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DYNAMIC BEHAVIOR OF A BRIDGE ON FLOATS

Theodore G. Konstantakopoulos

Civil Engineer PhD N.T.U.A., Greece
e-mail: theokons@teemail.gr

ABSTRACT: Bridges on floats are usually temporary structural systems carrying moving loads while consisting from at least two or more floating piers (pontoons). In this work, an analytical model suitable for the dynamic analysis of bridges on floats is presented. When a moving load is passing the bridge with constant velocity both the beams as well as the piers become in motion. The theoretical formulation is based on a continuum approach employing the modal superposition technique. Various cases of geometrical and loading parameters are studied and the analytical results obtained in this work are tabulated in the form of dynamic response diagrams.

KEY WORDS: Pontoon bridges; floats; moving loads; bridge dynamics.

1 INTRODUCTION

Pontoon bridges were used since ancient times to cross wide rivers. In our days floating structures are still in wide use in both military and civil constructions. Many floating bridges have been constructed across rivers and seas in several countries instead of conventional bridges based on piers and abutments. One of the most well known pontoon bridges in Europe is the double deck steel pontoon bridge in Istanbul over the Bosporus spanning, which was constructed in the beginning of the 20th century. Other worth mentioning floating bridges are the Lacey V. Murrow Bridge, the Hood Canal Bridge in USA, the Bergsoysundet Floating Bridge in Norway and the Daxie Island Floating Bridge in China. Some of their advantages compared to conventional structures include the reduced environmental impact, the ability of relocation and the significant low costs in deep water structures.

Two main different structural forms have been adopted up to now for floating bridges: a continuous pontoon type bridge which is made of closely connected pontoons and ramps pressed to the shore banks and a discrete pontoon type bridge with a beam deck and several discrete pontoons functioning as piers. In both cases, the support system for permanent and traffic loads of the bridge is due to the buoyancy of water.

Floating bridges usually are designed by applying the theory of elastic foundation neglecting hydrodynamic effects, or more realistically, by considering hydrodynamic effects taking into account hydrodynamic masses and dampers [1]. Fleischer & Park [2] used modal analysis to study the hydro-elastic vibration of a beam under a single-axis vehicle moving with constant speed. Seif & Inoue [3] investigated the dynamic behavior of a discrete-pontoon floating bridge under the condition of wave effects with the finite element method. We must also mention the studies of Chonan [4], Langen [5], Sneyd et al [6-8] and Wang et al [9]. The problem of floating bridges under moving loads was also studied by Langen [10], Qiu [11], as well as other researchers not mentioned herein.

Model testing can be used for the assessment of the dynamic behavior of floating bridges. However it is impractical for parametric studies and it is not capable of verifying the structural integrity of all possible loading effects. In addition, due to specific load conditions and environmental requirements, the floating bridge design will always differ from one site to another.

The problem can be divided into a hydrodynamic problem for the liquid flow and an elastic problem for the pontoons' oscillations.

For the hydrodynamic problem one can use the Potential Theory [12], in the field of naval architecture and ocean engineering, in order to take into account hydrodynamic effects on bridges for different water depths.

In this study the dynamic response of a bridge on floating piers under a moving load is presented. Using Laplace transformations, the analytical solutions for the dynamic deflections of the joints between pontoons are determined. The water surface is taken to be in calm and its level remains invariable. The floating piers are assumed to be non-deformable and are in dynamic equilibrium by taking into account, the moving load and the buoyancy forces developed.

Finally we note that the purpose of the present paper is to offer a simplified solution for the preliminary design of such a structure. Therefore, the following analysis is not so rigorous for hydraulic or naval architecture point of view, neglecting terms that have not important influence on the bridge's behavior, but which an hydraulic engineer would include them without fall.

2 THEORETICAL ANALYSIS

In order to analyze the dynamical behavior of bridges on floats, we consider a n-span bridge with n-1 intermediate floating piers as shown in *Fig. 1*. The first and the last span are simply supported at points 0 and n, respectively, which are immovable and at points 1 and n-1, which are resting on floats 1 and n-1 that are allowed to move only in the vertical direction. Consequently, all intermediate spans slide vertically and rotate as well. All parts of the bridge are connected to each other with hinged connections.

