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STRUCTURAL RESPONSE OF CONCRETE SKEW BOX-GIRDER BRIDGES

A State-of-the-Art Review

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ABSTRACT: Box shaped support girders are considered as the most efficient support section for bridges and flyovers, as they provide enhanced structural efficiency, superior stability and serviceability of the bridge, as well as pleasing aesthetic appearance and better economy. In case of multi-lane bridges, multi-spine and multi-cell box sections are commonly used. Sometimes, due to space constrain, skew bridges are built especially in congested metropolitan areas. Due to presence of skewness, the structural behavior of box-girder bridges is significantly affected. Several experimental and numerical studies have been made to examine the effect of skewness on the structural response of box-girder bridges subjected to static and/or dynamic loads. The aim of the current paper is to review the literature published on the structural behavior of skew box-girder bridges subjected to static & dynamic loads including seismic effects. Moreover, this study also reviews the effect of skewness on load distribution among the multi-spine/cell box-girders bridges and presence of diaphragms in the bridge.

KEYWORDS: Box-girder bridge, Seismic Response, Skew angle, Structural Response.

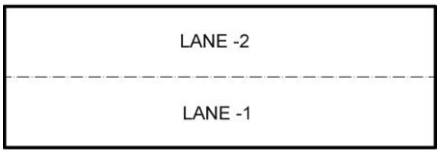
1 INTRODUCTION

Due to population growth and rapid urbanization, there has been an enormous growth in traffic volume on highways over the last few decades. In order to ensure smooth flow of traffic, numerous new highways and flyovers are being constructed. The use of box-girders has proven to be a very efficient structural solution for highway bridges and flyovers due to its high torsional rigidity, serviceability, economy, aesthetics and the ability to efficiently distribute the eccentric vehicular live load among the webs of the box-girder. For the multi-lane bridges, multi-spine/cell box-girders are most commonly adopted in order to limit the local deformations in the top slab of box. Sometimes, due to limitation to space, situation demands to go for intricate bridge geometry where the bridge is supported on skew supports. These types of bridges are referred as skewed bridges and the skewness of the bridge is expressed in terms of skew angle. Generally, angle of skew in non-curved sections is known as the angle between a line perpendicular to the centerline of the bridge and the centerline of the support section (abutment or pier). While for the bridges curved in plan, skew angle is defined as the angle between a centerline of support and a line along the radial direction of the bridge curvature.

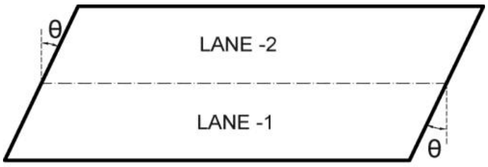
Skew support arrangements significantly alter the structural behavior of the bridge, which should be taken into account while designing the bridge (Chun 2010). Analysis and design of box-girder bridges in itself is complex since its behavior involves longitudinal & transverse bending, torsion, distortion, warping and shear lag and it becomes even more complicated when it is supported on skew supports. Generally, for right bridges analysis is done using techniques such as orthotropic plate theory, grillage method, folded plate method, finite difference method, finite strip method, or finite element method (Sennah and Kennedy 2001-2002). For small skew angles (< 20°-25°), bridges may be designed similar to right bridges with little modifications as suggested by different design guidelines for various countries (Chen and Duan 2014). However, for larger skew angles, which may go up to 75°, the detailed analysis of the bridge is generally performed using finite element method.

Moreover, for the design of multi spine/cell box-girder bridge, determination of design forces in an individual spine/cell is a complex issue in itself, further, presence of skewness all the more complicates the task of estimating the design actions in the spine/cell of the box-girder bridges. The distribution of vehicular live loads in different spines/cells is universally characterized by a Live Load Distribution Factor (LLDF), which is conceptually defined as a factor used to multiply the total longitudinal response of the bridge in order to determine the maximum response of the girder (Newmark 1948). Generally, LLDF is extensively determined using simplified approaches and codal guidelines, but many times such values turn out to be either over conservative or even dangerous in some situations.

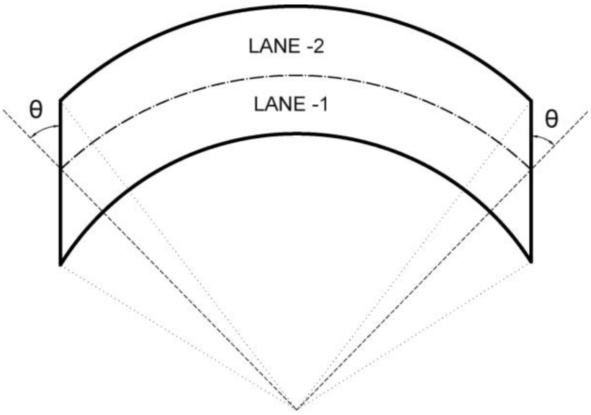
In past a number of analytical and experimental investigations have been made to understand the structural behavior of skewed box-girder bridges, as well as to develop the load distribution mechanism among the spines/cells of the multi spine/cell box-girders. However, most of these studies deal with the behavior of bridge in elastic range only ignoring nonlinear behavior or concrete. In order to fully understand the behavior of skewed box-girder bridges and live load distribution pattern, still there is a need of in-depth investigations. The aim of this paper is to provide current up to date literature review on the influence of skew angle on the structural behavior and live load distribution of box-girder bridges under static, dynamic and seismic loading. Also, present effort also reviews the extent to which skewness affect diaphragm positioning in box-girder bridges.



(a) Right Bridge



(b) Skewed Bridge



(c) Skewed and Curved Bridge

Figure 1. Plan view of various bridges showing angle of skew θ: (a) Right bridge – No skew; (b) Skewed bridge – No curvature; (c) Skewed and Curved Bridge

2 STRUCTURAL RESPONSE OF SKEWED BOX-GIRDER BRIDGES UNDER STATIC LOADING:

Since the inception of research on skew bridges in 1948, (Newmark et.al) it is well established that skew bridges are more susceptible to damage, since the force experienced by them gets significantly altered as compared to their right counterparts (for skew bridges having skew angle more than 15°). Unfortunately, due to lack of experimental and computational advancements, in initial years no research could clearly indicate the exact effect of skew on structural response of the bridge. With the advent of computers and advanced computational techniques, numerous researchers have studied the effect of skewness on bridges, however, most of the research is focused on the slab culverts and slab and girder bridges. Generally, it is observed that highly skewed bridges tend to attract more torsion, thus making the design more complicated, on the other hand the relieving fact is that as the skew angle increases, generally the longitudinal moment on the whole bridge reduces, thus providing the balancing effect. In the skewed bridges the load path follows the shortest route thus as a result the obtuse corner reactions are found noticeably large as compared to acute corner reactions, while the acute corners get the possibility of uplift also. Moreover, the obtuse corners are found to attract more negative (hogging) moments. Also, the transverse bending moments are found to be constantly increasing with increase in angle of skew.

