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SUSPENSION FOOTBRIDGE DYNAMIC RESPONSE TO PEDESTRIAN LOADING AND CORRESPONDING HUMAN COMFORT

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ABSTRACT: Pedestrian suspension footbridges have low stiffness, low mass, and low damping, making them prone to significant displacements, velocities, and accelerations under normal pedestrian loads. The present study analyzed two scaled, laboratory physical models and conducted simulations on forty numerical models to determine how particular design parameters affect modal frequencies and the dynamic response as compared to human comfort limits. The parametric study, validated through the physical model results, analyzed span length, cable sag, vertical stiffening, and lateral stiffening of the numerical models.

The present study established an accurate modeling methodology based on calibration with the scaled laboratory physical models. The modeling methodology was employed to conduct an extensive simulation parametric study. The study results indicate that modal frequencies of common pedestrian suspension bridges fall outside recommended ranges – the vertical velocities, lateral accelerations, and vertical accelerations of the structure under a single pedestrian load exceed published human comfort limits. The present study observed that shorter span lengths respond with higher modal frequencies and dynamic responses, that lower cable sag results in higher vertical frequencies and lower vertical dynamic responses, and that the addition of stiffening elements increases modal frequencies and decreases dynamic response. It was also concluded that limited cable stiffening, while influencing the bridge response, was not entirely sufficient to meet human comfort limits.

KEYWORDS: Suspension; Footbridge; Dynamic; Serviceability; Pedestrians.

1 INTRODUCTION

Strength is the critical design criterion; however, serviceability is also important, particularly for the acceptability of pedestrian suspension footbridges by users. Pedestrian loading induces a dynamic response that may result in public discomfort to the point where bridge users perceive the structure to be unsafe. The dynamic response of pedestrian suspension bridges has been an issue for many years and continues to be a problem, particularly pedestrian bridges that are very low mass, low stiffness, and relatively long span, as is typically the case with structures being constructed by aid organizations in developing countries. Even high profile pedestrian bridges, such as the Millennium Bridge in London, suffer from serviceability failures that occur as a result of not meeting deflection, velocity, and acceleration limits for pedestrian loading [1]. Pedestrian bridge dynamic response therefore is a very important consideration for design and must be evaluated so as to mitigate serviceability failures and maximize public acceptance of the bridge.

Dynamic response of pedestrian suspension bridges is most often problematic where pedestrian loading frequency is at or near the first modal frequency of the bridge. Typically the first six modal frequencies for common pedestrian suspension bridges are 2 Hz or less, with the first lateral mode having a frequency around 0.3 Hz and the first vertical mode having a frequency around 0.7 Hz. A typical human stride frequency, and also the fundamental load frequency for vertical excitation, is about 2 Hz. Human strides also induce a lateral force as a result of the way pedestrians shift their weight from side to side as they walk (Shi, 2013). The lateral force frequency due to walking is typically observed to be about 1 Hz.

The American Association of State Highway and Transportation Officials [3] specifies limits for pedestrian bridge fundamental frequencies *Guide Specification for the Design of Pedestrian Bridges*. The fundamental frequency in the vertical plane of a pedestrian bridge without live load must be greater than 3 Hz and the fundamental frequency in the transverse direction must be greater than 1.3 Hz. These fundamental frequency limits address the dynamic serviceability issue of resonance and effectively require that the dynamic response of pedestrian bridges be analytically predicted during the design process to limit dynamic motion problems [4].

The purpose of the present study is to determine how certain structural parameters affect displacements, velocities, accelerations, and modal frequencies of pedestrian suspension bridges to mitigate serviceability issues. There are many different types of suspension footbridges, however, the present study is based on Bridges to Prosperity design standards [5] because this type of pedestrian suspension bridge is representative and is being built in rural countries all around the world. In addition, vibration problems are known to be an issue for this type of structure. The objectives of the present study are to determine, through numerical simulations, how span, cable sag, vertical stiffness, and lateral stiffness influences the dynamic response, to determine ways to mitigate vibration problems, including displacements, velocities, and accelerations, to meet requirements for human comfort, and to evaluate schemes that shift the first several modal frequencies to meet the frequency limits for pedestrian bridges.

2 PHYSICAL MODELS

Two physical models, shown in *Figures 1a* and *1b*, were constructed to allow calibration and validation of the numerical models for the parametric study. Scaled models of a 40 m span bridge with 5 percent cable sag and an 80 m span bridge with 7.5 percent cable sag were constructed. These two span lengths and sags are the extremes for the bridges evaluated for the present study. In addition, the physical models are based on the standard dimensions from the Bridges to Prosperity Design Manual.

