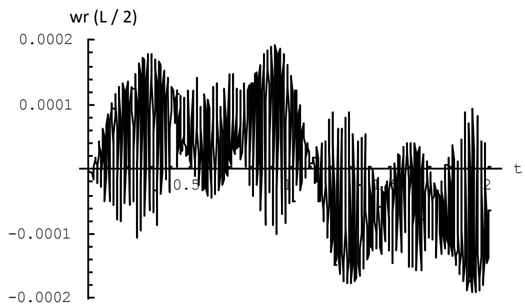
Figure 12. c) Deformation of the bridge due to roughness only

In the following figure 13 we see the plots showing the motion of the middle of the bridge due to the passage of a light vehicle moving with speed 10 m/sec on a surface with cobbled road $(d = 0.10\,m)$.



a) b)

Figure 13. a) Bridge without roughness b) Bridge with roughness



c)

Figure 13. c) Deformation of the bridge due to roughness only

In the two cases studied above, we observe that the trouble caused by the roughness of the bridge is slightly less than that caused by the roughness of the road.

The above observations and their causes, must be studied in detail taking into account the influence of the length of the bridge, the size of the vehicle, its speed, etc. The above is the subject of a future paper by the authors.

5 CONCLUSIONS

Passengers' trouble can be expressed by different ways.
The most usual and easy way is to use either the width of the motion due to the roughness, or the acceleration of this motion due to roughness alone, which happens in low periods. In this paper we follow the first way.
From the results of the models considered, one can draw the following conclusions:
Regarding the roughness on road:

- For smooth road surface with $d \le 0.01\,m$, the induced trouble can be considered as practically non-existent
- For middle road surface $0.01 \le d \le 0.02\,m$, the induced trouble is rather tolerable.
- For bad road surface $d \ge 0.03\,m$, the induced trouble is remarkable.
- Finally for a very bad road surface $d \ge 0.05\,m$, the trouble is extremely annoying.

Regarding the roughness on bridge:
We observe that the trouble caused by the roughness of the bridge is slightly less than that caused by the roughness of the road.

The above observations and their causes, must be studied in detail taking into account the influence of the length of the bridge, the size of the vehicle, its speed, etc. The above is the subject of a future paper by the authors.

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