Many researchers have proven above mentioned responses via various experimental, numerical and analytical studies, their works in case of skewed box-girder bridges can be summarized as follows:

In 1970, Sisodiya and his colleagues tested simply supported two-span continuous single cell box-girder bridge model made of aluminum alloy, which was curved in plan and was supported on skewed supports. The bridge model considered was having span 41 in (1.04 m), width 8 in (0.2 m) & depth 1.2 in (0.03 m) and was carrying a concentrated load (P) positioned at mid span of the inner web. Finally, they compared vertical deflection, tangential strains and maximum stresses present in webs at mid span via F.E results and experimental findings. Via 2-D finite element analysis, effect of aspect ratios chosen (1:1, 1:2, 1:4) for rectangular element was investigated. Results showed that there was not much noteworthy difference in the 2-D finite element results using rectangular elements with aspect ratio 1:1 and the results obtained from beam theory using shear deformation.

Brown and Ghali (1975) presented a semi-analytical procedure to analyze simply supported skew box-girder bridge using finite strip method and compared their results with the outcomes obtained via finite element method. They considered three examples out of which two were four-cell rectangular sections and the third was single cell trapezoidal section. The four-cell box-girder models were previously experimentally tested by Godden and Aslam (1971) and were having skew angle 30° and 45° and span 29.66 in (0.753 m) and 35.5 in (0.902 m) respectively. All the girders were idealized as a combination of parallelogrammic strips connected along nodal lines. They compared the results for deflection, longitudinal bending stresses and the transverse bending moments for different skew cases. The deflections were noted at 11 locations; in general, the deflections were found to be reduced by about 45% with the increase of skew angle from 30° to 45°. Moreover, the longitudinal bending stresses and the transverse bending moments were found in good agreement and thus, it was concluded that results obtained via finite strip method were as accurate as that obtained from finite element method and experimental study.

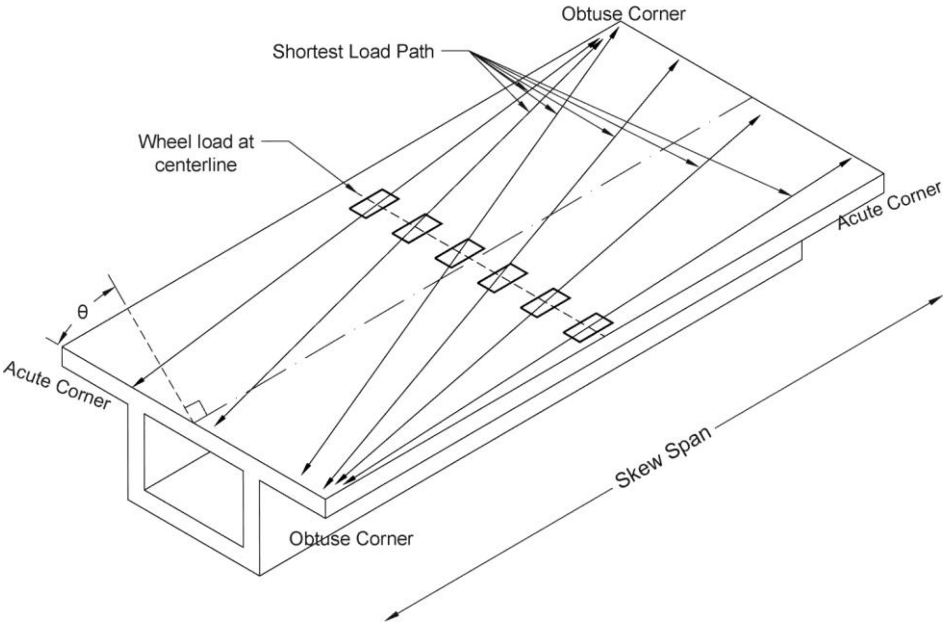


Figure 2. Wheel Load distribution path in skew box-girder bridges (Skew angle represented by θ).

Bouwkamp and his associates (1980) performed an experimental study to investigate the effect of skewness on a multi-cell two-span continuous box-girder bridge. They tested a 1: 2.82 scale model of a 61 m long two-lane, four-cell box-girder bridge located at California having 45° skew bent central support. The bridge model was subjected to concentrated loads at mid length of each span and the loads were gradually increased up to collapse of bridge. Based on the study conducted it was observed that at the obtuse corners attract high reactions as well as significant end moments and torsional moments, which reflected the tendency of the skew-bridge to span across the shortest load path between the supports. Moreover, it was suggested that the vertical reaction, bending moment and torsional moment may be represented by an eccentric resultant concentrated load. By comparing the results of right and skewed geometries they concluded that due to the skewness, mid-span and support moments (positive as well as negative moments) reduces for the live load as well as dead load case. It was also observed that experimental model sustained ultimate loading of dead load plus 4.5 times live load before failure. Such raising of collapse load was explained due to transfer of support failure zone as a consequence of skewness present in the bridge.

Scordelis and colleagues (1972, 1982) conducted an experimental study on a 45° skew reinforced concrete bridge model to check the adequacy of analytical design methods in terms of deflection, reaction, strain and moments generated in the bridge. They proposed the total moment in right mid-span section of simply supported 45° skew bridge to be 0.07WL (W being the weight and L being the total length of the bridge) which was 56% lesser than the simply supported right bridge case (0.125 WL), thus they advocated use of less number of longitudinal steel bars in the section. Further, they investigated the typical California continuous box-girder bridge tested by Bowcamp and results were taken for 4 load cases namely (a) dead loads (b) working stress loads (c) Overloads (d) failure loads. Based on the results it was predicted that skewness effect was dependent upon (a) Skew angle (b) span to width ratio (c) position of applied loads and (d) simply supported or continuous end conditions chosen. Researchers also noted that while point loads were placed on obtuse side they produced lower positive design moments while loads on acute side resulted in higher positive design moments. Above behavior was explained in terms of attraction of negative moments at obtuse end side due to presence of skewness. They also recommended that in skew bridges transverse moment is not uniformly distributed thus use of load distribution factor as per right bridge approximation should be avoided.

