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BOUNCING OF A VEHICLE ON AN IRREGULARITY

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ABSTRACT: This paper leads with the phenomenon of the bouncing of a vehicle due to an irregularity being on a road or on a bridge-deck. Attention is focused on the determination of the critical velocity for which the vehicle loses touch with the road's or the bridge-deck's surface following a missile's orbit and then striking on the road or the bridge during landing. If the vehicle moves with a velocity greater than the above critical one, we determine the corresponding time (and thus the point of the bridge) at which touch is lost. Afterwards, we determine also the landing point of the vehicle.

KEYWORDS: Bridge dynamics; Moving mass-loads; Bouncing; Irregularities.

1 INTRODUCTION

A lot of work has been reported during the last 100 years dealing with the dynamic response of railway bridges and later of highway bridges, under the influence of moving loads. Extensive references to the literature on this subject can be found in the excellent Frýba's book (1972).

Two early contributions, in this area, presented by Stokes (1849) and Zimmerman (1896) are very interesting. In 1905, Krýlov gave a complete solution to the problem of the dynamic behavior of a prismatic bar acted upon by a load of constant magnitude, moving with a constant velocity. In 1922, Timoshenko solved the same problem but for a harmonic pulsating moving force. Another pioneer work on this subject was presented in 1934 by Inglis, in which numerous parameters were taken into account. In 1951, Hillerborg gave an analytical solution to the previous problem by means of Fourier's method.

Despite the availability of high speed computers most of the methods used today for analyzing bridge vibration problems are essentially based on the Inglis's or Hillerborg's early techniques. Relevant publications are Saller's (1921), Jeffcot's (1929), Stending's (1934) ,Honda's and others'(1982), Gillespi's (1993), Green's and Cebon's (1994), Green's and others' (1995), Zibdeh's and Reckwitz's (1996), Lee's (1996), Michaltsos's and others' (1996), Xu's and Genin's (1997) , Foda's and Abduljabbar's (1998), Michaltsos (2001) and (2002).

On the other hand, in practice, in spite of the great number of works for over

50 years, bridges (as also other constructions which are acted upon by dynamic loads) have been designed to account for dynamic loads by increasing the design live loads by a semi-empirical “impact factor” or “dynamic load allowance”.

Recently, there have been many programs of research, discussing the effect of the characteristics of a bridge or a vehicle on the dynamic response of a bridge such as: the programs in U.S.A.(1977), in U.K. and Canada (1983), in the Organization for Economic Cooperation and Development (O.E.C.D.) (1992), in Switzerland (1972) etc.

Among the important studies in this field, we must especially refer to the important experimental research by Cantieri (1991dealing with different models of moving loads.

From the three factors (vehicle speed, matching of bridge and vehicle natural frequencies and irregularities and roughness of bridge surface deck) which affect the vibration of a bridge, the third is the one which has been more studied in the last years, mainly by experimental methods.

The literature counts in a lot of publications regarding the influence of deck roughness on the dynamic behavior of a bridge, but it is rather poor in number on the discussion of the irregularities’ effect on the vehicles’ and also on the bridge’s dynamic behavior.

We must refer to the studies of Chompooming and Yener (1995), that examines the influence of roadway surface irregularities and vehicle deceleration on bridge dynamic using the method of lines, which also ascertains the influence of the irregularities on the vehicle bouncing, Duaij et al. (1999), that examines the developed maximum acceleration of a moving single span bridge because the movement of a vehicle on even and uneven decks, of Michaltsos and Konstantakopoulos (2000), that studies the influence of an irregularity and the produced impact forces on the dynamic behavior of a bridge, of Pesterer and others (2004), that develops a technique to predict the dynamic contact forces arising after passing road surface irregularities by a vehicle, and finally of Stancioiu and others (2008), that examines the effect of bouncing of a moving oscillator on the vibrations of a beam.

In our earlier paper (Michaltsos – Konstantakopoulos 2000), was studied the influence of an irregularity on the dynamic behavior of a bridge under the assumption of a continuous touch vehicle and deck-surface.

The present paper examines the effect of an irregularity (lain on a road or on a bridge) on the vehicle behavior and also on the dynamic response of a bridge, without the above mentioned assumption.. Therefore, an irregularity with even entrance has been chosen and the two characteristic values of the vehicle’s velocity are determined. The first, above which the touch between vehicle and road or bridge deck surface is lost and therefore the vehicle flies like a missile launched, and the second, over whose the flying vehicle is landed beyond the end of the irregularity.

A 2-DOF model is considered for the solution of the bridge, while the theoretical formulation is based on a continuum approach, which has been used in the literature to analyze such bridges.

