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MODELING WIND LOADS ON LONG-SPAN BRIDGES

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ABSTRACT: In this paper, a critical review on modeling of wind loads on long-span bridges is traced. Starting from the theoretical background associated to the classical thin airfoil model and from a generalized quasi-steady wind-loading description, a unified consistent approach is proposed. The general frameworks of both frequency-domain and time-domain loading models are exploited, generalizing available approaches and eliminating commonly diffused inconsistencies, in order to preserve the main formal scheme of thin-airfoil-based classical results. The strong duality between time-domain and frequency-domain descriptions is clearly highlighted for both motion-related (aerolastic) and buffeting loads. Finally, in the same unified context, a brief overview of the main open topics and of possible effective strategies to account for some unconventional effects (e.g., related to flow three-dimensionality and nonlinear aerodynamics) is drawn.

KEYWORDS: Bridge aerodynamics; Bridge aeroelasticity; Thin airfoil theory; Bluff-body aerodynamics.

1 INTRODUCTION

In the early part of the last century, developments in aesthetics of bridge building and improvements in materials and technologies led to the construction of progressively longer, structurally more efficient and slender bridges. It was only after the Tacoma Narrows suspension bridge, collapsed shortly after its completion in 1940, that the potentially unstable behavior of long-span bridges under wind actions began to be investigated, highlighting that such structures can be highly sensitive to unsteady wind effects. Wind pressures acting upon a bridge deck are strongly time-dependent due to the local fluctuations of wind velocity, induced by both undisturbed and signature turbulence; the former being intrisic in the incident flow, the latter being initiated by the bridge itself. Moreover, when the bridge deforms under the wind loads, changes in structural configuration affect flow pattern and aerodynamic features, determining a fluid-structure coupling, usually referred to as aeroelastic interaction. As a result, in the case of unfavourable aerodynamic properties of the bridge deck, violent structural oscillations can occur, even at relatively mild wind speeds, and such an occurrence could not be diagnosed by considering only static analyses and steady wind forces. Therefore, unsteady wind loads are generally the most critical external loads which have to be considered in design of long-span bridges, aiming to avoid significant levels of wind-excited oscillations. These latter can be mainly of two types: limited amplitude (non-divergent) oscillations, generally produced by vortex shedding (signature effects) and/or by the random action of wind gusts (namely, buffeting effects, related to the intrinsic turbulence); divergent oscillations, produced by both galloping and flutter instabilities [1]. Generally, the former class of responses may be considered primarily as a serviceability problem, responsible mainly for excessive vibrations and having a potential for serious fatigue damage in the long term. On the other hand, the latter class, in particular flutter, may be considered as an ultimate design condition. In detail, flutter is a self-excited oscillatory instability (involving the interaction among aerodynamic, inertial and elastic structural forces), which corresponds, at certain critical wind speeds, to aerodynamic forces acting to feed energy into the oscillating structure, increasing its vibration magnitude, sometimes to catastrophic levels.

In order to design bridge structures against wind, the wind forces arising on possibly oscillating deck and coupled with the structural response have to be properly described. Actually, the theoretical milestone is surely still represented by the closed-form solutions obtained for the case of a zero-thick airfoil profile (namely, the thin airfoil) moving within an approaching inviscid two-dimensional flow under a small angle of incidence [2–5]. In detail, the time-domain formulation based on Wagner and Küssner indicial approaches [2, 3], and the mixed frequency-time description developed by Theodorsen and Sears [4, 5], were reorganized and further developed by many authors (e.g., [6–10]), revealing the main theoretical background needed to systematically explain some complex fluid-structure interaction phenomena, such as flutter instabilities. In this context, and among others, the pioneering work of Davenport [11] and Scanlan [8, 12] on bridge buffeting and flutter can be surely considered the most important effective attempt to pave the way towards modern bridge aerodynamics and aeroelasticity, giving almost realistic descriptions of both wind-induced forces acting upon long-span bridges and wind-structure interaction mechanisms.

Nevertheless, when unstreamlined bodies are considered, possible large flow separations, reattachments, recirculation zones and vortex shedding can occur, inducing significant unsteady effects and preventing to identify a thin and well defined boundary layer. Thereby, in these cases, the hypothesis of inviscid and fully attached flow, generally acceptable for streamlined bodies immersed in a flow with a small angle of attack, must be often rejected. Accordingly, the description of wind loads on cylindrical bodies with a bluff sectional geometry, such as typical bridge decks, can not be directly obtained by using the thin airfoil theory. In order to overcome such a drawback, several theoretical and computational approaches have been developed, based on both frequency-domain and time-domain descriptions [9]. Although proper circulatory and non-circulatory terms appear as clearly recognizable in the thin airfoil theory, in the context of bridge aerodynamics wind-induced forces are generally represented by distinguishing in-phase and out-of-phase components with respect to the structural motion and/or with respect to the wind gusts. Therefore, in this case a certain distinction can be made only referring to flow memory-dependent (pseudo circulatory) and independent (pseudo non-circulatory) contributions.

As regards frequency-domain approaches, Scanlan [8, 12–17] profitably exported some features of the Theodorsen results, by describing the wind loads induced by sectional harmonic motions via a linearized framework based on experimentally-evaluated frequency-dependent filter functions (namely, the flutter derivatives). These latter linearly relate the aeroelastic forces to the structural motion, and identify equivalent contributions describing aerodynamic stiffness and aerodynamic damping coupled with the structural dynamical features. Such an approach has been proven to be extremely effective in synthetically representing the aeroelastic response of long-span bridges and the corresponding wind-structure interaction mechanisms, allowing to straight estimate critical states for flutter onset [16–23], as well as to effectively face problems related to the bridge aeroelastic control [24] and to the energy harvesting from wind-induced bridge vibrations [25].