Wasti and colleagues (1984) conducted an experimental study on three large scale, two span, four cell, reinforced concrete box-girder bridge models which were subjected to dead load, working loads and overloads, including loading up to failure to compare various structural responses such as reactions, deflections and moments of right, skewed and curved bridges. The models tested were having same cross-sectional dimensions but three were different in plan i.e., one was right, other was skewed and third was curved. The dimensions of models were 72 ft (21 m) long along the longitudinal centre line, 12 ft (3.7 m) wide and 1.71 ft (0.52 m) deep. The skew bridge was having skew angle of 45° and the radius of curvature of curved bridge was 100 ft (30.48 m). It was observed that the skew bridge showed significant amount of changes in the vertical reactions depending on the transverse positioning of the load. When the acute side of span was loaded, the adjacent end reaction was smaller than that of the corresponding right and curved bridge cases. When the loads shift from the acute to the obtuse side of the skew span, the reaction increased drastically than that of the right and curved bridge models. The mid-span deflection and longitudinal bending moments in the skew bridge were even found dependent upon the transverse location of the load on the top deck. It was also shown that higher deflections and moments were developed, when the loads were placed on the acute side of the span, while lower moments and deflections were observed when the loads were placed on the obtuse side of the span. It was demonstrated that in the case of the skew bridge only, end moments and torsional moments were produced for all load cases considered, which led to conclusion that its behavior was most complex out of the three geometries considered.

Paavola (1992) developed a numerical model based on Vlasov's theory to analyze the thin walled box-girder and coded a computer program in FORTRAN77. His study included a continuous skewed concrete box-girder bridge of length 80.0 m, width 6.0 m and depth 3.0 m. The bridge was simply supported at the ends and at the mid-span it was resting on a skew support line whose angle was varied from 0° to 60°. The bridge was stiffened by diaphragms which restrained distortion at ends and at skew support line. The bridge was loaded symmetrically by two line loads above the webs having value of 100 kN/m. He studied the effect of skewed support line on the deflection and stresses. Based on the study it was shown that, as the angle of skew increases maximum deflection tends to decrease throughout the span, while axial stresses were not found much affected.

In 2005, Jun-Tao and his associates (2005) carried out the analysis of a slant-leg rigid frame 60° skew continuous prestressed concrete box-girder bridge which was not having any abutment, to study the lateral horizontal displacement and rotation of bridge at supports. They suggested three main forces which developed in-plane displacements and rotation in skew bridge namely: (i) longitudinal forces due to braking and seismic effects, (ii) lateral forces due to wind and seismic effect, and (iii) forces due to temperature variation, concrete shrinkage, creep, and prestressing force. They concluded that slant-leg rigid frame bridges without abutments by the virtue of having more horizontal stiffness have advantage over other bridge sections.

Ashebo and his colleagues (2007) evaluated the effect of skewness on longitudinal bending moment in an existing skew continuous box-girder bridge. They tested the Tsing Yi south twin-cell box-girder bridge which was continuous over three spans having first and last span as 23 m long while middle span was 27 m long. The carriageway width was 10.58 m and the bridge was having a 27° angle of skew. As a preliminary result of the study they observed that as higher skew angles were applied in F.E modelling of the bridge maximum span bending moment decreased, such decrease was swifter in cases when skew angles were increase more than 30°.

Grace and colleagues (2011) constructed, instrumented, and tested two 30° skew precast, prestressed concrete four cell half-scale box-girder bridge models which were reinforced with carbon-fiber-composite cable [CFCC] and conventional steel strands respectively, to study their structural responses. This investigation compared flexural behavior of above mentioned bridges in terms of the load-deflection response, load versus strain response, ultimate strength, failure mode and energy ratios. They also considered transverse post-tensioning (TPT) with TPT diaphragms, to check the excessive cracking in the longitudinal direction of the deck. Model bridges were 31 ft (9.5 m) long, 18 in. (0.46 m) wide and 11 in. (0.280 m) deep. The reinforced concrete bridge model was designed to be under-reinforced while CFCC-reinforced bridge model was designed to be over-reinforced. The bridge models were tested in two stages, namely un-cracked deck slab and cracked deck slab stage. The cracking load of the CFCC bridge model was found 25% higher as compared to conventional RC bridge model. The ultimate load of the CFCC bridge model was found 17%

higher when compared to conventional RC bridge model. Also, it was noted that CFCC model suffered a compressive failure due to crushing of concrete while the normal model had a tensile failure due to reinforcement yielding. Interestingly, both the models demonstrated approximately same deflection at ultimate load capacity.

Zhang and Lin (2012) proposed a thin walled box-girder element with ten degrees of freedom and coded 1-D finite element program SSCBA in FORTRAN. The accuracy of the program was tested with Ansys results using Shell63 element and the experimental data. They used this program to simulate the shear lag effect, torsion and warping state of a skewed box-girder bridge. For experimental investigation they built a plexi-glass three span continuous, 60° skewed, two cell box-girder bridge model in laboratory which was 3.2 m long and 0.4 m wide. The stress distribution was measured along the width of top & bottom plate at mid-span and at the junction of first & second span. Based on the stresses obtained they recommended that the structural behavior of skew continuous box-girder bridge, subjected to eccentric concentrated load is significantly affected due to skewness and its impact should not be ignored.