In order to analyze the system, we assume the following:

1. The system equilibrates in a horizontal position at time instant t=0 under its

self-weight only. The waves' influence is neglected.

2. When a load P with constant magnitude crosses the bridge with constant speed υ, the system is deformed as shown in *Fig. 1*.
3. The buoyancy forces at the floating supports are:

$$V_i = A_i \cdot s_i \qquad (i = 1,...,n-1) \tag{1}$$

where A_i is the area of the i^{th} pontoon's cross-section and s_i is its corresponding settlement (sinking).

4. Each span ″i″ of the bridge has mass per unit length m_i and bending stiffness EI_i (i=1,…,n).
5. The so-called ″influence functions″ which express the geometrical movement of an undeformed beam for unit settlement of each support (see *Fig. 2*) are the following:

$$g_{(i+1)i} = \frac{x_i}{\ell_i}\,, \qquad g_{i(i+1)} = \frac{\ell_i - x_i}{\ell_i} \tag{2}$$

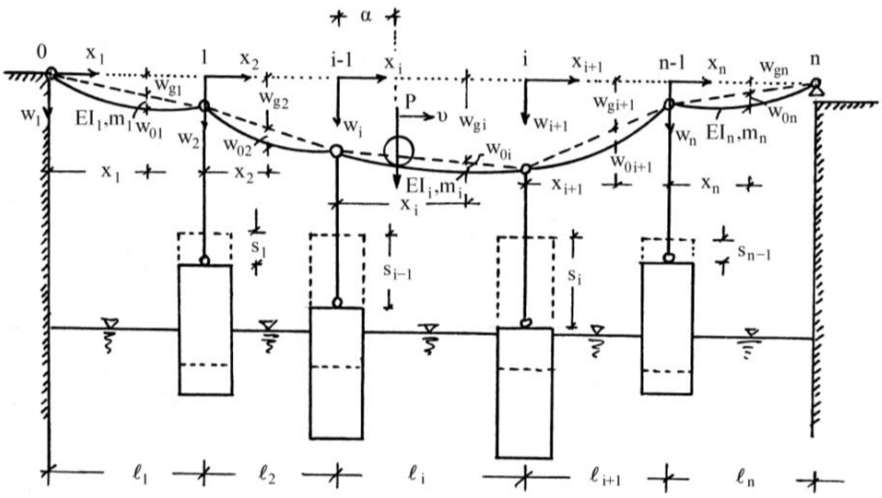


Figure 1. Deformed state of a bridge on floating piers

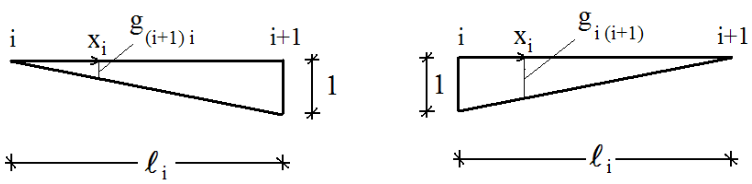


Figure 2. Influence functions for beam support unit settlements

We assume that a load P with constant magnitude moves on the i^{th} span. For the random span ρ according to *Fig. 1* and Eqs(2) we have:

span ρ: $$w_\rho = \frac{\ell_\rho - x_\rho}{\ell_\rho} s_{\rho-1} + \frac{x_\rho}{\ell_\rho} s_\rho + w_{o\rho} \tag{3}$$

The forces acting on the random joint ρ and on the joints (i-1),i are:

joint ρ: $$V_\rho = -\frac{m_\rho}{\ell_\rho}\int_0^{\ell_\rho} x_\rho \ddot{w}_\rho \, dx_\rho - \frac{m_{\rho+1}}{\ell_{\rho+1}}\int_0^{\ell_{\rho+1}} (\ell_{\rho+1} - x_{\rho+1})\, \ddot{w}_{\rho+1}\, dx_{\rho+1}$$