Attempts to define effective time-domain formulations, based on the definition of suitable indicial functions (describing the time evolution of the aerodynamic forces induced by a step variation in the effective angle of attack, as the result of a bridge step motion or of a sharp-edged gust), can be also found in recent literature [9, 17, 26, 27]. Nevertheless, time-domain approaches did not developed as much as the frequency-based models, due to the difficulties arising in the direct experimental (or numerical) evaluation of the aerodynamic response to proper, replicable and controllable step-wise motions or gusts [28, 29]. In the context of the bridge aerodynamics and following the classical results for the airfoil by Garrick [30] and Jones [31], Scanlan and co-workers [8, 32] firstly combined Fourier synthesis and rational approximation techniques for analytically-extracting a-posteriori approximations of indicial functions from experimentally-determined frequency-domain data (e.g., flutter derivatives). Starting from more refined formulations of the indicial response and considering generalized rational approximation procedures, similar approaches have been recently developed in [9, 26, 33–35]. Nevertheless, referring to motion-related wind loads, indicial responses which are indirectly estimated from flutter derivatives implicitly include non-circulatory contributions associated with the experimental procedures employed to determine flutter derivatives themselves. Therefore, when flow memory-independent effects are not negligible, the corresponding estimates of the motion-related indicial functions can not be generally considered as fully consistent with Wagner theory, that formally describes circulatory effects only.

As a matter of fact, the relative importance of non-circulatory contributions with respect to circulatory ones can be considered as problem dependent. For example, pseudo non-circulatory effects can be generally considered negligible for truss decks with large openings and grillages, or when the flow regime and the sectional geometry induce wide bluff features with large vortex structures. Nevertheless, many modern long-span bridges exhibit almost elongated and streamlined cross-sections, characterized by mildly-bluff performances. As a consequence, this occurrence on one hand does not allow to apply directly the ideal thin airfoil model and, on the other hand, can lead to non-circulatory effects generally not completely negligible with respect to circulatory ones [36]. This matter can be more evident when eccentricity between elastic and gravity axes, and/or small values of the reduced velocity (or equivalently high values of the reduced frequency) are considered. Moreover, when unstreamlined bodies are addressed, a fundamental aspect in time-domain formulations is related to the choice of the indicial responses to be considered in the loading description, and their relationships with downwash effects induced by the body motion. Partially borrowing the structure of the thin airfoil theory, available time-domain formulations have been generally developed by postulating the type of the indicial functions and of the downwash contributions to which they combine by convolution. As a result, such an axiomatic approach has led to different formulations, often not directly comparable each other and that can suffer from some consistency and/or effectiveness lack. In particular, addressing the case of bridge deck sections, downwash contributions given by the pitch rate are usually neglected, and this choice is justified by invoking the sectional bluffness. Nevertheless, referring to modern bridge sections, they can be not bluff enough to justify such an assumption.

In the context of motion-related wind loads, a general theoretical framework based on the main formal scheme of the classical results obtained by Theodorsen and Wagner, and developed without introducing a-priori simplifications, has been recently proposed in [27], contributing to overcome some consistency problems with respect to the thin airfoil model. In this way, the mutual role played by memory-independent terms and pseudo-circulatory downwash effects has been established, opening also to the possibility of drawing some insights on issues to which the thin airfoil theory does not provide suitable indications (for instance, for describing motion-induced drag force component).

In this paper, moving from the thin airfoil theory, the general frameworks of available frequency-domain and time-domain models for describing wind loads on bridge decks are discussed and generalized, by eliminating commonly diffused inconsistencies. As a result, a unified approach consistent in conventions, notation, assumptions and formulation is proposed. Finally, some open topics and advanced strategies aimed to account for some unconventional effects are critically addressed.

2 PROBLEM SETTING

The deck of long-span bridges can be regarded as a slender line-like cylinder and, by neglecting any effect related to possible skew winds (as it is customary in bridge aerodynamics), its longitudinal axis is assumed to be orthogonal to the mean wind flow. Accordingly, disregarding also three-dimensional local effects (induced for example by cables, barriers, dividers, pylons, and other possible deck details) and assuming a perfect flow correlation along the bridge span, the flow distribution around the girder can be generally considered, as a first approximation, as bi-dimensional in the plane of the bridge cross-section (wind load sectional model). Namely, the assumption of a fully correlated flow along the bridge span is indicated as strip assumption and it corresponds to assume that the span-wise flow and the pressure redistribution are negligible, so that pressures upon any section are due only to the wind incident on that section, without considering effects related to adjacent span strips [11, 37, 38]. Although such an assumption can fail in many real cases (namely, when the structure has dimensions comparable with the turbulence length-scales of the wind flow), it represents the basis of the current theories for describing wind forces acting upon bridge decks.

Accordingly, let a rigid cylinder-like body with an infinite span-length be considered, immersed in a low-speed wind flow orthogonal to the body axis, and characterized by an elongated cross-section $\mathcal{S}$, whose chord dimension is B. Furthermore, let ρ be the air density, ν the air kinematic viscosity, and U the mean velocity of the approaching flow. With reference to Fig. 1, the body has three degrees of freedom, corresponding to horizontal (p), vertical (h) and angular (α) displacements in the cross-sectional plane. Rotation is assumed to be about a chord point distant $aB/2$ from the chord midpoint (with $a > 0$ for a downstream rotation center). Moreover, let the reference configuration of the body be defined by the angle of attack α_o between the mean wind direction and the cross-section chord.