2. ANALYSIS

2.1. Irregularity on road

2.1.1. Assumptions

1. We assume that a mass-load, carried on a system of spring of constant k_o and of a damper of constant c_o, moves on the road with constant velocity υ_x (figure 1).

   At instant t=0, it meets the irregularity AB the shape of which is given by the equation

$$w = w_o(x) \tag{1}$$

2. Because of the limited length of such an irregularity, compared to the bridge length, one can assume that the velocity υ_x remains constant during crossing the irregularity.
3. Depending on the value υ_x of the velocity, the mass-load may either move in touch with the road (and the surface of the irregularity) or take-off following an orbit like the one of a missile-launched, and finally to land on point Δ (figure 1).
4. During flying the mass m_o of the wheel will be vibrating free, being hanged from the mass M, the orbit of which will be considered as the reference level for the above vibration of m_o.

2.1.2. Mathematical formulation

The total force acting on the road is:

$$F = M(g - \ddot{z}) + m_o(g - \ddot{w}_o) \tag{2}$$

Cutting at point G (figure 1a), and taking into account the equilibrium of forces, we get:

$$M\ddot{z} = -k_o(z - w_o) - c_o(\dot{z} - \dot{w}_o) \tag{3}$$

Due to equation (3) equation (2) becomes:

$$\left.\begin{array}{l} F = (M + m_o)g + (k_o z - c_o \dot{z}) - (k_o w_o + c_o \dot{w}_o + m_o \ddot{w}_o) \\ \text{where}: w_o = 0 \text{ for } x \le x_o \text{ or } x_o + e \le x \text{ and } w_o = w_o(a) \text{ for } x_o \le x \le x_o + e \end{array}\right\} \tag{4}$$

On the other hand, because $\dot{w}_o(x) = 0$, equation (3) may be written as follows:

$$\left.\ddot{z} + 2\beta_o\dot{z} + \gamma_o^2 z = \gamma_o^2 w_o\ , \quad \text{where}: 2\,\beta_o = \frac{c_o}{M}\ , \quad \gamma_o^2 = \frac{k_o}{M}\ \right\} \tag{5}$$

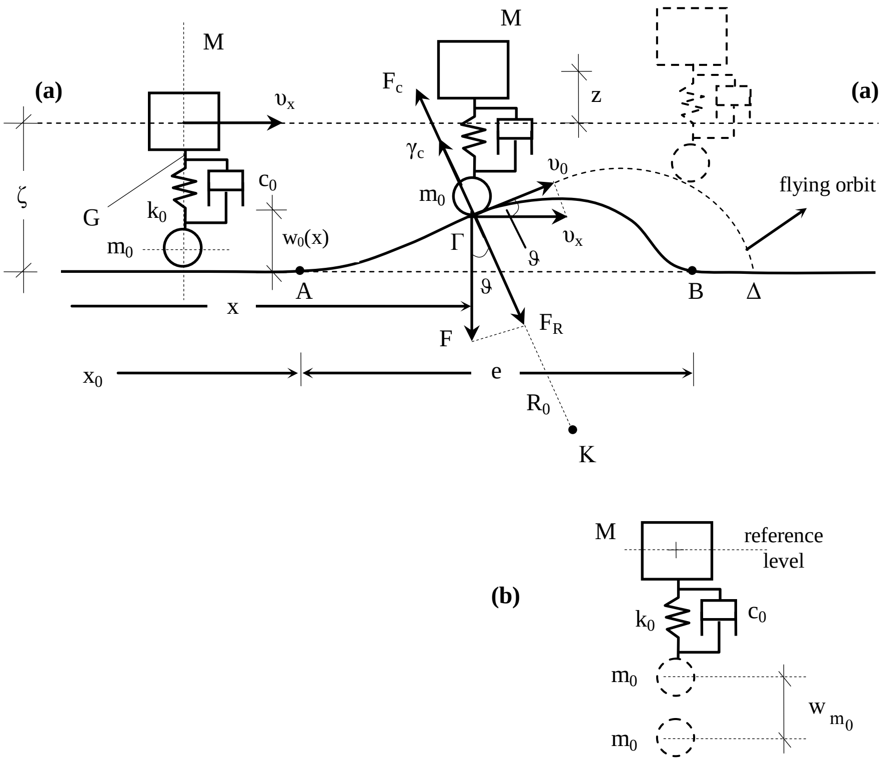


Figure 1. Irregularity AB on road crossed by a mass-load

The solution of equation (5), with initial conditions $z(0) = \dot{z}(0) = 0$, is given by the Duhamel's integral:

$$\left.\begin{aligned} & z(t) = \frac{1}{\omega_o}\int_0^t e^{-\beta_o(t-\tau)}\gamma_o^2 w_o \sin\omega_o(t-\tau)d\tau = \frac{\gamma_o^2 w_o}{\omega_o(\beta_o^2+\omega_o^2)}\left[\omega_o - e^{-\beta_o t}(\omega_o\cos\omega_o t + \beta_o \sin\omega_o t)\right] \\ & \text{where:}\quad \omega_o = \sqrt{\gamma_o^2 - \beta_o^2} \end{aligned}\right\} \quad (6)$$