joint i-1: $$V_{i-1} = \frac{P(\ell_i - \alpha)}{\ell_i} - \frac{m_{i-1}}{\ell_{i-1}}\int_0^{\ell_{i-1}} x_{i-1}\ddot{w}_{i-1}\, dx_{i-1} - \frac{m_i}{\ell_i}\int_0^{\ell_i} (\ell_i - x_i)\,\ddot{w}_i\, dx_i \tag{4}$$

joint i: $$V_i = \frac{P\cdot\alpha}{\ell_i} - \frac{m_i}{\ell_i}\int_0^{\ell_i} x_i \ddot{w}_i\, dx_i - \frac{m_{i+1}}{\ell_{i+1}}\int_0^{\ell_{i+1}} (\ell_{i+1} - x_{i+1})\,\ddot{w}_{i+1}\, dx_{i+1}$$

Because of Eqs(3) and Eq(1) and after some manipulation, Eqs(4) become:

joint ρ:

$$\begin{aligned} A_\rho s_\rho &+ \frac{m_\rho \ell_\rho}{6}\ddot{s}_{\rho-1} + \frac{m_\rho \ell_\rho + m_{\rho+1}\ell_{\rho+1}}{3}\ddot{s}_\rho + \frac{m_\rho}{\ell_\rho}\int_0^{\ell_\rho} x_\rho \ddot{w}_{o,\rho}\, dx_\rho + \\ &+ \frac{m_{\rho+1}}{\ell_{\rho+1}}\int_0^{\ell_{\rho+1}} (\ell_{\rho+1} - x_{\rho+1})\,\ddot{w}_{o,\rho+1}\, dx_{\rho+1} = 0 \end{aligned} \tag{5a}$$

joint i-1:

$$\begin{aligned} A_{i-1}s_{i-1} &+ \frac{m_{i-1}\ell_{i-1}}{6}\ddot{s}_{i-2} + \frac{m_{i-1}\ell_{i-1} + m_i \ell_i}{3}\ddot{s}_{i-1} + \\ &+ \frac{m_i \ell_i}{6}\ddot{s}_i + \frac{m_{i-1}}{\ell_{i-1}}\int_0^{\ell_{i-1}} x_{i-1}\ddot{w}_{o,i-1}\, dx_{i-1} + \\ &+ \frac{m_i}{\ell_i}\int_0^{\ell_i} (\ell_i - x_i)\,\ddot{w}_{o,i}\, dx_i = \frac{P(\ell_i - \alpha)}{\ell_i} \end{aligned} \tag{5b}$$

joint i:

$$\begin{aligned} A_i s_i &+ \frac{m_i \ell_i}{6}\ddot{s}_{i-1} + \frac{m_i \ell_i + m_{i+1}\ell_{i+1}}{3}\ddot{s}_i + \frac{m_{i+1}\ell_{i+1}}{6}\ddot{s}_{i+1} + \\ &+ \frac{m_i}{\ell_i}\int_0^{\ell_i} x_i \ddot{w}_{o,i}\, dx_i + \frac{m_{i+1}}{\ell_{i+1}}\int_0^{\ell_{i+1}} (\ell_{i+1} - x_{i+1})\,\ddot{w}_{o,i+1}\, dx_{i+1} = \frac{P\cdot\alpha}{\ell_i} \end{aligned} \tag{5c}$$

The equations of motion are:

$$\begin{aligned} &EI_\rho w''''_\rho + m_\rho \ddot{w}_\rho = 0 \qquad (\text{for } \rho \neq i,\ \rho = 1,...,n) \\ &EI_i w''''_i + m_i \ddot{w}_i = P \cdot \delta(x_i - \alpha) \end{aligned} \tag{6a}$$

The above because of Eqs(3) become:

$$\begin{aligned} &EI_\rho w''''_{o,\rho} + m_\rho \ddot{w}_{o,\rho} = -m_\rho \frac{\ell_\rho - x_\rho}{\ell_\rho} \ddot{s}_{\rho-1} - m_\rho \frac{x_\rho}{\ell_\rho} \ddot{s}_\rho \\ &EI_i w''''_{o,i} + m_i \ddot{w}_{o,i} = P \cdot \delta(x_i - \alpha) - m_i \frac{\ell_i - x_i}{\ell_i} \ddot{s}_{i-1} - m_i \frac{x_i}{\ell_i} \ddot{s}_i \end{aligned} \tag{6b}$$