The flow-induced pressures acting on the body can be reduced, with respect to the rotation axis, to the following generalized force components (Fig. 1): along-the-wind force (drag, D), across-the-wind force (lift, L), and pitching moment (M). They are usually expressed per unit span-length as:

$$D(t) = \mathcal{P}_o B C_D(t), \qquad L(t) = \mathcal{P}_o B C_L(t), \qquad M(t) = \mathcal{P}_o B^2 C_M(t) \tag{1}$$

where $\mathcal{P}_o = \rho U^2/2$ is the mean kinetic pressure, t is the time variable, and $C_g(t)$ (with $g = D, L, M$) are the dimensionless force coefficients, generally depending on the shape of $\mathcal{S}$, on the Reynolds number $Re = UB/\nu$, and on the angle of attack. Finally, as a notation rule, in the following quantities $C_g^o = C_g(\alpha_o)$ (with $g = D, L, M$) are used to indicate dimensionless force coefficients associated to steady mean loads in the reference configuration.

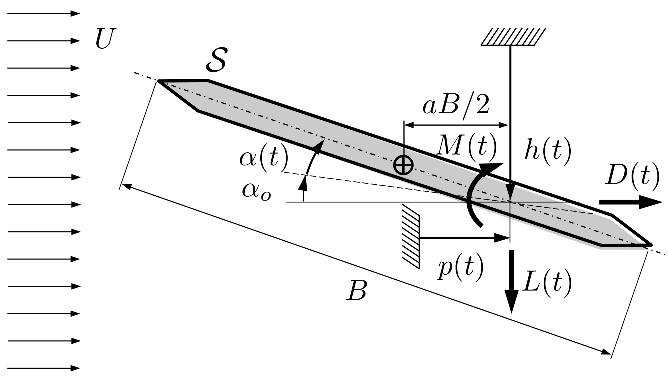

Figure 1. Wind load sectional model: notation and conventions

3 BACKGROUND: THIN AIRFOIL AERODYNAMICS

Let a rigid flat plate with vanishing thickness-to-width ratio (namely, the thin airfoil) be considered, immersed in a two-dimensional, incompressible and inviscid approaching flow. Let $x \in [-B/2, B/2]$ be the chord coordinate, with $x = -B/2$ at the airfoil leading edge and $x = B/2$ at the trailing edge. As a result of the potential flow theory [7], flow-induced pressure distribution on a fixed thin airfoil under a small incidence α_o reduces in: a zero drag force, an upward (negative) lift with $C_L^o = -2\pi\alpha_o$, and a clockwise (positive) pitching moment with $C_M^o = \pi(1/2 + a)\alpha_o = -C_L^o(1/2 + a)/2$. Accordingly, static aerodynamic forces are equivalent to the lift force applied at the upstream quarter-chord point (namely, the aerodynamic center or forward neutral point, at $x = -B/4$) and they vanish for $\alpha_o = 0$. It is worth pointing out that, as it is customary in bridge analysis [16], lift sign convention herein employed is opposite with respect to the classic aeronautical one.

The problem of characterizing the aerodynamic forces acting on the thin airfoil harmonically oscillating in the flow about $\alpha_o = 0$ was solved by Theodorsen [4]. Small oscillations about the mid-chord axis, and described by $h(t) = \tilde{h}e^{i\omega t}$ and $\alpha(t) = \tilde{\alpha}e^{i\omega t}$, are herein considered, where ω is the oscillation circular frequency, $\tilde{h}$ and $\tilde{\alpha}$ are the small motion amplitudes, and i is the imaginary unit.

Generalized self-excited loads per unit length have been described by Theodorsen as the superposition of the following circulatory (c, depending on the frequency of oscillations and accounting for flow unsteady effects) and non-circulatory (nc, frequency-independent and including inertial effects due to the moved fluid mass) contributions:

$$L_{nc}(s) = \mathcal{P}_o B\, C_{L/\alpha} \left(\frac{h''}{B} + \frac{\alpha'}{2} - \frac{a}{2}\alpha'' \right) \tag{2}$$

$$L_c(k,s) = \mathcal{P}_o B\, C_{L/\alpha}\, \mathcal{C}(k)\, \hat{\alpha}(s) \tag{3}$$

$$M_{nc}(s) = -\mathcal{P}_o B^2 \frac{C_{M/\alpha}}{2\left(\frac{1}{2}+a\right)} \left[\left(\frac{1}{2} - a\right)\alpha' + \left(\frac{1}{8} + a^2\right)\alpha'' - 2a\frac{h''}{B} \right] \tag{4}$$

$$M_c(k,s) = \mathcal{P}_o B^2\, C_{M/\alpha}\, \mathcal{C}(k)\, \hat{\alpha}(s) = -\frac{B}{2}\left(\frac{1}{2} + a\right) L_c(k,s) \tag{5}$$

where $s = 2Ut/B$ is the dimensionless time, $k = B\omega/(2U)$ is the reduced frequency of oscillation, $C_{g/\alpha} = \partial C_g^o/\partial\alpha_o$, and f' denotes the first derivative of f with respect to s, the first time derivative resulting in $\dot{f} = 2Uf'/B$. It is worth remarking that, as it is customary in the aeronautic field, quantities k and s are normalized with respect to the half-chord length.