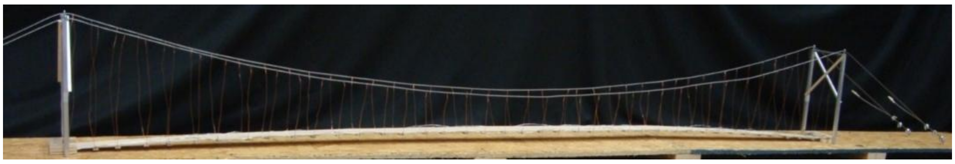


Figure 1a. Scaled 40 m physical model

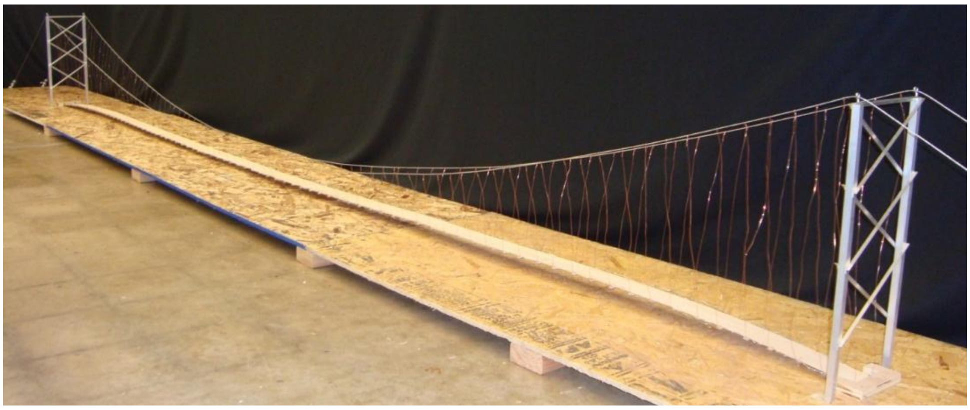


Figure 1b. Scaled 80 m physical model

The scaled physical models were designed at 1:18 scale of suspension footbridges. Scale factors depend on the scaled parameter; therefore, the scale factor was determined as presented in *Table 1*. The controlling parameter for the present study is mass; however, the model materials, including aluminum tubing and angles, steel cable, copper wire, aluminum plates, and basswood pieces, were chosen to closely match the full scale bridge materials in all engineering criteria.

Table 1. Scale factors for dynamic testing [6]

Quantity	**Dimensions**	**Scale Factor**
Length	L	S
Mass	M	S^3
Time	T	S
Stress	$ML^{-1}T^{-2}$	1
Velocity	LT^{-1}	1
Acceleration	LT^{-2}	1/S
Force	MLT^{-2}	S^2
Stiffness	MT^{-2}	S
Damping	MT^{-1}	S^2
Natural Frequency	T^{-1}	1/S

Laboratory testing of the model bridges consisted of a symbolic pedestrian load. The symbolic pedestrian was made of a plastic cylinder with eight scaled, evenly spaced feet around the circumference. Because velocity is scaled by unity, the symbolic pedestrian walking speed used was 1 m/s consistent with a 0.6 m (3.33 cm scaled) pedestrian stride [7]. An average pedestrian foot is slightly over 25 cm long (1.2 cm scaled). The symbolic pedestrian crossed the physical bridge model at a constant velocity to simulate the loading.

The response data from the physical model test was collected through with a high-speed video camera at 300 frames per second. Readily observable targets were attached to the model at points of interest based on the fundamental mode shapes derived from numerical models. *Figure* 2 presents one of the physical models with seven targets and the symbolic pedestrian set up for testing. The video testing was conducted a minimum of three times for each bridge to allow comparison of results and evaluation of repeatability. Any data set that contained outliers was discarded and the test was repeated.