The system of Eqs(5) and Eq(6b) gives the unknowns $w_{o,i}$ and s_i (i=1 to n). We are searching a solution in the form of separate variables in the form:

$$w_{o,i} = \sum_k X_{ik}(x_i) \cdot T_{ik}(t) \tag{7}$$

where X_{ik} are the shape functions of the i^{th} beam, while T_{ik} are the time functions to be determined. Introducing the above expressions (7) into Eqs(6a) and Eqs(5) and taking into account that the shape functions of a simply supported beam are $\sin(k\pi x_i / \ell_i)$ and satisfy the equations for free motion, we obtain:

$$m_\rho \sum_k X_{\rho k} \ddot{T}_{\rho k} + m_\rho \sum_k \omega_{\rho k}^2 X_{\rho k} T_{\rho k} = -m_\rho \frac{\ell_\rho - x_\rho}{\ell_\rho} \ddot{s}_{\rho-1} - m_\rho \frac{x_\rho}{\ell_\rho} \ddot{s}_\rho \quad (\rho = 1,...,n,\ \rho \neq i) \tag{8a}$$

$$m_i \sum_k X_{ik} \ddot{T}_{ik} + m_i \sum_k \omega_{ik}^2 X_{ik} T_{ik} = P \cdot \delta(x_i - \alpha) - m_i \frac{\ell_i - x_i}{\ell_i} \ddot{s}_{i-1} - m_i \frac{x_i}{\ell_i} \ddot{s}_i \quad (\rho = i) \tag{8b}$$

$$\begin{aligned} &\frac{m_\rho}{\ell_\rho} \sum_k (\ddot{T}_{\rho k} \int_0^{\ell_\rho} x_\rho X_{\rho k} dx_\rho) + \frac{m_{\rho+1}}{\ell_{\rho+1}} \sum_k (\ddot{T}_{\rho+1,k} \int_0^{\ell_{\rho+1}} (\ell_{\rho+1} - x_{\rho+1}) X_{\rho+1,k} dx_{\rho+1}) = \\ &\qquad -A_\rho s_\rho - \frac{m_\rho \ell_\rho}{6} \ddot{s}_{\rho-1} - \frac{m_\rho \ell_\rho + m_{\rho+1}\ell_{\rho+1}}{3} \ddot{s}_\rho - \frac{m_{\rho+1}\ell_{\rho+1}}{6} \ddot{s}_{\rho+1} \end{aligned} \tag{8c}$$

$$\begin{aligned} &\frac{m_{i-1}}{\ell_{i-1}} \sum_k (\ddot{T}_{i-1,k} \int_0^{\ell_{i-1}} x_{i-1} X_{i-1,k} dx_{i-1}) + \frac{m_i}{\ell_i} \sum_k (\ddot{T}_{i,k} \int_0^{\ell_i} (\ell_i - x_i) X_{i,k} dx_i) = \\ &\qquad \frac{P(\ell_i - \upsilon t)}{\ell_i} - A_{i-1} s_{i-1} - \frac{m_{i-1}\ell_{i-1}}{6} \ddot{s}_{i-2} - \frac{m_{i-1}\ell_{i-1} + m_i \ell_i}{3} \ddot{s}_{i-1} - \frac{m_i \ell_i}{6} \ddot{s}_i \end{aligned} \tag{8d}$$

$$\begin{aligned} &\frac{m_i}{\ell_i} \sum_k (\ddot{T}_{i,k} \int_0^{\ell_i} x_i X_{i,k} dx_i) + \frac{m_{i+1}}{\ell_{i+1}} \sum_k (\ddot{T}_{i+1,k} \int_0^{\ell_{i+1}} (\ell_{i+1} - x_{i+1}) X_{i+1,k} dx_{i+1}) = \\ &\qquad \frac{P\upsilon t}{\ell_i} - A_i s_i - \frac{m_i \ell_i}{6} \ddot{s}_{i-1} - \frac{m_i \ell_i + m_{i+1}\ell_{i+1}}{3} \ddot{s}_i - \frac{m_{i+1}\ell_{i+1}}{6} \ddot{s}_{i+1} \end{aligned} \tag{8e}$$