He and his associates (2012) presented the testing and analysis results of a 1:8 scale model of a 45° skewed prestressed concrete continuous box-girder bridge, having span lengths as 40 m, 70 m and 40 m. The scaled model produced in laboratory had 19 m overall span having 8.75 m main span and 5 m each side spans. Measured stresses and computed results ascertained that the tested bridge remained in elastic zone only. Test results showed that unsymmetrical loading (live load) produced lower torsional load than symmetrical loading case (live load), also the twist effect were observed lower in center span. Similarly, the vertical reactions at the pier clearly indicated that obtuse corner attracted much force while in unsymmetrical loading case one of the acute angle bearing got uplifted. Further to check the influence of skew angle on several response parameter authors generated finite element models ranging from 0° skew to 60° skew angle. Based on the results they found that as the skew angles increased, the vertical bending moments and deformations decreased, however, torsional stresses and deformations increased as well as the differential reaction levels were found enhanced with increase in skew angle. Thus they concluded that bridges having skew angle more than 45° are not suitable of high speed railway construction. Also the authors recommended the use of multiple moving load travelling in opposite direction in skew bridges for checking the possible uplifts. Moreover, the results of parametric study revealed that as width to span ratio was increased, vertical displacement and torsional deformation decreased but torsional stresses increased.

Soliman and his colleges (2000) presented an experimental study on skew box-girder bridges to find out the effect of skewness on warping displacements and vertical reactions. Based on experimental verification by a specimen model they developed a non-linear finite element program to find out the reactions and warping displacements, and based on which they presented results for three different skew angled bridges namely 0°, 25° and 40°. They considered a simply supported 2.6 m long, 0.40 m wide and 0.26 m deep single box-girder section which carried same reinforcement in each of the skew case considered. A concentrated eccentric load was considered at the mid-span section of outer web in each case. They noted that with increasing torsional moments, warping displacement increased which was elucidated due to decrease in overall stiffness of the skew bridges. It was also found out that bottom slab attracted higher warping displacements as compared to top slab irrespective of skew angle considered. Moreover, it was observed that acute angle corners attract less warping displacements than obtuse corner of the bridge, but in general with increasing skew these displacements were found to be increasing. Also, this study reaffirmed that obtuse corner attract higher vertical reactions than acute corners and there exist the possibility of uplift at acute corners in eccentrically loaded skew bridge.

3 DIAPHRAGMS IN SKEW BOX-GIRDER BRIDGES

Construction of diaphragms in skew bridge is always considered as a challenge. In general, for bridges having skew angle 20° or less either orthogonal or skewed intermediate diaphragms can be provided, however, for highly skewed bridges intermediate diaphragms normal to the girders are recommended in staggered formation (Chen et al 2014). Dilger and his associates (1988) have made an interesting study which showed that skew box-girder bridges behave better in absence of any kind of diaphragm. The study included comparison between traditional skew diaphragms, orthogonal diaphragms and the bridge structure without any diaphragm. They considered a two span continuous 45° skewed concrete box-girder bridge (total length 80 m, width 12 m, depth 2 m) with above mentioned three different diaphragm conditions. In addition to dead load, a concentrated live load of 1000 kN and prestressing loads were implemented in the computer modeling using grillage as well as F.E. modeling of the bridge. It was observed that bridge without any diaphragm attracted highest bending moments and it had lowest torsional rigidity, while orthogonal diaphragm case attracted highest obtuse corner support reaction and lowest acute corner support reaction and provides highest torsional rigidity. Although in case of bridges without diaphragms high torsional moments were observed near obtuse corner thus requiring extra thickness of end slab still this configuration was advocated by researchers as it provides ease of construction and easy interior access of the box section.

Scordelis et al (1982) also observed that un-diaphragmed span suffered more deflection than the diaphragmed span, however, super positioning of deflection still held true. Further the same study was continued by the authors and they overloaded the bridge model to study the effect of stiffness degradation and cracking in the section and finally to study the collapse behavior of skew box-girder bridge. They concluded that skew box-girder bridge did not reduced load carrying capacity. As the allowable stress in steel was increased from 24 ksi (165 MPa) to 60 ksi (414 MPa), still the end reaction values and girder moment distribution were found in same range by both experimental investigations and via cell program values (although the program was based on un-cracked section theory). For loading up-to total collapse, first un-diaphragmed span was loaded for which first yielding occurred in mid-span bottom longitudinal steel at 160 kip (712 kN) while total collapse occurred at 207.1 kip (921.2 kN) and 208.5 kip (927.5 kN) for first and second load cycle, while for diaphragmed span first yielding occurred at same load but complete collapse load was observed to be 243 kip (1081 kN). However, diaphragmed span sustained 16.2 in (0.413 m) total deflection before collapse as compared to 14.3 in (0.363 m) in un-diaphragmed section. They also concluded that presence of skewness in bridge reduced total girder moments in both central support region and span thus total collapse load were increased to a significant level as compared to the right bridge.

4 LIVE LOAD DISTRIBUTION IN SKEW BOX-GIRDER BRIDGES

Multi-cell box-girder bridges are widely used for multi-lane highway bridges. Due to the application of vehicular live loads on the bridge decks, all the boxes undergo longitudinal as well as transverse bending moments and shear forces. As an effect, the webs tend to twist along the major axis to maintain the comprehensive stability of deflection at the girder and slab interfaces. The participatory action of a box-shaped cell depends on span length, spacing of cells, stiffness of the slabs and webs. The calculation of this participatory effect of the cells in resisting the applied loads is known as live load distribution factor which is accepted by many bridge design codes [AASHTO, AASHTO-LRFD, OHBDC, CHBDC, AUSTROADS]. The distribution of live load shear and moment is crucial in the design of new bridges as well as it is an important parameter considered in the evaluation of the load carrying capacity of existing bridges. A brief description of the past research on the estimation of load distribution and load carrying capacity of the skew box-girder bridge is presented below:

Schaffer (1967) conducted series of field tests to evaluate the lateral distribution of live load in four spine 45° skewed prestressed concrete box-girder bridge which was designed using a live load distribution factor of S/5.5 (S being the lateral spacing of girders). The bridge considered in this study was 64 ft 10.5 in (19.77 m) long, and 28 ft (8.534 m) wide and it was subjected to three-axle truck loading which closely resembled the AASHTO HS20 loading. Since the modulus of elasticity of tested bridge was not known the author expressed the moments in terms of moment coefficient (M/E) defined as experimental moments divided by elastic modulus. In general, maximum moment coefficient was noted when the vehicle was in vicinity of skew mid-span, irrespective of the moving direction of the vehicle. Moment coefficient in the skew bridge were found 13% lesser moment on exterior girder and 19% lesser on interior girder as compared to corresponding girders in right bridge. Thus it was concluded that girders in a skew bridge which were designed treating the bridge as a right bridge, would in-fact be stressed to lower levels than their right bridge counterparts. In general, the measured deflections of the skew bridge were found smaller than those in the right bridge, however, it was also noted that the deflection under the directly loaded girder may exceed the deflection at analogous location of similar right bridge. The measured strains in the skew bridge were found very close to those recorded strains on the right bridge, but in general they were on the slightly lower side. Moreover, he also found that skew angle results in eccentric distribution of girder support reactions.