Finally, equation (4), becomes:

$$F = (M+m_o)g + \frac{2k_o(\gamma_o^2-\beta_o^2)}{\gamma_o^2}w_o - \frac{\gamma_o^2 w_o}{\omega_o}e^{-\beta_o t}\left[\frac{k_o}{\beta_o^2+\omega_o^2}(\omega_o\cos\omega_o t + \beta_o\sin\omega_o t) + c_o \sin\omega_o t\right] \quad (7)$$

During the irregularity crossing, the mass-load takes the tangential speed υ_o (fig.1), that is given by the following relation:

$$\upsilon_o = \upsilon_x \cos\vartheta = \frac{\upsilon_x}{\sqrt{1+\tan^2\vartheta}} = \frac{\upsilon_x}{\sqrt{1+w_o'^2}} \quad (8)$$

The irregularity's radius of curvature is: $R_o = \frac{(1+w_o'^2)^{3/2}}{w_o''}$, and thus the developed centrifugal acceleration will be:

$$\gamma_c = \upsilon_x^2 \cdot \frac{w''_o}{(1+w'^2_o)^{5/2}} \tag{9}$$

The developed centripetal force, which causes the deviation of the vehicle is $F_c + (M+m_o)\gamma_c$, or finally:

$$F_c = (M+m_o)\upsilon_x^2 \frac{w''_o}{(1+w'^2_o)^{5/2}} \tag{10}$$

The restoring weight-force F_R, is given by the following relation:

$$F_R = F \cdot \cos\vartheta = \frac{F}{(1+w'^2_o)^{1/2}} \tag{11}$$

and therefore, the condition for a safe crossing of the irregularity without lost of the touch between wheel and road surface will be:

$$F_R \geq F_c \tag{12}$$

with F, given in equation (7).
From the above equation (12), we find the first critical speed υ_{1cr}, which determines the vehicle's behavior. On the other hand, from equation (12), for a known speed $\upsilon_x > \upsilon_{1cr}$, one can find the time t_{cr} and the point Γ of the vehicle's take-off , through the relation: $x_\Gamma = t_{cr} \cdot \upsilon_x$. At point $\Gamma(x_\Gamma, w_\Gamma)$, the vehicle loses the touch and thus follows an orbit like the one of a missile launched with initial speed υ_o at initial angle ϑ_Γ (figure 1). The equations of the orbit, in parametric form, are the following:

$$\left. x = x_\Gamma + \upsilon_o t \cos\vartheta_\Gamma\,, \quad h = w_{o\Gamma} + \upsilon_o t \sin\vartheta_\Gamma - \frac{g t^2}{2} \right\} \tag{13}$$

A vehicle moving with a speed greater than a value (depended on the irregularity's form), it will land beyond the point B, end point of the irregularity. This speed is an important parameter, that from now-on we will call "second critical speed" υ_{2cr}. υ_{2cr}. Eliminating the time t from the first of equations (13) we get:

$$h = w_{o\Gamma} + (x - x_\Gamma)\tan\vartheta_\Gamma - \frac{g(x-x_\Gamma)^2}{2\upsilon_o^2 \cos^2\vartheta_\Gamma} \tag{14}$$

Putting x=e and h=0 the above equation gives the 2nd critical speed as follows:

$$\upsilon_{2cr} = \frac{(e - x_\Gamma)\sqrt{g}}{\sqrt{2(e-x_\Gamma)\tan\vartheta_\Gamma + 2w_{o\Gamma}}} \tag{15}$$