The frequency-dependent function $\mathcal{C}(k)$ introduced in Eq. (3) is the Theodorsen's complex circulatory function, equal to [4]:

$$\mathcal{C}(k) = \mathcal{F}_T(k) + \mathrm{i}\mathcal{G}_T(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + \mathrm{i}H_0^{(2)}(k)} \tag{6}$$

where $H_\cdot^{(2)}$ are Hankel functions of the second kind. It is possible to show [7] that for low frequency regimes (namely, quasi-stationary motions) the imaginary part $\mathcal{G}_T(k) = \mathrm{Im}[\mathcal{C}(k)]$ vanishes, and the real part $\mathcal{F}_T(k) = \mathrm{Re}[\mathcal{C}(k)]$ tends to 1 (Fig. 2). It can be noted that unsteady wind loads expressed by Eqs. (3) and (5) are defined in terms of a mixed formulation involving both time and frequency, and that the circulatory contributions depend on the effective instantaneous angle of attack $\hat{\alpha}$ at the three-quarter chord point (namely, the rear neutral point, at $x = B/4$)

$$\hat{\alpha} = \alpha + \frac{\dot{h}}{U} + \frac{B}{2}\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U} = \alpha + 2\frac{h'}{B} + \left(\frac{1}{2} - a\right)\alpha' \tag{7}$$

corresponding to the instantaneous downwash dimensionless velocity of a fluid particle at the rear neutral point.

The aerodynamic loads induced by an arbitrary motion of the thin airfoil in a potential flow can be expressed moving from the results proposed by Wagner [2],

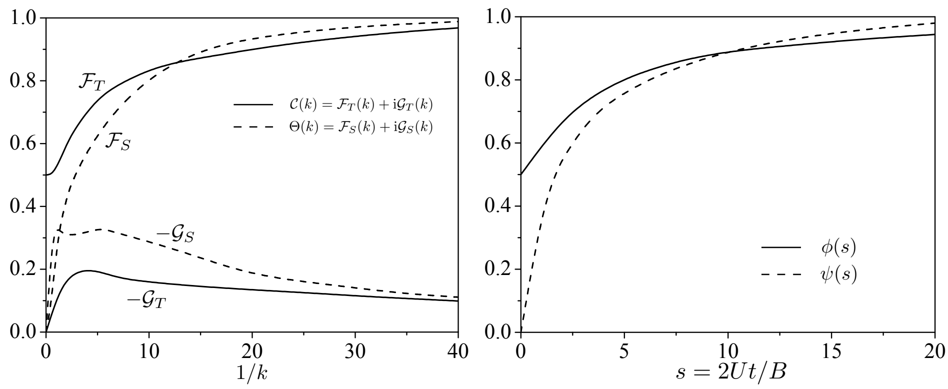

Figure 2. The thin airfoil model. Theodorsen and Sears functions (on the left). Wagner (ϕ) and Küssner (ψ) indicial functions (on the right)

who solved the problem of an abrupt change of the angle of attack from $\alpha_o = 0$. In detail, considering $\alpha(t) = \mathcal{H}(t)\tilde{\alpha}$, where $\mathcal{H}(t)$ is the Heaviside function and $\tilde{\alpha}$ is a small finite value so that the first-order approximation of all physical quantities can be applied, the non-steady circulatory lift can be expressed as:

$$L_c(s) = \mathcal{P}_o B\, C_{L/\alpha}\, \phi(s)\, \tilde{\alpha} \tag{8}$$

where $C_{L/\alpha} = -2\pi$ and $\phi(s)$ is the Wagner's indicial lift-growth function, describing the transient evolution of the lift force up to its static value, and characterized by $\phi(0) = 0.5$ and $\phi(s)$ tending to 1 for s approaching infinity (Fig. 2). Due to the special simplicity of the thin airfoil theory, the instantaneous angle of attack $\hat{\alpha}(s)$ and the Wagner's indicial function suffice to define both lift and moment in the case of an arbitrary motion of the airfoil, involving also the vertical displacement h. In this case, invoking the linear superposition principle and assuming that thin airfoil moves from the rest at $s = 0$ (with $\phi(s) = 0$ for $s < 0$), circulatory terms L_c and M_c in the time domain can be expressed by the following Duhamel's convolution integrals [6]:

$$\begin{aligned} L_c(s) &= \mathcal{P}_o C_{L/\alpha} B \int_{-\infty}^{s} \phi(s-\tau)\hat{\alpha}'(\tau)\, d\tau \\ &= \mathcal{P}_o C_{L/\alpha} B \left[\phi(s)\hat{\alpha}(0) + \int_0^s \phi(s-\tau)\hat{\alpha}'(\tau)\, d\tau\right] \end{aligned} \tag{9}$$

$$M_c(s) = \mathcal{P}_o C_{M/\alpha} B^2 \int_{-\infty}^{s} \phi(s-\tau)\hat{\alpha}'(\tau)\, d\tau = -\frac{B}{2}\left(\frac{1}{2} + a\right) L_c(k,s) \tag{10}$$

or equivalently by

$$L_c(s) = \mathcal{P}_o \int_{-\infty}^{s} \hat{\alpha}(\tau) I_{\hat{\alpha}}(s-\tau)\, d\tau \tag{11}$$

where $I_{\hat{\alpha}}(s) = B\,C_{L/\alpha}\,[\phi(0)\delta(s) + \phi'(s)]$ is the thin-airfoil's impulse response function (with $I_{\hat{\alpha}}(s) = 0$ for $s < 0$), corresponding to $\hat{\alpha}(s) = \delta(s)$, $\delta(s)$ being the Dirac's delta function.