Ashebo and his colleagues (2007) also evaluated the effect skewness on transverse load distribution in existing three-span continuous skew Tsing Yi south box-girder bridge (whose dimensions are described earlier in this paper). They measured analytically relative percentage changes in the distribution factors for various skew angles. Up to 30° skew angle distribution factor change rate remained slow but as higher skew angles were considered this rate became rapid. In general, they concluded that for bridges having skewness up to 30° load distribution is not much affected by the virtue of skew supports.

Frissen and his associates (2005) analyzed multi-beam box-girder bridges, using three different approaches to find the cross-sectional forces in the joints of skewed pre-stressed box-girder bridge. The 3-D finite element analysis of the bridge was carried out using (a) brick elements, (b) degenerated shell elements and, (c) orthotropic plate model consisting 8-noded quadrilateral plate elements. For the analysis they selected Heultsedreef Highway Bridge located in south Netherlands, which consisted of five prestressed and prefabricated box girders spanning 27.225 m. In addition to dead load and prestressing, they considered live load at critical position which was determined via influence field analysis using shell element model. On comparison of linear analysis result via above mentioned three different models authors found very small difference in the results. A comparison of results obtained for right and skew bridge models having similar geometry, revealed that although skew bridge experiences the same amount of maximum transverse moment in a given joint but, its location along the span changed drastically indicating the need of special design procedure for the same. Authors recommended nonlinear analysis especially for skew case as orthotropic plate models were unable to predict its exact behavior.

Hodson and associates (2011) performed live load and dynamic load tests on 12° skew, five span, prestressed concrete box-girder bridge which was consisting of four-cells. Also they carried out a finite element analysis to compare the analytical results with the test data to calibrated finite element results. This calibrated FE model was then employed to determine load distribution factors and theoretical load ratings of the bridge. Results from the study indicated that the distribution factors taken from AASHTO LRFD specifications were 29% to 46% conservative as compared to those obtained from the finite element model for an interior girder while for exterior girder they were 2% to 9% un-conservative. Further, Hodson and his colleagues (2011) conducted static live-load tests on cast-in-place 8° skew, prestressed concrete bridge, consisting of four box-girders which were continuous over two equal spans each measuring 39.35 m & having 12.8 m width. In total, they conducted sixteen quasi-static live-load tests along five different load paths to estimate flexural live load distribution factor experimentally. Further, using the eight noded solid elements (CSI, 2009) maximum distribution factor for the one, two, and three loaded lanes were also obtained. As expected, AASHTO LRFD distribution factors were found significantly higher than FEM results for the interior girders, however, for the exterior girder, when the parapets were included in the analysis, the AASHTO LRFD distribution factors were found somewhat un-conservative. Consequently, they concluded that the presence of parapets severely affects distribution factor values. Also, the study indicated that diaphragms did not have much effect on the moment distribution factors, as with or without diaphragms distribution factors changed only by 1%. To investigate the effect of span length, girder spacing and overhang on load distribution, the span length of the bridge model was varied from 18.3 m to 73.2 m, girder spacing was varied from 2.1 m to 4 m and overhang was varied from 0.3 m to 1.2 m. Results suggested that a decrease take place in moment distribution factors when span length in increased, while the increase in girder spacing resulted an increase in the distribution factors, but at the same time increase in deck overhang distance did not considerably affected the interior girder, but it raised the distribution factor of exterior girder. Change in deck thickness also failed to affect any distribution factors. Further, for vast parametric study, they varied the skew angle from 0° to 60°. They concluded that distribution factors tend to decrease for both girders when the skew angle reaches 15° and higher and at the same time the ratio of AASHTO to FEM distribution factors decreased for interior girder while it increased for exterior girder.

Mohseni and Khalim (2012) developed skew correction factor equations for distribution of live load in multi-cell box-girder bridges by performing several parametric studies. They chose four different existing multi-cell continuous box-girder bridges for their study and employed AASHTO HS20-44 truck loading. The results for moment distribution factor from the FEM model were compared with grillage method results, AASHTO specification and AASHTO LRFD specification, taking various span length and skew angles as the subject of comparison. They concluded that current AASHTO specifications overestimates the moment distribution for external girder by 40% and underestimates for internal girders by 25%. They also pointed that live load distribution factor as per AASHTO LRFD specification was highly conservative for exterior girders when the angle of skew was varied from 0° to 60°, but it underestimated moment distribution factor for interior girder when skew angle reached above 30°.

Mohseni and colleagues (2012-13) recently also investigated the effect of skewness on maximum deflection and distribution factors for tensile and compressive stresses in the multi-cell box-girder bridges under AASHTO truck loads. They developed 240 finite element models using SAP2000v12 and compared the results with AASHTO and AASHTO LRFD guidelines. They varied the skew angle, span length, number of boxes and number of lanes for the investigation. Using the statistical approach many empirical equations were deduced to give corrections in case of skew box-girder bridges. The skew angles considered in their study were 30°, 45° and 60° and they varied the span length from 30 m to 90 m at the interval of 15 m. They concluded that for skewed bridges, maximum compressive stress occurs at the intermediate support line as found in right bridges, but the maximum tensile stress was found at the section which passes through the center of each lane and was parallel to skewed abutment, which was higher in magnitude than the right bridge. Via the parametric study, they concluded that for the same span length if the number of loaded lanes is increased then both positive and negative stress distribution factor increases. Also, it was noted that in shorter span, stress and deflection distribution factors had more magnitude than longer ones, however, stress and deflection distribution factors decreases as the number of boxes were increased. Skew angle was found to have insignificant effect on positive stress distribution factor, but the distribution factors for negative stress and maximum deflection increased with increase in skew angle. Moreover, based on the stresses found out using least square regression, they provided equations for distribution factors for negative stress and maximum deflection. Further, the equations were found in good agreement with finite element analysis and non-orthogonal grillage analysis thus authors suggested that non-orthogonal grillage analysis as a good tool for determining bridge response.