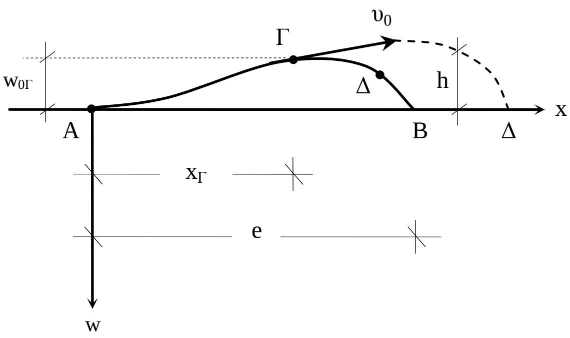


Figure 2. The orbit of a vehicle moving by speed greater than υ_{2cr}

The vehicle will arrive on point Δ in time t_Δ, which is obtained by the solution of the following equations:

$$\left.\begin{aligned} h(t_\Delta) = w_\Gamma + \upsilon_o t_\Delta \sin\vartheta_\Gamma - \frac{g t_\Delta^2}{2}, \text{for} \quad \upsilon_x < \upsilon_{2cr} \quad w_\Gamma + \upsilon_o t_\Delta \sin\vartheta_\Gamma - \frac{g t_\Delta^2}{2} = 0, \quad \text{for} \quad \upsilon_x > \upsilon_{2cr} \end{aligned}\right\} \tag{16}$$

Thus, the point Δ, on which the vehicle lands, is determined by the relations:

$$\left. x_\Delta = \upsilon_x t_\Delta, \quad h_\Delta = w_o(t_\Delta), \text{for} \quad \upsilon_x < \upsilon_{2cr} \quad \text{and} \quad h_\Delta = 0, \quad \text{for} \quad \upsilon_x > \upsilon_{2cr} \right\} \tag{17}$$

2.2 Bridge’s irregularity

2.2.1 Assumptions

In addition to the assumptions of §2.1.1, the following ones are considered:

1. We assume that on the deck of a single-span beam there is the irregularity AB of length e, that starts at point A ($x=x_A$) and ends at point B ($x=x_A+e$). The above irregularity has a form given by equation (1).
2. A mass-load like the one described in §2.1 moves on the beam with constant velocity.
3. The mass M, before its entering on the beam, is moving on the level a-a, from which are measured its displacements (figure 3a).
4. Moreover we assume that the mass-load enters the irregularity normally, i.e. without the appearance of impact forces.
5. Finally, we assume that the irregularity does not affect the beam’s characteristics (I and m), and that the critical speeds υ_{1cr} and υ_{2cr} can be determined by using equations (12) and (17) respectively.

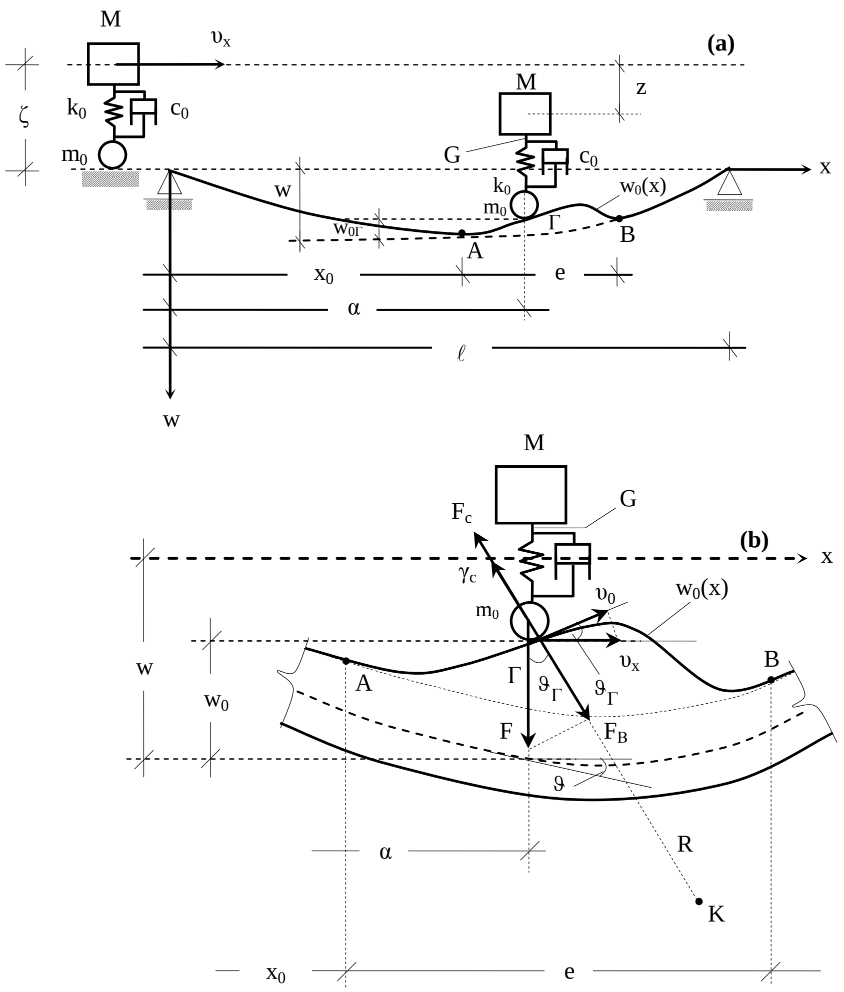


Figure 3. Bridge irregularity AB

2.2.2 Mathematical formulation

The acting forces on a bridge having at $x = x_o$ an irregularity are:

a. The force of the moving load as it is disturbed when it enters the irregularity:

$$F = M(g - \ddot{z}) + m_o[g - (w - w_{o\Gamma})^{\cdot\cdot}] = M(g - \ddot{z}) + m_o g - m_o \ddot{w} \tag{18a}$$

And $$z(t) = \frac{1}{\omega_o} \cdot \int_0^t e^{-\beta_o(t-\tau)}[\gamma_o^2 w(x,\tau) - \gamma_o^2 w_o + 2\beta_o \dot{w}(x,\tau)] \sin\omega_o(t-\tau) d\tau \tag{18b}$$

b. The impact force when the load exits the irregularity and so a speed $\bar{\upsilon}$ is added:

$$\bar{\upsilon} = \sqrt{\upsilon_o^2 + (\dot{w}_m + \dot{w})^2 + 2\upsilon_o(\dot{w}_m + \dot{w})sin\vartheta_\Delta}\,, \quad \hat{\alpha} = arcsin\left[\frac{\upsilon_o}{\bar{\upsilon}}cos\vartheta_\Delta\right] \quad (19)$$

Then the equations of motion will be:

$$\left.\begin{array}{ll}
a)\ \underline{\text{For } \upsilon_x \le \upsilon_{1cr}} EI_y w'''' + c\dot{w} + m\ddot{w} = [(M + m_o)g + \Phi_0^t(x,t)]\,\delta(x-a) & \text{for} \quad 0 \le t \le \ell/\upsilon_x \\
b)\ \underline{\text{For } \upsilon_x \ge \upsilon_{1cr}} EI_y w'''' + c\dot{w} + m\ddot{w} = [(M + m_o)g + \Phi_0^t(x,t)]\,\delta(x-a) & \text{for} \quad 0 \le t \le x_\Gamma/\upsilon_x \\
EI_y w'''' + c\dot{w} + m\ddot{w} = 0 & \text{for} \quad x_\Gamma/\upsilon_x \le t \le x_\Delta/\upsilon_x \\
EI_y w'''' + c\dot{w} + m\ddot{w} = [(M + m_o)g + \Phi_{t_\Delta}^t(x,t)]\,\delta(x-a) + & \\
\qquad + P_{imp}\,\delta(t - t_\Delta)\delta(x - x_\Delta) & \text{for} \quad x_\Delta/\upsilon_x \le t \le \ell/\upsilon_x
\end{array}\right\}$$

(20a)

where:

$$\left.\begin{array}{l}
P_{imp} = (1+\varepsilon)\cdot\bar{\upsilon}\,m_o\cos\alpha \quad \text{the impact force that acts at the landing instant} \\
t_\Gamma: \quad \text{is the time needed for the arrival of mass-load on point } \Gamma \text{ (lost of touch)} \\
t_\Delta: \quad \text{is the time needed for the landing of mass-load on point } \Delta \text{ (end of flying)} \\
w_o = 0 \qquad \text{for} \quad t \le x_\Gamma/\upsilon_x \quad \text{or} \quad t \ge x_\Delta/\upsilon_x \\
w_o = w_o(x) \quad \text{for} \quad x_\Gamma/\upsilon_x \le t \le x_\Delta/\upsilon_x
\end{array}\right\}$$

(20b)

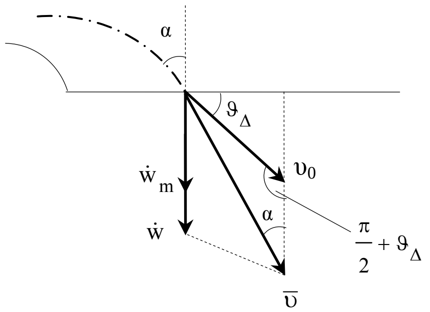


Figure 4. Impact velocity and impact angle

Clearly, a closed form solution of equations (20a) is not possible. However, one can seek approximate solutions, based on previous pertinent works (Kounadis 1985).