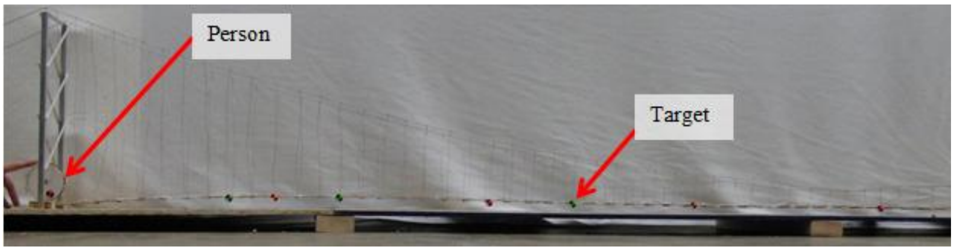


Figure 2. Physical bridge model testing set-up

The collected raw video data were imported into Tracker [8], a free video analysis and modeling tool built on the Open Source Physics Java framework. Tracker was used to determine the displacement of each crosshair for every frame during the bridge testing. The raw video data was calibrated by placing an object with a known length near the center of the video and setting its dimensions in Tracker. In addition, the frame rate was adjusted to 300 frames per second in Tracker to match the camera rate. Following processing, displacement versus time graphs were exported from Tracker into Excel; the graphs were overlaid for all three trials at every point and zeroed to each other to synchronize symbolic person release. Next, a power spectral density (PSD) program was written in Matlab to average displacements versus time graphs for each point and determine the modal frequencies of the bridge models. The PSD is calculated through the use of a Fast Fourier Transform (FFT) and identifies frequency content. The modal frequencies of the models were determined by averaging the PSD results from the analysis points along each bridge. The mode shapes are determined based on the numerical models. The results are averaged for the three calibration modes: 1) vertical mode with 1 wave; 2) vertical mode with 1.5 waves; and 3) vertical mode with 2 waves. The scaled modal frequency is found by taking the average from all participating locations and dividing it by the scale factor of 18. The scaled frequencies for the 40 m model footbridge are 0.45 Hz, 0.87 Hz, and 1.09 Hz for the first three vertical modes. The scaled modal frequencies for the 80 m model footbridge are 0.41 Hz, 0.53 Hz, and 0.63 Hz. These scaled modal frequencies were used to calibrate the numerical models and validate the parametric study.

3 NUMERICAL MODELS

Numerical models were analyzed in SAP2000 to determine the significance of selected design parameters relative to the dynamic response of pedestrian suspension bridges. Material engineering properties are based on information available in regions where these bridges are common. Model geometry is based on widely accepted configurations and standard pedestrian suspension bridges. The numerical model elements, including frame and cable elements were chosen to represent, as accurately as possible, the behavior of full scale bridge elements. The main cables, suspenders, and stiffening cables were modeled with cable elements. Nailers (wood members attached to cross-beams) and the safety fence were modeled as a distributed load and joint masses, respectively, rather than as structural elements. Boundary conditions were idealized as follows: support tower columns are pin connected; suspension cables are pin connected at the anchors; deck ends are supported by 200 kgf/mm (11.2 kips/inch) longitudinal and lateral springs.

A dynamic analysis was conducted for each of the forty bridges to determine fundamental frequencies. In addition, maximum displacements, velocities, and accelerations under pedestrian loading were calculated using a nonlinear, direct-integration, time-history analysis. These results were used to evaluate the bridge performance relative to established human comfort criteria. Pedestrian dynamic load is modeled as a moving vertical load of 81.6 kgf (180 lb) and a lateral load of 3.06 kgf (6.7 lb) applied in the direction away from the pedestrian's center of mass placed at 0.6 m (1.97 ft) intervals; the footfall loads are placed 0.2 m (7.9 inches) apart. The load vs. time functions are based on available pedestrian footfall force data and are presented in Figure 3. The vertical and lateral load functions act simultaneously with the sequence repeating 0.5 seconds after the start of the previous step. This timing scheme results in a short overlap of foot placement. Both the modal analysis and time-history analysis were initiated at the conclusion of the nonlinear dead load analysis. The modal damping ratio is defined as 1 percent, which is recommended for outdoor footbridges in AISC Design Guide 11 [9], and the Rayleigh damping frequencies are defined as the first and tenth modal frequencies for the time-history analysis.

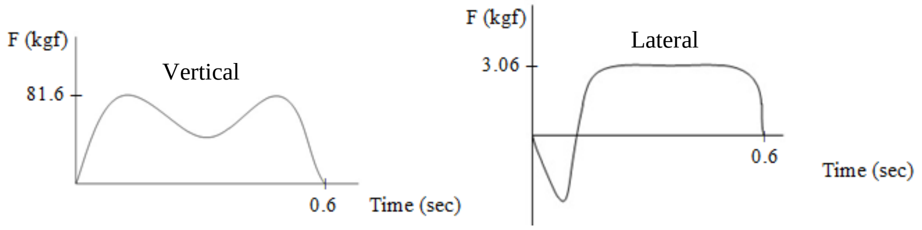


Figure 3. Pedestrian force-time functions

After the numerical simulation results were available, the physical model results for three vertical mode shapes (1 wave, 1.5 waves, and 2 waves) were used to calibrate the numerical model connection rigidity, material stiffness, and mass distribution to improve the numerical simulation to reflect the behavior of the physical models. The numerical models were configured to match the exact mass of the physical models, the properties of the wood, and the additional mass from connections.