By using Fourier synthesis, a strong duality between time-domain and frequency-domain descriptions has been proven by Garrick [30], resulting in the following relationships among the Theodorsen's circulatory function $\mathcal{C}(k)$, the Wagner's indicial function $\phi(s)$ and the impulse response function $I_{\hat{\alpha}}(s)$:

$$\phi(s) = \frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{\infty} \frac{\mathcal{C}(k)}{k} \mathrm{e}^{\mathrm{i}ks}\, dk \tag{12}$$

$$\mathcal{C}(k) = \mathrm{i}k \int_{0}^{\infty} \phi(\tau) \mathrm{e}^{-\mathrm{i}k\tau}\, d\tau = \frac{\overline{I_{\hat{\alpha}}}}{B\,C_{L/\alpha}} = \phi(0) + \overline{\phi'}(k) \tag{13}$$

the frequency-dependent function $\overline{f}(k)$ being the Fourier transform of the time-dependent function $f(s)$.

Frequency-domain formulation allows to relate the power spectrum $S_{L_c}(k)$ of the circulatory lift to the one ($S_{\hat{\alpha}}(k)$) of the angle of attack at the three-quarter chord point as [32]:

$$S_{L_C}(k) = (\mathcal{P}_o B\,C_{L/\alpha})^2 |\chi_T(k)|^2 S_{\hat{\alpha}}(k) \tag{14}$$

where $|\chi_T(k)|^2 = \mathcal{C}(k)\mathcal{C}^*(k) = \mathcal{F}_T^2(k) + \mathcal{G}_T^2(k)$ is the so-called Theodorsen's aerodynamic admittance function, $\mathcal{C}^*$ denoting the complex conjugate of $\mathcal{C}$. An excellent approximation of the Wagner's function was proposed by Jones [31], resulting to be very useful because of its simple Fourier transform

$$\phi(s) \cong a_o - \sum_{j=1}^{n} a_j \mathrm{e}^{-b_j s} \tag{15}$$

with $n = 2$, $a_o = 1$, $a_1 = 0.165$, $a_2 = 0.335$, $b_1 = 0.0455$, $b_2 = 0.3$. Equation (15) includes a steady term a_o corresponding to the limit of $\phi(s)$ for s tending to infinity, and n exponential functions that can be regarded as a cascade of filters. Accordingly, by combining Eqs. (12), (13) and (15), a rational approximation of $\mathcal{C}(k)$ results from:

$$\mathcal{F}_T(k) = a_o - k^2 \sum_{j=1}^{n} \frac{a_j}{b_j^2 + k^2}, \qquad \mathcal{G}_T(k) = -k \sum_{j=1}^{n} \frac{a_j b_j}{b_j^2 + k^2} \tag{16}$$

The influence of the intrinsic turbulence in the wind was historically accounted for in the thin airfoil model by considering only vertical wind gusts acting upon the thin airfoil with a zero incidence, that is by superimposing to the mean wind flow only a vertical (i.e., orthogonal to the mean wind direction) turbulent wind-speed component v, herein assumed positive if downwards. In detail, by assuming $v \ll U$ to be fully correlated along the airfoil span (namely, strip assumption) and by considering the wing entering a sinusoidal gust $v(x,s) = \tilde{v}\mathrm{e}^{\mathrm{i}k(s-2x/B)}$, the aerodynamic forces acting upon the airfoil reduce to the following lift force acting at $x = B/4$ [7]:

$$L(k,s) = -\mathcal{P}_o B\, C_{L/\alpha} \frac{\tilde{v}}{U} \Theta(k) \mathrm{e}^{\mathrm{i}ks} \tag{17}$$

where $\Theta(k)$ is the frequency-dependent Sears' function [5]

$$\Theta(k) = [J_0(k) - \mathrm{i}J_1(k)]\mathcal{C}(k) + \mathrm{i}J_1(k) = \mathcal{F}_S(k) + \mathrm{i}\mathcal{G}_S(k) \tag{18}$$

$J_\cdot(k)$ being Bessel functions of the first kind, and $\mathcal{F}_S(k)$ and $\mathcal{G}_S(k)$ indicating the real and imaginary parts of $\Theta(k)$, respectively.

The case of the thin airfoil moving with speed U and $\alpha_o = 0$, and entering an arbitrary vertical wind gust can be addressed starting from the case of a sharp-edged gust $v(x,t) = \mathcal{H}(t - x/U)\tilde{v}$. In this case, the resulting lift force was determined by Küssner [3] as:

$$L(s) = -\mathcal{P}_o B\, C_{L/\alpha}\, \psi(s)\, \frac{\tilde{v}}{U} \tag{19}$$

where $\psi(s)$ is the Küssner's indicial function (with $\psi(s) = 0$ for $s \leq 0$ and $\psi(s)$ tending to 1 for s approaching infinity, Fig. 2), describing the transient evolution of the lift force up to the static value associated to an effective angle of attack $\hat{\alpha} = -\tilde{v}/U$. If an arbitrary function $v(s)$ is considered for $s \geq 0$ as the gust velocity encountered by the airfoil's leading edge ($x = -B/2$) at the instant $t = sB/(2U)$, Duhamel's linear superposition gives, as in Eqs. (9) and (11), the following expression for the unsteady lift:

$$\begin{aligned} L(s) &= -\mathcal{P}_o C_{L/\alpha} B \left[\psi(0) \frac{v(s)}{U} + \int_0^s \psi'(\tau) \frac{v(s-\tau)}{U}\, d\tau \right] \\ &= \mathcal{P}_o \int_{-\infty}^{s} I_v(s-\tau) \frac{v(\tau)}{U}\, d\tau \end{aligned} \tag{20}$$

where $I_v(s)$ is the thin-airfoil's impulse response function corresponding to $v(s) = \delta(s)U$, and equal to

$$I_v(s) = -C_{L/\alpha} B[\psi(0)\delta(s) + \psi'(s)] \tag{21}$$

Even in the case of aerodynamic forces induced by vertical wind gusts, a strong duality between time-domain and frequency-domain descriptions can be stated, similarly to the case of Theodorsen and Wagner functions. In detail, Fourier synthesis enables to show the strong duality between Sears and Küssner functions

$$\psi(s) = \frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{\infty} \frac{\Theta(k)}{k} \mathrm{e}^{\mathrm{i}ks} \, dk \tag{22}$$

$$\Theta(k) = \mathrm{i}k \int_{0}^{\infty} \psi(\tau) \mathrm{e}^{-\mathrm{i}k\tau} \, d\tau = \psi(0) + \overline{\psi'}(k) = -\frac{\overline{I_v}(k)}{C_{L/\hat{\alpha}} B} \tag{23}$$

and thereby to express the power spectral density of $L(s)$ as [32]:

$$S_L(k) = (\mathcal{P}_o B \, C_{L/\alpha})^2 |\chi_S(k)|^2 S_v(k)/U^2 \tag{24}$$

where $|\chi_S(k)|^2 = \Theta(k)\Theta^*(k) = \mathcal{F}_S^2(k) + \mathcal{G}_S^2(k)$ is the Sears' aerodynamic admittance function and $S_v(k)$ is the power spectral density of the gust function $v(t)$. A convenient empirical approximation of the function $|\chi_S(k)|^2$ can be put in the form $|\chi_S(k)|^2 = (1 + \lambda k)^{-1}$, with $\lambda = 5.0$ (in agreement with Scanlan [15]) or $\lambda = 2\pi$ (following the statistical approach adopted by Liepmann [39]). Accordingly, when gust frequency is significantly greater than zero, the gust-induced fluctuating lift results to be significantly smaller than the quasi-steady value, the latter attained for k approaching to zero.

By replacing $\phi(s)$ by $\psi(s)$ in Eq. (15), also the Küssner's function can be approximated through exponential filters. An excellent approximation is obtained by considering $n = 2$, $a_o = 1$, $a_1 = a_2 = 0.500$, $b_1 = 1.000$, and $b_2 = 0.130$, [7, 32]. Accordingly, real and imaginary parts of the Sears' function $\Theta(k)$ can be evaluated by employing a rational approximation as in Eqs. (16).

It is worth remarking that Eq. (18) allows to state a theoretical relationship between buffeting and self-excited loads acting upon the thin airfoil, that is between Sears and Theodorsen circulatory functions (in the frequency domain), as well as between Küssner and Wagner functions (in the time domain). In detail, Eq. (18) clearly suggests that, even in the ideal case under investigation, indicial responses to a step-wise vertical gust (namely, ψ) and to a step-wise change in the angle of attack (namely, ϕ) are formally different. In other words, and as confirmed by analyzing curves in Fig. 2, although the asymptotic behavior associated to the steady limit is the same in both cases (when the effective angle of attack is the same), the transient local flow patterns arising around the profile as induced by a wind gust or by an airfoil motion are significantly different.

4 QUASI-STEADY WIND LOAD MODEL

In some cases, wind loads acting upon on a cross-section $\mathcal{S}$ can be effectively represented by means of a quasi-steady model. Accordingly, forces induced by a turbulent approaching wind flow are assumed at each instant to be equal to the steady loads corresponding to both the structural configuration and the effective angle of attack in a representative point at that instant, resulting independent from any memory effects related to motion and wind fluctuations at earlier times [40]. Quasi-steady approach thereby holds if [11, 37, 40–42]:

- The characteristic dimension of $\mathcal{S}$ is small when compared with the turbulence length scales in the approaching flow, thereby the turbulent fluctuations can be assumed as perfectly correlated around $\mathcal{S}$ and dependent only on time and not on space;
- Signature turbulence (associated to wake-induced effects) has a harmonic content with a characteristic frequency range higher than the undisturbed turbulence. Accordingly, undisturbed and signature turbulence can be considered uncorrelated, and the corresponding wind-force contributions can be assumed as independent and thereby linearly superimposable; such an assumption corresponds to assume that the cross-section $\mathcal{S}$ is characterized by high values of the dimensionless Strouhal number $St = B\omega_w/(2\pi U)$, ω_w being the circular frequency of full cycles of vortex shedding in the wake of $\mathcal{S}$ [16];
- The section $\mathcal{S}$ moves from a reference configuration slowly and with small oscillation amplitudes into the approaching wind flow (namely, quasi-static assumption), and the fluctuating wind components are small with respect to the mean wind speed (namely, quasi-steady assumption).