3. NUMERICAL EXAMPLES AND DISCUSSION

The purpose of this paper is to study the influence of the following parameters on the dynamic response of a bridge:

a) The shape and the position on the road or on the bridge of an irregularity.

b) The model used, its constant of spring, and the critical velocity on which the vehicle strikes (or not) on the bridge.

One should note also the following:

- The vehicles are supposed to move along the center line of the bridge.
- The displacements in the middle of the span of the bridge are determined.
- For the bridge's oscillations only the six first flexural modes are taken into account.

3.1 Data

3.1.1 Type of irregularity

The type of irregularity used, is shown in figure 5, while its form is given by the following equation (see and figures 1 and 2):

$$w_o(x) = \frac{f}{\alpha^4}(x - x_o)^4 - \frac{4f}{\alpha^3}(x - x_o)^3 + \frac{4f}{\alpha^2}(x - x_o)^2 \qquad (18)$$

Two types of irregularity of the above form are used. The type 1, a rather smooth irregularity, with a=2.50m and f=0.10m, and the type 2, a rather sharp one, with a=2.50m and f=0.30m. Considering their position on each quarter of the span of the bridge, we studied their effect on the behavior of the bridge.

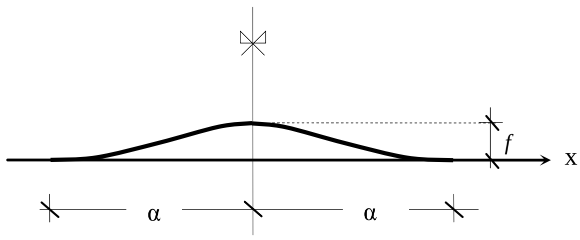


Figure 5. Shape of the irregularities used

3.1.2 The vehicle

As it is proved [5], for a wheelbase smaller than the 1/6 of the bridges span, one can use, without loss of the accuracy, the one-axis model instead of the real with two-axes. In this paper we use a light vehicle with the following data: M=200kg, m_o=20kg, k_o=2000dN/m, c_o=100dN sec/m. The vehicle's velocity used depends on the irregularity shape or, in other words, on the velocities υ_{1cr} , and υ_{2cr} .

3.1.3 The bridge

We consider a one-span bridge of length $\ell = 100\text{m}$, $I = 0.65\text{m}^4$ and $m = 1000\text{kg/m}$.

3.2 Irregularity on road

3.2.1 Irregularity of type 1

Let us consider now that the vehicle of §3.1.2 moving on a flat road with constant velocity υ_x, meets an irregularity of type 1.

From equations (12) and (15) we find respectively: $\upsilon_{1cr} = 12.6162\,m/sec$, and $\upsilon_{2cr} = 15.2973\,m/sec$.

1. We assume that the vehicle moves with velocity: $\upsilon_{1cr} < \upsilon_x = 14\,m/sec < \upsilon_{2cr}$.
   Thus, according to equation (12), the vehicle will lose the touch with the road at point Γ with: $x_\Gamma = 2.495\,m$, at time $t_\Gamma = 0.1997\,sec$, while according to equation (17) it will land on point Δ (within the irregularity) with $x_\Delta = 3.5666\,m$, at time $t_\Delta = 0.25476\,sec$.
2. We assume now, that the vehicle moves with velocity $\upsilon_x = 25\,m/sec > \upsilon_{2cr}$.
   Thus, according to equation (12), the vehicle will lose the touch with the road at point Γ with: $x_\Gamma = 1.2549\,m$, at time $t_\Gamma = 0.0502\,sec$, while according to equ. (17) it will land on point Δ (beyond the end of irregularity) with $x_\Delta = 9.7268\,m$, at time $t_\Delta = 0.3891\,sec$.

3.2.2 Irregularity of type 2

We consider now that the vehicle of §3.1.2 moving on a flat road with constant velocity υ_x, meets an irregularity of type 2.

From equations (12) and (15) we find respectively: $\upsilon_{1cr} = 7.5173\,m/sec$ and $\upsilon_{2cr} = 8.9872\,m/sec$.

1. We assume that the vehicle moves with velocity $\upsilon_x = 14\,m/sec$, which in the present case is $\upsilon_x > \upsilon_{2cr}$. Thus, according to equation (12) the vehicle will lose the touch with the road at point Γ with: $x_\Gamma = 1.2898\,m$ at time $t_\Gamma = 0.09213\,sec$, while according to equation (17) it will land on point Δ (beyond the end of the irregularity) with $x_\Delta = 9.1797\,m$ at time $t_\Delta = 0.6557\,sec$.
2. We assume now that the vehicle moves with velocity $\upsilon_x = 25\,m/sec > \upsilon_{2cr}$.
   According to equation (12), the vehicle will lose the touch with the road at point Γ with: $x_\Gamma = 1.1252\,m$ at time $t_\Gamma = 0.04501\,sec$, while according to equ. (17) it will land on point Δ (beyond the end of irregularity) with $x_\Delta = 24.9799\,m$ at time $t_\Delta = 0.9992\,sec$.