Table 2 presents calibration results for the 40 m and 80 m models. The differences in the results between the modal frequencies is due to a number of factors, the two most prominent being: 1) construction imperfections in the physical model that are not present in the numerical model; and 2) suspender wire in the physical models is not perfectly straight, distributing the loads unevenly. Although the numerical model is unable to fully predict the physical model behavior, the numerical modeling techniques were significantly improved and much was learned for the parametric study. Through constructing the physical models, the numerical modeling methodology, including boundary condition behavior and member connectivity, was improved and the numerical models were determined to accurately represent the full scale structures.

Table 2. Comparison of numerical and physical model calibration results

40 m Model			
Vertical Mode	**Physical Model Freq (Hz)**	**SAP Model Freq (Hz)**	**Percent Difference**
1 wave	0.45	0.545	21.1
1.5 waves	0.87	0.993	14.1
2 waves	1.09	1.082	0.73

80 m Model			
Vertical Mode	**Physical Model Freq (Hz)**	**SAP Model Freq (Hz)**	**Percent Difference**
1 wave	0.41	0.332	19.0
1.5 waves	0.53	0.551	3.96
2 waves	0.63	0.644	2.22

4 PARAMETRIC STUDY

Numerical simulations were conducted for a parametric study to determine to what extent cable sag, vertical stiffness, and lateral stiffness affects 40 m, 50 m, 60 m, 70 m, and 80 m pedestrian bridges. Cable sag magnitudes considered are 5 percent and 7.5 percent. Vertical stiffening considered is cable cross-bracing between suspenders from the deck to the main cables. Lateral bracing considered is cable cross-bracing under the deck. Four models were created for one cable sag for one span length to evaluate no stiffening, lateral stiffening only, vertical stiffening only, and vertical and lateral stiffening. Two cable sag types were considered for five span lengths, resulting in a total of forty numerical models.

The vertical and lateral stiffening is 6.4 mm (¼") diameter cable with a pretension of 120 kgf (270 lbs). The stiffening cables connect at the main cables or crossbeams. The stiffening schemes evaluated were determined to have the greatest stiffening effect based on the mode shapes; cables were added in the areas with the largest distortions of rectangular geometry. Vertical stiffening is present over 60 percent of the structure: 20 percent on each end and 20 percent centered on the bridge. Vertical stiffening cables connect the ends of the crossbeams to the main cable. *Figure 4* presents the stiffening pattern for the 40 m span bridge. Lateral stiffening is between 40 percent of the crossbeams: 10 percent on each end and 20 percent centered at the middle of the bridge. *Figure 5* presents the lateral stiffening cable pattern under the deck for a 60 m span.

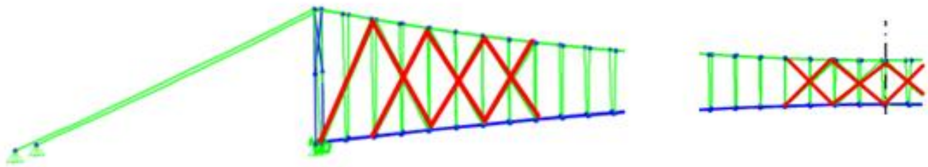


Figure 4. Vertical stiffening pattern for 40 m span

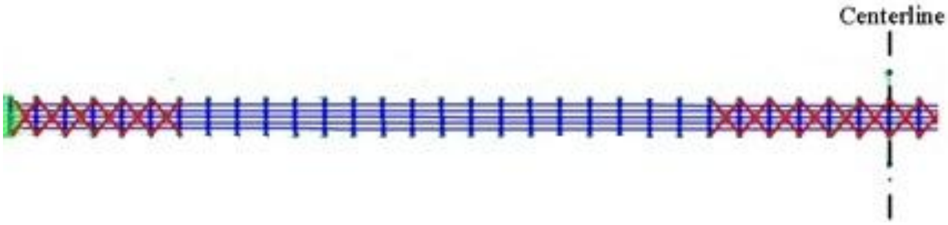


Figure 5. Lateral stiffening pattern for 60 m span

5 RESULTS AND DISCUSSION

The parametric study modal analysis and time-history analysis results were compared to established human comfort limits. Pedestrian bridges with vertical modal frequencies of 1.3 to 2.3 Hz and lateral modal frequencies of 0.5 to 1.2 Hz are known to elicit human discomfort. The vertical velocity criteria for human comfort is 1 cm/s (0.033 ft/s), and the vertical acceleration limit is 0.7 m/s^2 (2.3 ft/s^2). The lateral displacement limit is 45 mm (1.75 inches), and the lateral acceleration limit is 0.3 m/s^2 (1.0 ft/s^2).