Multiplying Eq(8a) by $X_{\rho 1}$, $X_{\rho 2}$,…, $X_{\rho n}$, successfully, integrating the outcome from 0 to ℓ_ρ (ρ=1,…,n), then Eq(8b) by X_{i1}, X_{i2},…, X_{in} and integrating the outcome from 0 to ℓ_i etc., we conclude to the following system:

$$\frac{m_\rho \ell_\rho}{2}\ddot{T}_{\rho k} + \frac{m_\rho \ell_\rho \omega_{\rho k}^2}{2} T_{\rho k} = -\frac{m_\rho \ell_\rho}{k\pi}\ddot{s}_{\rho-1} - (-1)^{k+1}\frac{m_\rho \ell_\rho}{k\pi}\ddot{s}_\rho \tag{9a}$$

$$\frac{m_i \ell_i}{2}\ddot{T}_{ik} + \frac{m_i \ell_i \omega_{ik}^2}{2} T_{ik} = P\cdot\sin\Omega_i t - \frac{m_i \ell_i}{k\pi}\ddot{s}_{i-1} - (-1)^{k+1}\frac{m_i \ell_i}{k\pi}\ddot{s}_i \tag{9b}$$

$$\begin{aligned}&\sum_k (-1)^{k+1}\frac{m_\rho \ell_\rho}{k\pi}\ddot{T}_{\rho k} + \sum_k \frac{m_{\rho+1}\ell_{\rho+1}}{k\pi}\ddot{T}_{\rho+1,k} = \\ &\quad -A_\rho s_\rho - \frac{m_\rho \ell_\rho}{6}\ddot{s}_{\rho-1} - \frac{m_\rho \ell_\rho + m_{\rho+1}\ell_{\rho+1}}{3}\ddot{s}_\rho - \frac{m_{\rho+1}\ell_{\rho+1}}{6}\ddot{s}_{\rho+1}\end{aligned} \tag{9c}$$

$$\begin{aligned}&\sum_k (-1)^{k+1}\frac{m_{i-1}\ell_{i-1}}{k\pi}\ddot{T}_{i-1,k} + \sum_k \frac{m_i \ell_i}{k\pi}\ddot{T}_{i,k} = \\ &\quad \frac{P(\ell_i - \upsilon t)}{\ell_i} - A_{i-1}s_{i-1} - \frac{m_{i-1}\ell_{i-1}}{6}\ddot{s}_{i-2} - \frac{m_{i-1}\ell_{i-1} + m_i \ell_i}{3}\ddot{s}_{i-1} - \frac{m_i \ell_i}{6}\ddot{s}_i\end{aligned} \tag{9d}$$

$$\begin{aligned}&\sum_k (-1)^{k+1}\frac{m_i \ell_i}{k\pi}\ddot{T}_{i,k} + \sum_k \frac{m_{i+1}\ell_{i+1}}{k\pi}\ddot{T}_{i+1,k} = \\ &\quad \frac{P\upsilon t}{\ell_i} - A_i s_i - \frac{m_i \ell_i}{6}\ddot{s}_{i-1} - \frac{m_i \ell_i + m_{i+1}\ell_{i+1}}{3}\ddot{s}_i - \frac{m_{i+1}\ell_{i+1}}{6}\ddot{s}_{i+1}\end{aligned} \tag{9e}$$

Where: $\Omega_i = \dfrac{i\pi\upsilon}{\ell_i}$.

The above system of equations (9) with unknowns the terms $T_{\rho 1}$, $T_{\rho 2}$,…, $T_{\rho k}$ (k=1 to n) and s_1, s_2,…, s_{n-1} and can be solved using the Laplace transformation. Thus, we set:

$$\begin{aligned}LT_{\rho k}(t) &= \overline{T}_{\rho k}(p) \\ Ls_i(t) &= \overline{s}_i(p)\end{aligned} \tag{10}$$