3.3 Irregularity on bridge

We will study the above two types of irregularity (see §3.1.1), which lie on the bridge and whose their start A is located on points $\ell/4,\ \ell/2,\ 3\ell/4$.

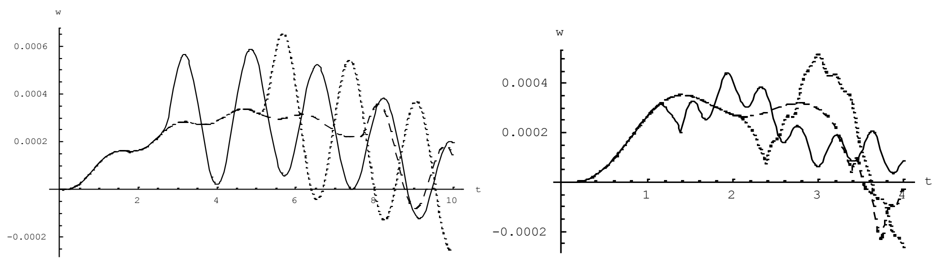
For each one of the above irregularities we will study the influence on the dynamic behavior of the bridge for the cases of an under critical speed and of an overcritical one, especially the influence of the developed impact forces..

3.3.1 Irregularity of type 1

The diagram of figure 6 shows the oscillations of the middle of the bridge produced by a vehicle moving with velocity υ_x =10 m/sec (under critical speed), or υ_x =25m/sec (overcritical speed).

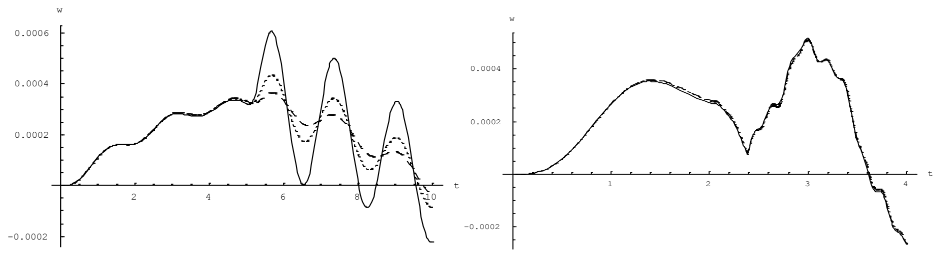
Note also that for υ_x =10 m/sec the position of the irregularity strongly affects the above deflections. The more unfavorable positions are around the middle of the bridge.

We ascertain, for υ_x =25m/sec, that the interposed time of about 0.339 sec, during which the bridge performs free vibrations (due to the flight of the vehicle), decreases the influence of the vehicle impact and, of course, the amplitude of the deflections of the bridge.



a) υ_x =10 m/sec b) υ_x =25 m/sec

Figure 6. Irregularity of type 1 (a=2.5, f=0.1) xo=L/4 (___), xo=L/2 (....), xo=3L/4 (-----) for ko=2000, co=100



a) υ_x =10 m/sec b) υ_x =25 m/sec

Figure 7. Irregularity of type 1 (a=2.5, f=0.1) Deformations of the middle of the bridge for ko=2000 (___), ko=1000 (…..), ko=500 (---), co=100, xo=L/2

The diagram of figure 7 shows the influence of the spring constant k_o on the oscillations of the middle of the bridge produced by a vehicle moving with velocity υ_x =10 m/sec (under critical speed) or υ_x =25m/sec (overcritical speed), and an irregularity with $x_o = \ell/2$. We see that for υ_x =10 m/sec and stiffer springs increase the deflections up to 80%, while for υ_x =25m/sec the influence is negligible.