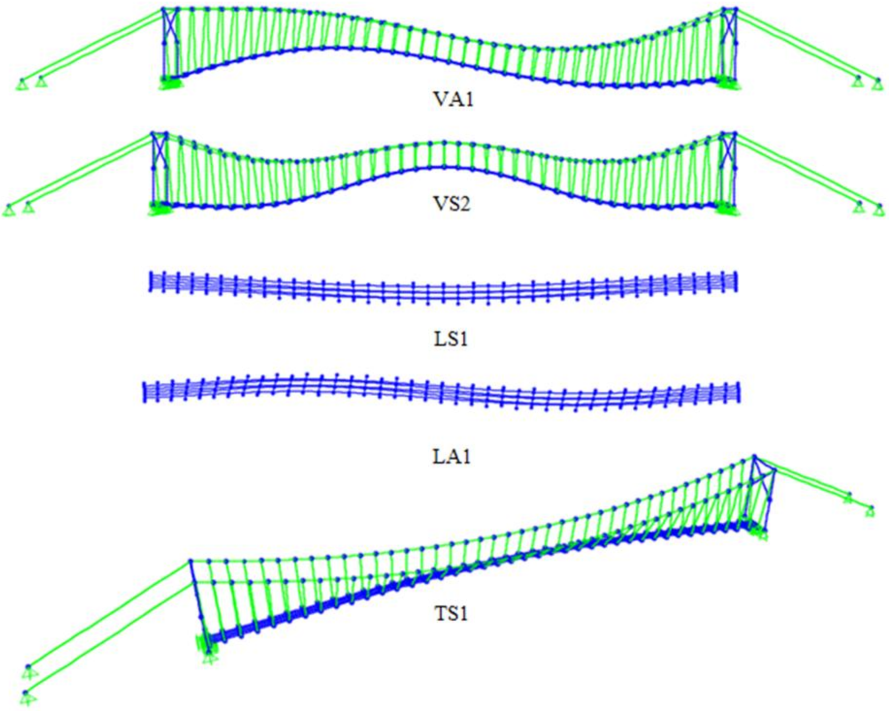
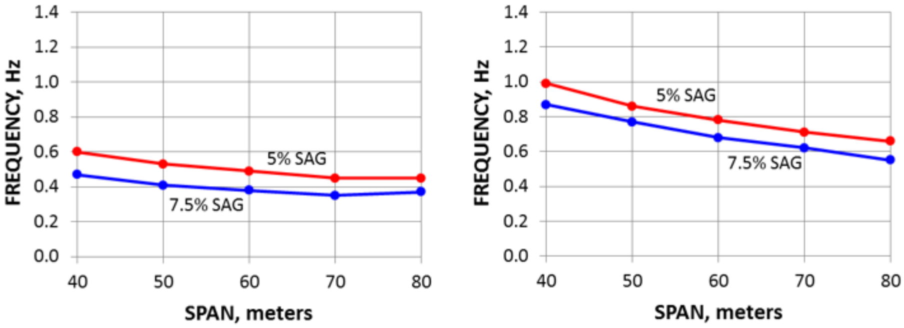


Figure 6. Vertical, horizontal, and torsional mode shapes

The numerical parametric study results were evaluated to determine the influence of each parameter: span length, cable sag, vertical stiffening, lateral stiffening, and both vertical and lateral stiffening. Five mode shapes, including two vertical, two lateral, and one torsional, were studied as presented in *Figure 7*. Four dynamic response quantities: 1) lateral displacements; 2) vertical velocities; 3) lateral accelerations; and 4) vertical accelerations, were evaluated and compared to human comfort criteria. Lateral displacements due to one person walking in no case results in exceedance of human comfort criteria; therefore, these results are not reported.