From the above Eq(10) we get:

$$\begin{aligned}L\ddot{T}_{\rho k}(t) &= p^2\,\overline{T}_{\rho k}(p) - p\,T_{\rho k}(0) - \dot{T}_{\rho k}(0) \\ L\ddot{s}_i(t) &= p^2\,\overline{s}_i(p) - p\,s_i(0) - \dot{s}_i(0)\end{aligned} \tag{11}$$

where $T_{\rho k}(0), \dot{T}_{\rho k}(0), s_i(0), \dot{s}_i(0)$ are the initial conditions, which are known. Therefore, from Eqs (9), (10) and (11) we conclude to the following system:

$$\begin{aligned}&\left(\frac{m_\rho \ell_\rho}{2}p^2 + \frac{m_\rho \ell_\rho \omega_{\rho k}^2}{2}\right)\overline{T}_{\rho k} + \frac{m_\rho \ell_\rho}{k\pi}p^2\,\overline{s}_{\rho-1} + (-1)^{k+1}\frac{m_\rho \ell_\rho}{k\pi}p^2\,\overline{s}_\rho = \\ &\quad \frac{m_\rho \ell_\rho}{2}[pT_{\rho k}(0) - \dot{T}_{\rho k}(0)] - \frac{m_\rho \ell_\rho}{k\pi}[p\,s_{\rho-1}(0) + \dot{s}_{\rho-1}(0)] + (-1)^{k+1}\frac{m_\rho \ell_\rho}{k\pi}[p\,s_{\rho-1}(0) + \dot{s}_{\rho-1}(0)]\end{aligned} \tag{12a}$$

$$(\frac{m_i\ell_i}{2}p^2+\frac{m_i\ell_i\omega_{ik}^2}{2})\bar{T}_{ik}+\frac{m_i\ell_i}{k\pi}p^2\bar{s}_{i-1}+(-1)^{k+1}\frac{m_i\ell_i}{k\pi}p^2\bar{s}_i=\frac{P\Omega_i}{p^2+\Omega_i^2}+$$
$$+\frac{m_i\ell_i}{2}[pT_{ik}(0)-\dot{T}_{ik}(0)]-\frac{m_i\ell_i}{k\pi}[ps_{i-1}(0)+\dot{s}_{i-1}(0)]+(-1)^{k+1}\frac{m_i\ell_i}{k\pi}[ps_{i-1}(0)+\dot{s}_{i-1}(0)] \quad (12b)$$

$$\sum_k(-1)^{k+1}\frac{m_\rho\ell_\rho}{k\pi}p^2\bar{T}_{\rho k}+\sum_k\frac{m_{\rho+1}\ell_{\rho+1}}{k\pi}p^2\bar{T}_{\rho+1,k}+[\frac{m_\rho\ell_\rho}{6}p^2\bar{s}_{\rho-1}+(\frac{m_\rho\ell_\rho+m_{\rho+1}\ell_{\rho+1}}{3}p^2+A_\rho)\bar{s}_\rho+$$
$$+\frac{m_{\rho+1}\ell_{\rho+1}}{6}p^2\bar{s}_{\rho+1}]=\sum_k(-1)^{k+1}\frac{m_\rho\ell_\rho}{k\pi}[pT_{\rho k}(0)+\dot{T}_{\rho k}(0)]+\sum_k\frac{m_{\rho+1}\ell_{\rho+1}}{k\pi}[pT_{\rho+1,k}(0)+\dot{T}_{\rho+1,k}(0)]+ \quad (12c)$$
$$+\{\frac{m_\rho\ell_\rho}{6}[ps_{\rho-1}(0)+\dot{s}_{\rho-1}(0)]+\frac{m_\rho\ell_\rho+m_{\rho+1}\ell_{\rho+1}}{3}[ps_\rho(0)+\dot{s}_\rho(0)]+\frac{m_{\rho+1}\ell_{\rho+1}}{6}[ps_{\rho+1}(0)+\dot{s}_{\rho+1}(0)]\}$$