5 SEISMIC BEHAVIOR OF SKEW BOX-GIRDER BRIDGES

Damages in past earthquakes such as 1971 San Fernando earthquake, 1994's Northridge, 1995's Great Hanshin, 1999 Chi-Chi earthquake of Taiwan, 2010's Chile's or Haiti's earthquakes have again and again showed the susceptibility of skewed bridges for attracting much more damage than the right counterparts. The codal guidelines are often insufficient to predict the actual seismic response of skew bridges, thus attracting much focus of researchers to inspect this special nature of such bridges. A brief summary is discussed below to understand the consequences of skewness on seismic response of skew box-girder bridges:

Ghobarah and Tso (1973) presented their study on dynamic response of the Foothill Boulevard Undercrossing, located in the San Fernando Valley, California which was subjected to devastating San Fernando earthquake in 1971. Study used a spine-line model of the bridge which had intermediate supports and was capable of taking flexural and torsional deformations. Total span of the modeled spine was 185 ft. (56.38 m) with line of intermediate supports at an angle 55° to line perpendicular to center line of bridge and at a distance 82 ft (25 m) from left support. The vertical acceleration data were scaled down to 70% and used as a ground motion data at base of the bridge, they observed that due to skewness of intermediate columns, the effect of torsional vibration was equally significant as the effect of flexural vibration. Therefore, they recommended that while designing the bridges with skew supports, the combined effect of torsional and flexural vibrations must be considered. However, the use of fixed boundary conditions at the end of spine model was a questionable consideration, as the bridge suffered in plane horizontal translation and rotation at the ends.

Wakefield and his colleagues (1991) also presented a case study on Foothill Boulevard Undercrossing. In 1971, during the earthquake this bridge suffered damage at intermediate supports and also an in-plane offset of about 3 in. (0.08 m) at the abutments. They performed free vibration analysis, linear and nonlinear time-history analyses, using two different F.E. models namely: beam model and built-up plate model. In contrast with Ghobarah's study authors provided only vertical restraints as the end boundary conditions. Free vibration analysis showed built up model predicted higher natural frequencies than beam model for all four modes investigated, thus proving built up model as a superior alternative of modelling. Initial three modes were found out to be rigid-body motions with very little bending, but fourth mode suddenly showed a large bending and torsional deformation in the deck. These results showed that the bridge deck behaved essentially as a rigid body. For the earthquake analysis, they considered lateral component of earthquake excitation as the ground input motion. The linear time history showed the maximum in-plane lateral displacement of the bridge deck approximately 5.6 in (0.14 m) at the outside corner of the bridge, which was higher than the actually noted value for which the authors reasoned that as the linear analysis fails to capture the plastic failure of columns, thus it resulted in higher prediction. While Nonlinear time-history analysis stopped after 7.92 seconds, further started diverging due to the plastic failure of the bridge column. Via this method maximum computed lateral displacement as out as 3.58 in (0.09 m), which was in close sync with the observed value, thus proving non-linear time history as better method. The time-history analysis results also confirmed that rigid-body modes dominated the earthquake response of the bridge under dynamic loads, which led to conclusion different from Ghobarah's study that short, stiff skewed bridges behave essentially as rigid bodies provided that the deck is not rigidly fixed to the abutments.

Chen (1994) performed an in depth seismic response study for two-span continuous, four celled pre-stressed concrete box-girder bridge. The length of each span was 109 ft (33.23 m) while the deck width was 38 ft (11.58 m). He employed time-history mode superposition and dynamic transient analysis methods of analysis for determination of longitudinal displacement, total shear in column, total bending moment in column, axial compression, longitudinal and transverse shear of abutment with respect to the abutment centerline. The bridge was analyzed for an ATC-6 type earthquake which consisted of four cases, namely 1.0L, 1.0T, 1.0 L+0.3T, and 0.3 L+1.0T, (Here L refers to longitudinal motion and T transverse motion). Skew angles were increased up to 80° at an interval of 20°. Author performed linear and nonlinear analysis in conjunction with single-mode, multi-mode, and transient methods of analysis. The effect of skewness was spotted very prominently as longitudinal deflection profile showed a constant decrease with increase in skew angle in all the analysis, while axial compression, shear and bending moment in column was observed to decrease for skew angles up to 40°, but beyond that opposite behavior was observed, which is a major cause of concern in the design of high skew bridges as the forces increased even up to 1.6 times as compared to the orthogonal bridges. A comparison between single-mode analysis (SMA) and multi-mode analysis (MMA) revealed that former underestimates earthquake response as SMA cannot model the coupled action. On an average SMA's results were about 43.5% lower than MMA's. Thus he concluded that SMA is a good tool for estimating response of right/lowly-skewed (up to 20°) bridges and for higher skew angle MMA should be used. He also suggested that dynamic transient analysis is the best tool for calculating earthquake response of complex bridge systems.

Meng and Lui (2000) carried out response spectrum analyses for foothill boulevard undercrossing (details mentioned previously) using finite element approach. The used three different bridge models namely: elastic deck, rigid deck and stick models. Based on the study they concluded that flexibility of bridge deck plays a key role in determination of internal forces and displacement of deck & supporting columns. Neglecting flexibility of superstructure during dynamic analysis may result in underestimation of above mentioned quantities. It was emphasized that the seismic response of the skewed bridge is also dependent upon boundary conditions chosen for columns and the overall skewness of the bridge. The use of the rigid deck or stick model was not recommended for highly skewed bridges, as these models failed to calculate the correct axial forces generated in the columns due to earthquake. Moreover, they observed, that, large skewness present in bridge, might induce vibratory modes such as torsion and lateral flexure, which are known to increase axial forces, shear forces, moments and torques in the supporting columns and deck displacement. They concluded that major causes of failure of bridge columns were due to inadequate shear strength, insufficient cross-sectional sizes and transverse torsional reinforcement for the middle bent columns.