Addressing the notation introduced in Fig. 3 and assuming for the sake of simplicity $a = 0$ (see Fig. 1), $\mathcal{S}_o$ denotes the reference section configuration and $\mathcal{S}$ the actual one, the latter obtained as a rigid transformation of $\mathcal{S}_o$. Moreover, $\mathsf{F}_o \equiv (O, x_o, y_o)$ is a fixed Cartesian frame centered in the centroid O of $\mathcal{S}_o$, (x_o,y_o) being principal inertial axes for $\mathcal{S}_o$. Let $\mathsf{M}_o \equiv (\hat{O}, \hat{x}_o, \hat{y}_o)$ be a local Cartesian frame, rigidly moving with $\mathcal{S}$ (such that $\mathsf{F}_o \equiv \mathsf{M}_o$ when $\mathcal{S}_o \equiv \mathcal{S}$), and $\mathsf{F}_w \equiv (O, x, y)$ a Cartesian frame whose axis x is aligned with the mean wind direction, angled α_o with respect to x_o, and $\alpha_o + \alpha(t)$ with respect to $\hat{x}_o$, with $\alpha(t)$ denoting the rotation of $\mathcal{S}$ with respect to $\mathcal{S}_o$. The translation transforming O in $\hat{O}$ is described by the time-dependent displacement functions $p(t)$ and $h(t)$ along x (along-wind direction) and y (cross-wind direction) axes, respectively.

Let two reference points P_f and P_m be introduced, in order to describe lift and drag forces (point P_f), and the moment with respect to $\hat{O}$ (point P_m). In detail, we assume $(P_r - \hat{O}) = B(\beta_{rx}\,\hat{\mathbf{i}}_o + \beta_{ry}\,\hat{\mathbf{j}}_o)$, with $r = f, m$ and ($\hat{\mathbf{i}}_o$, $\hat{\mathbf{j}}_o$) the unit vectors along the coordinate axes of M_o. These latter are such that

$$\{\mathbf{i}, \mathbf{j}\}^T = \mathbb{R}(\alpha_o + \alpha)\,\{\hat{\mathbf{i}}_o, \hat{\mathbf{j}}_o\}^T, \qquad \mathbb{R}(\vartheta) = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix} \tag{25}$$

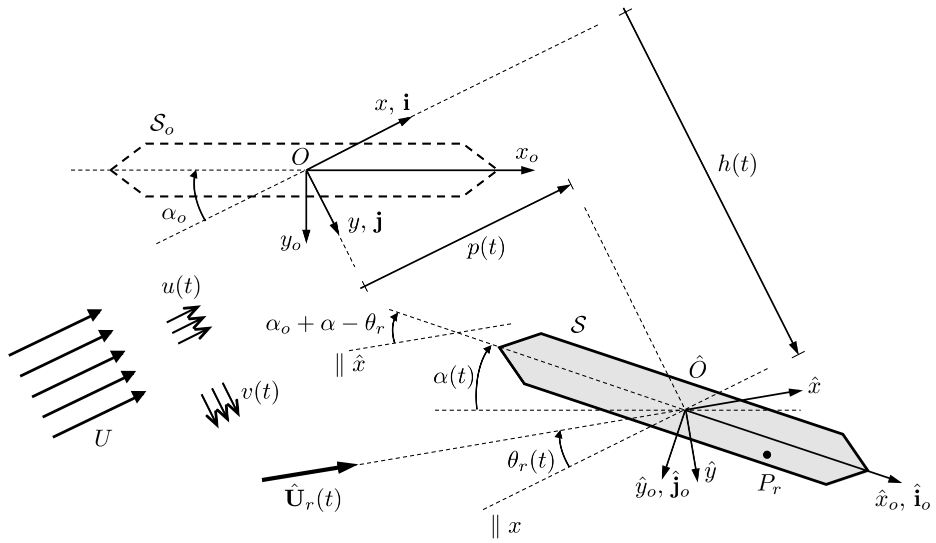

Figure 3. Quasi-steady wind load model: notation

where ($\mathbf{i}$, $\mathbf{j}$) are the unit vectors along the coordinate axes of F_w, and $\mathbb{R}(\vartheta)$ is the in-plane orthonormal rotation matrix corresponding to the angle ϑ. Accordingly, at the reference points P_f and P_m the instantaneous angle of attack is:

$$\hat{\alpha}_r = \alpha_o + \alpha(t) - \theta_r(t), \qquad \theta_r(t) = \arctan\left[\frac{\hat{\mathbf{U}}_r \cdot \mathbf{j}}{\hat{\mathbf{U}}_r \cdot \mathbf{i}}\right] \tag{26}$$

θ_r denoting the angle between the relative wind speed $\hat{\mathbf{U}}_r$ at P_r (with $r = f, m$) and the mean wind direction $\mathbf{i}$, with $\hat{\mathbf{U}}_r$ being expressed in F_w as:

$$\hat{\mathbf{U}}_r = \left\{ \begin{array}{c} U + u - \dot{p} \\ v - \dot{h} \end{array} \right\} - B\,\dot{\alpha}\,\mathbb{R}(\alpha_o + \alpha)\mathbf{b}_r, \qquad \mathbf{b}_r = \{-\beta_{ry},\, \beta_{rx}\}^T \tag{27}$$

where $u(t)$ and $v(t)$ are the time-dependent turbulent wind components along axes x and y, respectively.