$$\sum_k(-1)^{k+1}\frac{m_{i-1}\ell_{i-1}}{k\pi}p^2\bar{T}_{i-1,k}+\sum_k\frac{m_i\ell_i}{k\pi}p^2\bar{T}_{i,k}+[\frac{m_{i-1}\ell_{i-1}}{6}p^2\bar{s}_{i-2}+(\frac{m_{i-1}\ell_{i-1}+m_i\ell_i}{3}p^2+A_{i-1})\bar{s}_{i-1}+$$
$$+\frac{m_i\ell_i}{6}p^2\bar{s}_i]=\sum_k(-1)^{k+1}\frac{m_{i-1}\ell_{i-1}}{k\pi}[pT_{i-1,k}(0)+\dot{T}_{i-1,k}(0)]+\sum_k\frac{m_i\ell_i}{k\pi}[pT_{i,k}(0)+\dot{T}_{i,k}(0)]+ \quad (12d)$$
$$+\{\frac{m_{i-1}\ell_{i-1}}{6}[ps_{i-2}(0)+\dot{s}_{i-2}(0)]+\frac{m_{i-1}\ell_{i-1}+m_i\ell_i}{3}[ps_{i-1}(0)+\dot{s}_{i-1}(0)]+\frac{m_i\ell_i}{6}[ps_i(0)+\dot{s}_i(0)]\}+$$
$$+P\frac{p\ell_{i-1}-\upsilon}{p^2\ell_{i-1}}$$

$$\sum_k(-1)^{k+1}\frac{m_i\ell_i}{k\pi}p^2\bar{T}_{i,k}+\sum_k\frac{m_{i+1}\ell_{i+1}}{k\pi}p^2\bar{T}_{i+1,k}+[\frac{m_i\ell_i}{6}p^2\bar{s}_{i-1}+(\frac{m_i\ell_i+m_{i+1}\ell_{i+1}}{3}p^2+A_i)\bar{s}_i+$$
$$+\frac{m_{i+1}\ell_{i+1}}{6}p^2\bar{s}_{i+1}]=\sum_k(-1)^{k+1}\frac{m_i\ell_i}{k\pi}[pT_{i,k}(0)+\dot{T}_{i,k}(0)]+\sum_k\frac{m_{i+1}\ell_{i+1}}{k\pi}[pT_{i+1,k}(0)+\dot{T}_{i+1,k}(0)]+ \quad (12e)$$
$$+\{\frac{m_i\ell_i}{6}[ps_{i-1}(0)+\dot{s}_{i-1}(0)]+\frac{m_i\ell_i+m_{i+1}\ell_{i+1}}{3}[ps_i(0)+\dot{s}_i(0)]+\frac{m_{i+1}\ell_{i+1}}{6}[ps_{i+1}(0)+\dot{s}_{i+1}(0)]\}+$$
$$+P\frac{\upsilon}{p^2\ell_i}$$

The above system of Eqs(12) is a linear system with respect to the unknowns $\bar{T}_{\rho k}, \bar{s}_{\rho k}$ (ρ=1,…,n). Solution of the above system gives the unknowns in the form:

$$\bar{U}(p)=\frac{N(p)}{M(p)}$$

where N(p) and M(p) are polynomials with respect to p, with M(p) of equal or higher order than N(p). Hence, Heaviside's rule can be applied, which leads finally to the following expression for the unknowns U(t):

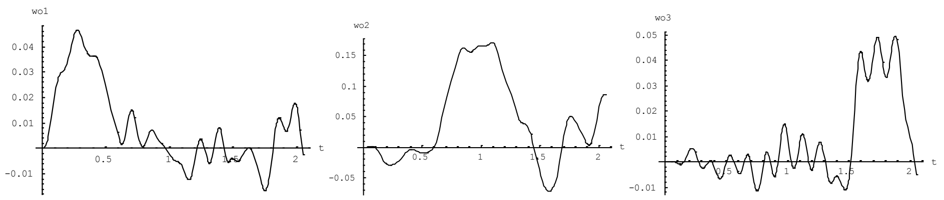
$$U(t)=L^{-1}\bar{U}(p)=L^{-1}\frac{N(p)}{M(p)}=\sum_{j=1}^{r}\frac{N(\theta_i)e^{\theta_i t}}{M'(\theta_i)} \quad (13)$$

where θ_i are the r roots (j=1 to r) of the polynomial M(p).