Mohti and Pekcan (2008) selected a benchmark bridge to study the effect of various parameters interacting with the skew angle. They considered a continuous three-span box-girder bridge having first and third span 30.48 m (100 ft) and middle span 36.576 m (120 ft) and angle of skew 30°. Moreover, they performed a parametric variation of this bridge changing skew angles from 0° to 60°. For the analysis they used two methods viz. Finite element method and simplified beam-stick models of bridge and for both the models nonlinear static pushover, linear and nonlinear time history analyses were performed. In their study, the bridges were analyzed for four different types of ground motions and each input ground accelerations were scaled to have peak ground acceleration of 0.3g. For larger skew angles, the results from the nonlinear time history analyses agreed with the observed yield mechanism in the columns from the pushover analyses. It was observed that the Beam-stick model successfully captured the effect of skewness and the significant modes needed for further analysis. However, the finite element method was recommended for bridges with large skew angles (> 30°).

Kalantari and Amajadian (2010) presented an approximate method suitable for hand calculations for the dynamic analysis of skewed highway bridges. In this method the bridge deck was assumed to be rigid in its own plane during earthquake loading, consequently, the deck was allowed to move horizontally but the vertical movements were restrained. The south-eastern bridge of the Foothill Boulevard Undercrossing subjected to San Fernando earthquake loading was analyzed using this method. They presented their results in term of natural frequency and internal forces namely shear forces in longitudinal and perpendicular direction and displacements. The results obtained from this approximate method were compared with the findings of a rigid finite element model and a flexible finite element model, and it was observed that results of natural frequency were very close to rigid finite element results and were able to capture the true mode shapes up-to 3rd mode. Moreover, all the results for all the internal forces considered in this study were found on conservative side even for very severe skew angle.

Kaviani and colleagues (2012) analyzed three models based on concrete box-girder bridge in California, by varying structural parameters such as symmetry of span arrangement, column-bent height and abutment skew angle. First two bridges considered were continuous over two spans having span length as 33 m, 34 m and 47 m, 44 m respectively while the third one was a continuous over three spans having span lengths 47.5, 44 m, and 36 m. The width of the three bridges considered was 8.3 m, 23 m and 23.5 m respectively while the skew angles considered for these bridges were 0°,15°,30°,45° and 60°.

By performing nonlinear time-history analyses using three sets of forty ground motions applied along six different incidence angle for each bridge totaling in mammoth 43200 models, it was found that parameters like deck rotation and column drift ratio, were higher for skew-type abutment bridges than those for orthogonal bridges. On examining various response parameters and ground motion characteristics, the skew abutment bridges were found to have higher probability of collapse due to excess rotations. To prevent these deck rotations, they recommended the use of shear keys, which may reduce the probability of collapsing. Interestingly the results also indicated that arrangement of span didn't affect the seismic response of such bridges. This study also indicated an inverse relationship between the collapse potential and abutment skew.

Mohti and Pekcan (2013) further examined the effect of various parameters such as skew angle, ground motion intensity, soil condition, abutment support conditions, bridge aspect ratio, and foundation-base conditions on the overall seismic response of a skewed continuous box-girder bridges. They studied a two-span span box-girder bridge with length 40.85 m and having skew angle of 52° with aspect ratio of approximately 0.3. This benchmark bridge was treated as base model upon which many parametric variations were done using 3-D Finite element modeling technique and 1-D improved Beam Stick modeling technique. They chose six different skew angles 0°, 20°, 30°, 45°, 52° (benchmark), and 60° with three different aspect ratios of 0.3, 0.54, and 1.1. They used 1940 El-Centro S00E record (scaled to PGA of 0.6 g) to measure the accuracy of beam stick model with FE model via time history analysis and, the result of which yielded close match between the two modeling techniques. Thus refined beam stick modeling was considered as an efficient tool to include large skew in modeling and capturing dynamic response accurately with simplicity. They considered two different soil types, two different ground motion intensities and six different ground motions from PEER strong motion database for each case to perform a vast parametric study. It was noted that as the angle of skew increased, irrespective of soil type or intensity of ground motion chosen, in general, longitudinal displacement increased at all locations on the bridge, as on increasing skew angle longitudinal stiffness decreases. While the deck's transverse displacement was noted small at the abutment, but increasing towards the bent, thus causing transverse bending of the deck its plane.

Ahmed and colleagues (2013) studied the behavior of diagonal deformation and modal frequencies upon seismic loading on a joint less skew integral box-girder bridges. They considered 0° to 75° skew angles at an interval of 15° for their investigation. A 3-D finite element model was generated for the continuous two-span three-cell integral box-girder bridge using shell elements in SAP2000. Earthquake loading used for the model was applied based on time history function generated from Los Angeles century city [LACC]. They observed that modal frequency up to 4th mode increases as the skew angle increases from 0° to 60°, but further found to decrease when the skew angle moved to 75°. It was quite strangely noticed, that modal frequencies of skew angle 15° and 75° were very close in magnitude. The diagonal deformation, indicating in-plane distortion of the bridge deck, was found more or less same for all the modal modes for the orthogonal bridges but with increase in skew angle, the diagonal deformation raised for higher modes. Moreover, they observed increase in bending moment with increase in skew angle up to 45° and a decrease afterward. The maximum vertical shear was found for orthogonal bridges, while the maximum horizontal shear as well as maximum torsion was found for 45° skew bridges. Moreover, axial force values increased with the increasing skew angle and due to the external dynamic excitations. Based on the results obtained they recommended that as vertical shear magnitude comes very small, therefore, horizontal shear value should be considered in combination with torsion to account for shear-torsion effect of the bridge. A general trend discovered for shear torsion revealed that shear torsion effect increased up-to 45° skewness but it decreased further.

Skewed bridges are susceptible to seismic damage due to their intricate dynamic responses and need to be retrofitted to mitigate damages. Zakeri et.al (2015) developed fragility curves for different seismic retrofitting techniques for single-framed skew RC box-girder bridges and compared their performance. They considered several retrofitting strategies including steel jacketing, shear keys, restrainer cables, seat extenders and their various combinations in their study to examine the performance of retrofitted skewed box-girder bridges. Comparison of the fragility of bridges with different skew angles revealed that the most effective retrofit strategy for a right bridge might not always be the most effective retrofit strategy for a highly skewed bridge due to its intricate dynamic response. The study concluded that alone steel jacketing can be applied as a suitable retrofit measure in low-skew (less than 30°) bridges. The emphasized that a combination of retrofits, particularly including steel jacketing of columns, is more beneficial rather than one retrofitting technique to overcome the impact of skew and improve the performance of highly skewed bridges. Authors also suggested that a combination of steel jacketing, shear key and restrainer cable work as best suited combination for skew bridges.