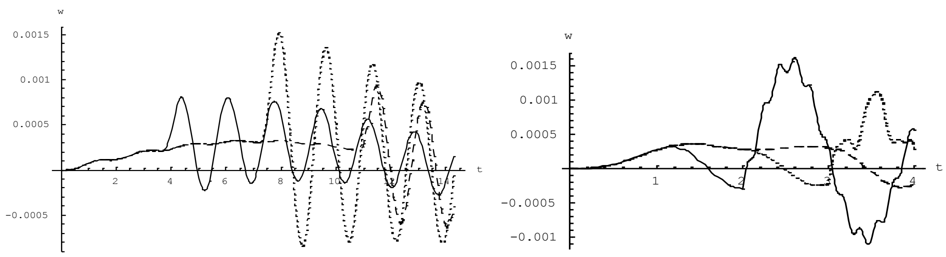
3.3.2 Irregularity of type 2

The diagram of figure 8a shows the oscillations of the middle of the bridge produced by a vehicle moving with velocity υ_x =7 m/sec (under critical speed). Clearly the deflections are increased from 1.5 times (for $x_o = 3\ell/4$) to 3 times (for $x_o = \ell/2$) the deflections of the middle of the bridge with a deck without irregularity. The more unfavorable positions are around the middle of the bridge, while favorable ones are around the third quarter of the span of the bridge.

The diagram of figure 8b shows the oscillations of the middle of the bridge produced by a vehicle moving with velocity υ_x =25m/sec (overcritical speed).

We see that though the interposed time of about 0.9542 sec (during which the bridge performs free vibrations because of the flight of the vehicle) is greater in the present case, it is not able to decrease the influence of the vehicle impact and, of course, the amplitude of the deflections of the bridge.

On the other hand we observe that due to the longer flight, the influence of irregularities (lain beyond the middle of the bridge) is smaller while it becomes negligible for $x_o > 3\ell/2$.

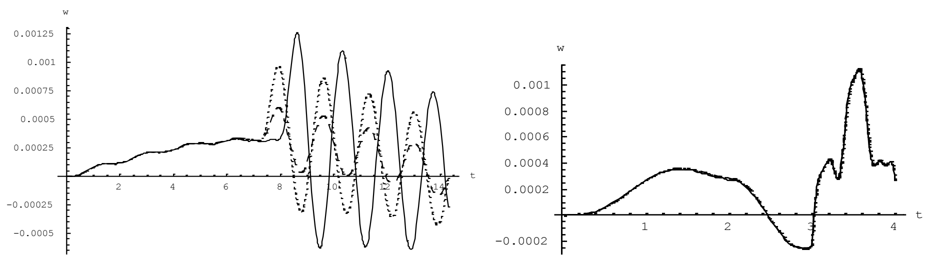


a) υ_x =7 m/sec b) υ_x =25 m/sec

Figure 8. Irregularity of type 2 (a=2.5, f=0.3) xo=L/4 (_ __), xo=L/2 (….), xo=3L/4 (-----) for ko=2000, co=100, υ_x =7 m/sec

The diagram of figure 9a shows the influence of the spring constant k_o on the oscillations of the middle of the bridge produced by a vehicle moving with velocity υ_x =7 m/sec (under critical speed) and an irregularity with $x_o = \ell/2$.We see that, in this case of a sharp irregularity, the stiffer springs increase much more (up to 150%) the deflections.

Finally, the diagram of figure 9b shows the influence of the spring constant k_o on the oscillations of the middle of the bridge produced by a vehicle moving by velocity υ_x =25 m/sec (over critical speed). Clearly the influence in this case is, also, negligible.



a) υ_x =7 m/sec b) υ_x =25 m/sec

Figure 9. Irregularity of type 2 (a=2.5, f=0.3) Deformations of the middle of the bridge for ko=2000 (___), ko=1000 (…..), ko=500 (----), co=100, xo=L/2 and υ_x =7 m/sec

4. CONCLUSIONS

From the results of the model considered, one can draw the following conclusions:

1. Two velocities, the υ_{1cr}, υ_{2cr}, that are depended on the shape of the irregularity and produce the take-off or not of a vehicle moving on the bridge, are determined.
2. The influence of the existence of the irregularity on the developed deflections of the bridge is significant even if the vehicle moves with velocity less than the υ_{1cr} .
3. If a vehicle moves with velocity greater than υ_{1cr}, and υ_{2cr}, loses touch with the deck-bridge and flies following an orbit like the one of a launched missile. At the end of this flight, the vehicle lands with impact on the bridge. The so-developed impact forces produce deflections much more greater than the ones caused by the same vehicle crossing the same irregularity without taking into account the loss of touch with the bridge.
4. The value of the spring constant k_o affects the dynamic behavior of the bridge. Soft springs produce small oscillations while stiffer ones may increase the amplitude of oscillations up to 80%.
5. The position of the irregularity strongly affects the deformation of the bridge.

For under critical velocities the more unfavorable positions are around the mid-span of the bridge while the favorable ones are around the third quarter of the span of the bridge. For overcritical velocities the unfavorable positions remain the same, but the increase of the oscillation amplitudes is much more higher, while the influence of irregularities (lain beyond the $3\ell/5$ of the span) is practically negligible.

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