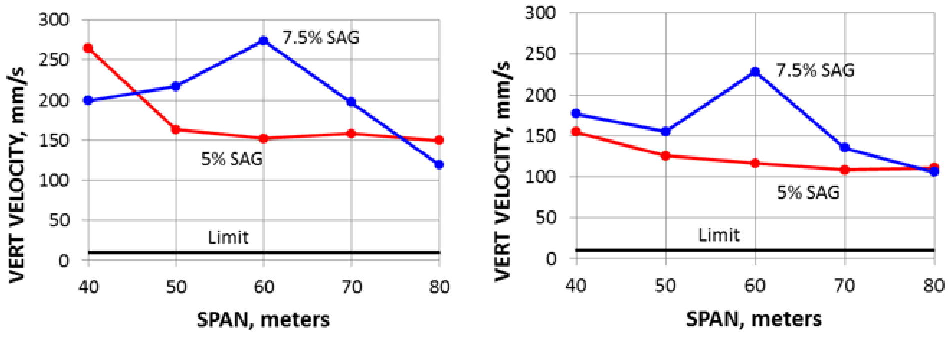
5.1 Span length and cable sag

Corresponding modal frequencies are higher for shorter spans and higher vertical mode frequencies correspond to 5 percent cable sag as can be observed from *Figure 8*. The lateral and torsional modes are dependent on the deck stiffness; therefore, the cable sag does not affect these modes. Displacements, velocities, and accelerations tend to decrease as the span length increases. The vertical dynamic response is lower for 5 percent cable sag as experience for the walking pedestrian and the bystander – defined as a stationary person located away from the walking pedestrian, as observed from *Figure 9*. These results are expected as less cable sag corresponds to higher bridge stiffness. Exceptions to these general observations include the vertical velocity for a 60 m span with 7.5 percent cable sag due to structure frequency and pedestrian walking frequency matching.



a) VA1 Frequency vs. Span   b) VS2 Frequency vs. Span

Figure 7. Vertical modal frequencies vs span



a) Walker Vert Velocity vs. Span b) Bystander Vert Velocity vs. Span

Figure 8. Walker and Bystander vertical velocity vs span

5.2 Vertical stiffening

Vertical stiffening through inclined cables from the main cable to the deck causes the first vertical mode (VA1) frequency to increase by 14 to 42 percent. The second vertical mode (VS2) frequency increases by 5 to 10 percent. The VA1 frequency increase due to stiffening appears to be effective as a result of the strategic location to mitigate the effects of this mode. However, VA1 stiffening is also located in areas where VS2 is mitigated. Vertical stiffening does not influence the lateral mode frequencies because lateral frequencies are primarily a function of deck stiffness.

Adding vertical, diagonal stiffening cables results in decreased vertical velocities and accelerations for most span lengths. The average vertical velocity experienced by a walking pedestrian is 10 to 15 times the comfort limit and the vertical velocity experienced by a bystander is 3 to 4 times the comfort limit. For simulations were vertical, diagonal, stiffening cables are included, the average vertical accelerations experienced by a walking person are typically 6 times greater than the comfort limit and for a bystander, less than the comfort limit. The bridge movement experienced by the bystander is largely mitigated because the overall bridge response is reduced, however, the walking pedestrian, as the forcing function, continues to generate a significant, localized motion. The lateral accelerations are marginally influenced by the vertical, diagonal, cable stiffening.

5.3 Lateral stiffening

Lateral, diagonal, cable stiffening increases the lateral modal frequencies by as much as 13 percent. The first lateral mode (LS1) increases by 3 to 13 percent. The second lateral mode, (LA1) increases by 0 to 5 percent. The first lateral mode is most influenced because the stiffening is located to strategically limit the displacements for this mode. Longer bridge spans required additional stiffening to influence the dynamic response and, therefore, a slightly greater frequency increase was observed.

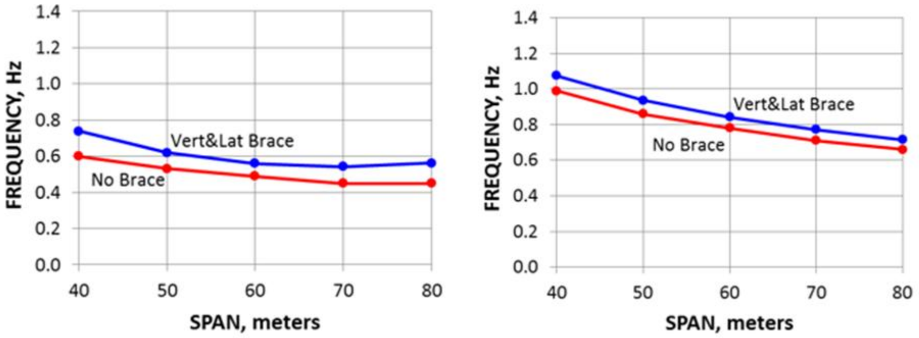
For 40, 50, 60, and 80 meter span lengths, the lateral accelerations decrease when lateral stiffening is included. However, the 70 m span lateral accelerations increased for both the walker and bystander because the modal frequency of LA1 is close to the walker lateral frequency.