6. DYNAMIC BEHAVIOR OF SKEW BOX-GIRDER BRIDGES

The understanding of the dynamic behavior of skew bridges under moving vehicle loads is an important aspect of bridge design. Moving vehicles exert a dynamic impact on bridge which results in increase from static load effect. Deng (2014) conducted a literature review for dynamic impact factor used in highway bridge design and found out that dynamic impact factors are affected by large number of variables such as skewness and there still exists a need of a much more sensitive study to exactly know the extent of effect of these variables on dynamic impact factor.

McCallen and his associates (1994) investigated the dynamic response of a two span continuous box-girder bridge and 40° skewed bridge located at Painter street overpass via the numerical simulation using beam stick modeling technique as well as 3-D finite element modeling. They modeled three different cases for the bridge which contained a comparable right bridge, and other two as skewed geometries with fixed base support. Results revealed that in vertical and longitudinal mode skew bridge demonstrated higher frequency, while in torsional and transverse modes the right bridge had higher frequency. Mode shapes in skew bridge model were found out to be combined flexural-torsional, which was considered much complex than the response given by equivalent right bridge model since they exhibited directional coupling. They observed approximately 15% difference in modal frequencies of skewed and comparable right bridge. Finally, they recommended that for accurate seismic modeling of the bridge around 20% to 30% modal damping should be used, and for analysis of such complex bridges a nonlinear detailed finite element modeling approach must be used.

Meng and Lui (2002) proposed a refined stick model for the initial dynamic analysis of skew box-girder bridges. The results obtained were compared with those from numerical solutions obtained via skew plates. They showed that this dual-beam stick model was superior to the conventional single-beam model in predicting natural frequencies and mode shapes of the bridge as well as in estimating the displacements of the superstructure and internal forces in the substructure. They recommended dual beam stick model for the initial dynamic analysis of skewed highway bridges because of its simplicity and relative accuracy and also because the conventional single beam stick model often fails to capture certain predominant vibration modes that are important in obtaining the true dynamic response of the bridge.

Since, Meng and Lui’s studies were just model studies, performed in a laboratory or numerical simulations, Ashebo and his colleagues (2007) conducted a full-scale field test to study the effect of dynamic loads on existing skew continuous box-girder bridges. Firstly, they conducted a full-scale field test for finding the impact of parameters such as speed of vehicle, weight of vehicle, position of vehicle etc. on the dynamic load factor of bridge. They carried four kinds of tests namely calibration test, modal test, moving force identification test and dynamic load factor test on the Tsing Yi bridge (whose geometry is described earlier in this paper). Three modal tests on the bridge were carried out to determine natural frequencies, mode shapes and damping ratios. Also a FEM model of the skewed bridge was modeled in SAP2000 by considering skew angles range from 0° to 60°. For skew angles varying from 0° to 45°, the fundamental mode was found to be the bending vibration type, whereas, a torsional mode was obtained for 60° skew angle model. Thus they concluded that the tested bridge dynamic behavior was similar to a beam, because the span of the bridge was much greater than its section width. Change in the natural frequency up to skew angle of 30° was noted as very small. The magnitude of the natural frequency increased in the range of 30° to 45° skew angle, but was noted minimum for 60° skew. Based on their study, it was concluded that the torsional mode dominates in high skew angle range, whereas the bending mode dominates in the low skew angle regime. Finally, they recommended that the influence of skew on dynamic behavior of the bridge is not significant for skew angles in the range of 0° to 30°.

Mohseni and his associates (2014) investigated the influence of skewness on dynamic responses of box-girder bridges. They conducted an extensive parametric study on three-equal span, multi-cell box- girder bridge model which was subjected to AASHTO standard truck HS 20-44 live loads with angle of skew varying from 0° to 60°. They observed that the dynamic impact factor for bending moment decreases with increase in skew angle, however, it was found difficult to ascertain a particular trend to describe relationship between skew angle and dynamic impact factor for shear and reactions. Mohseni and his colleagues further expanded this study (2014) to estimate the fundamental frequency of skew box-girder bridges using numerical analysis. They investigated impact of span length, number of boxes and skew angle on the natural frequency of the bridge. They varied span length from 30 m to 90 m similarly number of boxes were changed from two to six and the skew angle was varied between 0° to 60° for a two equal span continuous RC box-girder bridge. Results for a 60 m long three box section bridge for which the skew angle was varied from 0° to 60° skew angle showed that fundamental frequency it increased up to 42% with increase in skew angle. They also commented that estimation of first fundamental frequency for skewed box-girder bridges via different codal guidelines is highly conservative.

7 CONCLUSIONS

1. Bridges with skew angle lower than 20° are simple enough to design by few modifications in right bridge guidelines, however, for bridges with high skew angle a careful in-depth analysis is needed.
2. Width to span ratio play a major role in deciding the extent to which skew angle will affect the response of the bridge. Very long bridges tend to negate the skew effect but in short bridges high skew angle can generate a variety of extra forces which must be accounted in while designing.
3. Although the presence of orthogonal diaphragms is proved to be most advantageous in skew box-girder bridges, as they reduce structural actions to great extent still, due to construction difficulties they might be omitted in some cases.
4. Live load distribution factors in multi-cell skew box-girder bridges predicted by some of the codal provisions are found either way over-conservative or sometime risky also, especially in skew box-girder bridges. Efforts have

been made by various researchers to distribute the live load among spines/webs of multi-spine/cell box-girder in a simplistic way to facilitate the manual design of girders.

5. Studies on seismic analysis of box-girder bridges reveal that skew box-girder bridges are more vulnerable due to coupling of different vibration modes at very early stages. To prevent loss of life, suggested countermeasures should be used in designing the new skew box-girder bridges and if there is a need to retrofit existing once appropriate retrofitting technique must be used.
6. Natural frequency of highly skew bridges (45° or more) tends to decrease with respect to right bridges while for moderately skew bridges (30°-45°) it increases. For bridges with skew angle lower than 30° dynamic effects are not found severe.
7. Although a good amount of research work has already been done to understand the behavior of skew box-girder bridges, however, still there are no exclusive guidelines available for selecting the optimum cross-section dimensions for different skew angle.

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