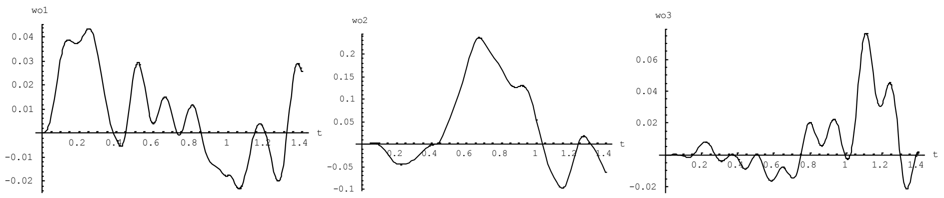
3 NUMERICAL RESULTS

We consider a three-span bridge with lengths L_1=L_3=10m and L_2=15m which rests on two pontoons with area A=40m^2. The beams are made from steel with modulus of elasticity E=2.1X10^6 dN/cm^2, moment of inertia I=0.00045m^4 and mass per unit length m=100kg. At time t=0sec, a load P=20000 dN enters the bridge moving with speed υ.

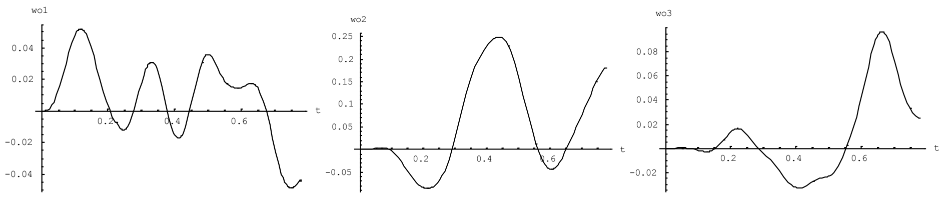
Applying the equations of the preceding paragraphs for different values of the speed υ (55, 90, 160km/h), we obtain the diagrams in *Fig. 3* and *Fig. 4*, which show the oscillations of the middle of the three spans w_{o1}, w_{o2}, w_{o3}, and the sinking s_1 and s_2 of the two pontoons, respectively.



(a) 55km/h – 15m/s



b) 90km/h – 25m/s



c) 160km/h – 45m/s

Figure 3. Mid-span oscillations w_{o1}, w_{o2}, w_{o3} for various speeds υ of the moving load P

4 CONCLUSIONS

A very simple approach to determine deflections of a bridge on floats based on closed form solutions is presented. From the above results, one can draw the following conclusions: The influence of the moving load velocity to the dynamic response of the bridge is significant, especially for the intermediate span. As the load velocity increases, the maximum deflection also increases and becomes maximum after the load pass. The oscillation of the intermediate span is strongly affected by the ones of the neighboring spans due to the pier sinking.

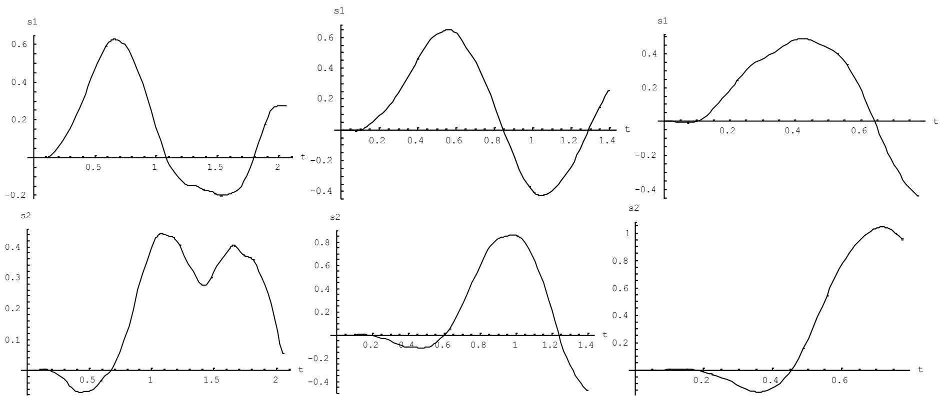


Figure 4. Pontoon sinking s_1 (first raw) and s_2 (second raw) for speeds υ (55, 90 and 160km/h) of the moving load P

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