5.4 Vertical and lateral stiffening

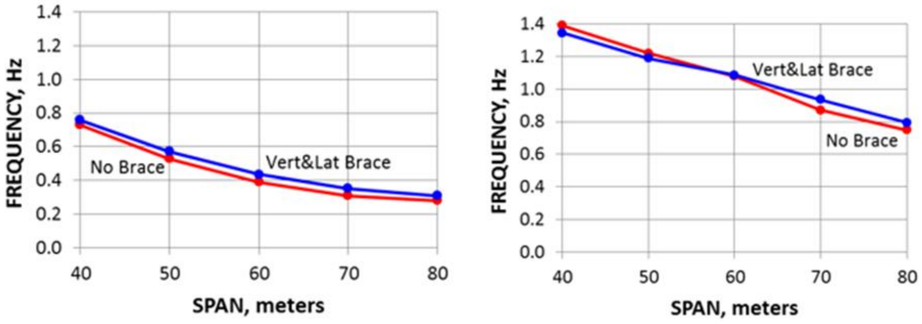
Numerical bridge models that include both vertical and lateral diagonal cable stiffening responded primarily at the first vertical modal frequency, VA1, as observed from *Figure 10* for models with 5 percent cable sag. The response of bridges with 7.5 percent cable sag is similar to 5 percent. VA1 frequencies increase by 15 percent to as much as 46 percent. The second vertical mode, VS2, increases by 6 percent to 11 percent. The first lateral mode (LS1) frequency increases by up to 13 percent. The torsional response mode is not significantly influenced by either vertical or lateral diagonal cable stiffening.

Vertical and lateral stiffening generally decreases all of the dynamic responses of displacement, velocity, and acceleration. The vertical velocities, lateral accelerations, and vertical accelerations for a walker and bystander are significantly improved for most bridge spans and geometries with 5 percent cable sag as observed from Figure 10. The response is similar for with 7.5 percent cable sag. The average lateral accelerations for 70 m span increase when diagonal cable stiffening is present, but all other dynamic frequencies for all other spans decrease. The lateral accelerations increase for 70 m span length because the second lateral modal frequency is very close to the walker lateral frequency.

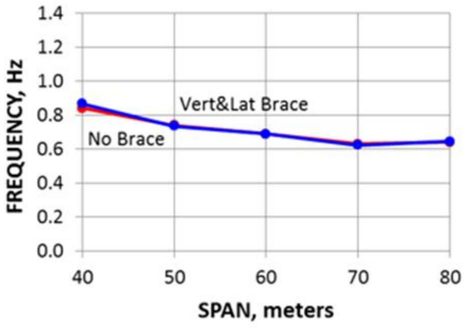
In general, the bridge response after stiffening improves the bystander condition most because the stiffening is very effective on a global scale rather than in the area local to the walker. The bystander average vertical velocities are as much as 5 times the comfort limit when vertical and lateral stiffening are provided. The bystander average lateral accelerations are within 20 percent of the comfort limit. The bystander average vertical accelerations are within 10 percent of the comfort limit. The vertical velocities, lateral accelerations, and vertical accelerations experienced by the walker are not affected to the same degree by diagonal cable stiffening as for the bystander; however, typically the response is improved.



a) VA1 Frequency vs. Span b) VS2 Frequency vs. Span

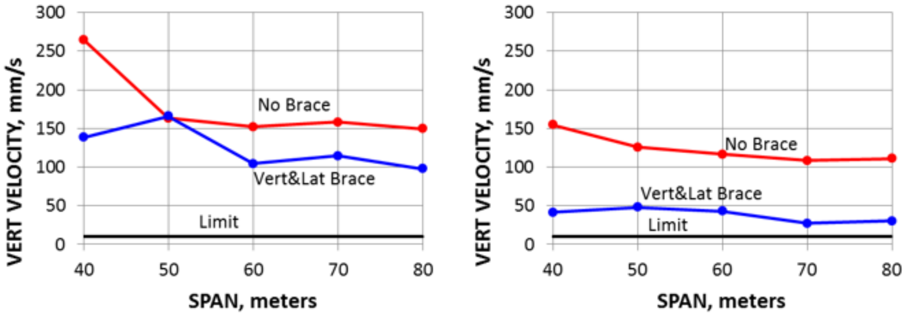


c) LS1 Frequency vs. Span d) LA1 Frequency vs. Span



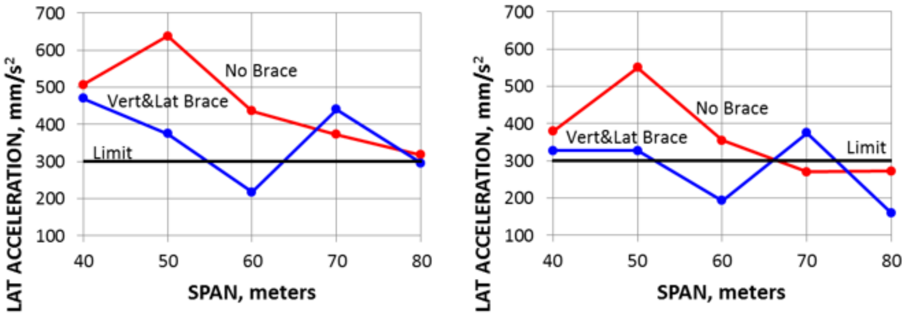
e) TS1 Frequency vs. Span

Figure 9. Modal frequencies vs span with combined vertical and lateral bracing



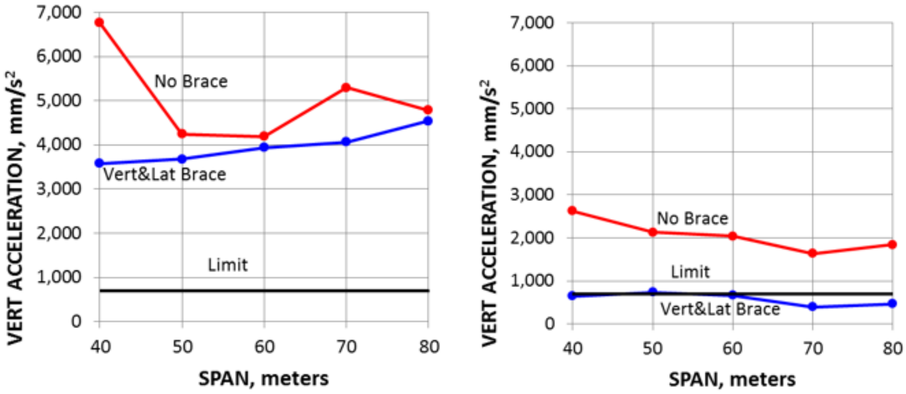
a) Walker Vert Velocity vs. Span

b) Bystander Vert Velocity vs. Span



c) Walker Lat Acceleration vs. Span

d) Bystander Lat Acceleration vs. Span



e) Walker Vert Acceleration vs. Span

f) Bystander Vert Acceleration vs. Span

Figure 10. Dynamic response of models with vertical and lateral stiffening

6 SUMMARY AND CONCLUSIONS

This study calibrated the numerical model methodology against physical, scaled laboratory physical models. An extensive simulation parametric study was conducted to evaluate the dynamic bridge response to pedestrian walking loads, with and without cable stiffening. The study results indicate that modal frequencies of common pedestrian suspension bridges fall outside recommended ranges – the vertical velocities, lateral accelerations, and vertical accelerations of the structure under a single pedestrian load exceed published human comfort limits. The simulation results from the parametric study support several conclusions:

1) Shorter span lengths have higher modal frequencies.
2) Less cable sag results in higher vertical modal frequencies.
3) Vertical, diagonal, cable stiffening increases the first two vertical modal frequencies.
4) Lateral, diagonal, cable stiffening increases the first two lateral modal frequencies and slightly increases the first torsional modal frequency.
5) Combining vertical and lateral, diagonal, cable stiffening causes all modal frequencies to increase except the torsional modal frequency.
6) Shorter span lengths typically experience higher dynamic responses; however, spans with a modal frequency close to the pedestrian walking frequency experience a higher dynamic response.
7) Vertical, diagonal, cable stiffening causes the vertical velocities and vertical accelerations to decrease and a corresponding improvement in the vertical accelerations felt by a bystander to within the comfort limits.
8) Combining vertical and lateral diagonal cable stiffening improves the response felt by both a walker and bystander for most spans.
9) For most span lengths, and most notably a 50 m span, the second lateral mode is close to the 1 Hz lateral frequency of a normal pedestrian walk and is therefore easily excited under normal walking conditions.
10) For all span lengths, the lateral displacements resulting from one pedestrian walking are within the limits for human comfort, and the lateral accelerations only slightly exceed the limit for human comfort. However, the vertical velocities and vertical accelerations greatly exceed the human comfort limits; therefore, the vertical vibrations are a greater concern.
11) Vertical and lateral, diagonal, cable stiffening does not decrease the dynamic response of the bridge to meet human comfort limits; however, the response is improved with stiffening, particularly with respect to the response felt by a bystander.

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