With respect to the moving Cartesian frame $\mathsf{M}_w \equiv (\hat{O}, \hat{x}, \hat{y})$ (such that $\mathsf{F}_w \equiv \mathsf{M}_w$ when $\mathcal{S}_o \equiv \mathcal{S}$), whose axis $\hat{x}$ is aligned with $\hat{\mathbf{U}}_r$, the following aerodynamic forces can be introduced:

$$\hat{D} = \mathcal{P}_f B\, C_D(\hat{\alpha}_f), \qquad \hat{L} = \mathcal{P}_f B\, C_L(\hat{\alpha}_f), \qquad \hat{M} = \mathcal{P}_m B^2\, C_M(\hat{\alpha}_m) \tag{28}$$

where $\hat{D}$ acts along $\hat{x}$, $\hat{L}$ along $\hat{y}$, $\hat{M}$ is reduced with respect to $\hat{O}$, and $\mathcal{P}_r = \rho\|\hat{\mathbf{U}}_r\|^2/2$ is the instantaneous kinetic pressure in P_r (with $r = f, m$).

Introducing the vector of the generalized forces $\mathbf{F}(t) = \{D,\ L,\ M\}^T$ with components referred to F_w, it results in:

$$\mathbf{F}(t) = \begin{bmatrix} \mathbb{R}(\theta_f) & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \begin{Bmatrix} \hat{D}(\hat{\alpha}_f) \\ \hat{L}(\hat{\alpha}_f) \\ \hat{M}(\hat{\alpha}_m) \end{Bmatrix} \tag{29}$$

By enforcing quasi-static and quasi-steady assumptions, that is $|g/U| \ll 1$ and $|q/B| \ll 1$ with $g = u, v,\dot{p}, \dot{h}, \dot{\alpha}B$ and $q = p, h, \alpha B$, the nonlinear relationship in Eq. (29) can be linearized around the reference configuration $\mathcal{S}_o$:

$$\mathbf{F}(t) \cong \mathbf{F}^o(\alpha_o, t) = \mathbf{F}_s(\alpha_o) + \mathbf{F}^o_b(\alpha_o, t) + \mathbf{F}^o_a(\alpha_o, t) \tag{30}$$

where the dependency on the steady angle of attack α_o has been emphasized and, by indicating $C^o_g = C_g(\alpha_o)$ and $C_{g/\hat{\alpha}} = \partial C_g/\partial\hat{\alpha}|_{\hat{\alpha}=\alpha_o}$ (with $g = D, L, M$), where:

- $\mathbf{F}_s(\alpha_o)$ is a constant vector collecting steady mean forces:

$$\mathbf{F}_s(\alpha_o) = \mathcal{P}_o B\ \{C^o_D \quad C^o_L \quad B\, C^o_M\}^T \tag{31}$$

- $\mathbf{F}^o_b(\alpha_o, t)$ denotes the quasi-steady buffeting contribution, linearly depending on the turbulence velocity vector $\mathbf{V}(t) = \{u(t),\ v(t)\}^T$ and expressed via the constant 3×2 buffeting matrix $\mathbb{C}^b(\alpha_o)$:

$$\begin{aligned} \mathbf{F}^o_b(\alpha_o, t) &= \mathbb{C}^o_b(\alpha_o)\mathbf{V}(t) \\ &= \frac{\mathcal{P}_o B}{U} \begin{bmatrix} 2C^o_D & -(C_{D/\hat{\alpha}} + C^o_L) \\ 2C^o_L & -(C_{L/\hat{\alpha}} - C^o_D) \\ 2BC^o_M & -BC_{M/\hat{\alpha}} \end{bmatrix} \begin{Bmatrix} u(t) \\ v(t) \end{Bmatrix} \end{aligned} \tag{32}$$

- $\mathbf{F}^o_a(\alpha_o, t)$ describes the quasi-static self-excited or aeroelastic forces, expressed in terms of the cross-section's generalized displacement ($\mathbf{q}$) and velocity ($\dot{\mathbf{q}}$) vectors, and in terms of the constant 3×3 aerodynamic damping and stiffness matrixes, $\mathbb{C}^o_a$ and $\mathbb{K}^o_a$, respectively:

$$\begin{aligned} \mathbf{F}^o_a(\alpha_o, t) &= -\mathbb{C}^o_a(\alpha_o)\dot{\mathbf{q}}(t) - \mathbb{K}^o_a(\alpha_o)\mathbf{q}(t) \\ &= -\left[\mathbb{C}^o_b \quad \mathbf{c}\right]\dot{\mathbf{q}}(t) + \mathcal{P}_o \begin{bmatrix} 0 & 0 & C_{D/\hat{\alpha}} \\ 0 & 0 & C_{L/\hat{\alpha}} \\ 0 & 0 & BC_{M/\hat{\alpha}} \end{bmatrix} \mathbf{q}(t) \end{aligned} \tag{33}$$

  where $\mathbf{q}(t) = \{p(t),\ h(t),\ B\alpha(t)\}^T$ and where the vector $\mathbf{c}$ is componentwise defined as

$$[\mathbf{c}]_i = [\mathbb{C}^o_b\, \mathbb{R}(\alpha_o)\, \mathbf{b}_r]_i \tag{34}$$

  with $r = f$ for $i = 1, 2$, and $r = m$ for $i = 3$.

If the steady angle of attack α_o is assumed to be small itself, then the linearized relationships previously introduced may be rearranged substituting $\mathbb{R}(\alpha_o)$ in the Eq. (34) with the 2×2 identity matrix and considering: $C_g^o = C_g(0)$, $C_{g/\hat{\alpha}} = \partial C_g/\partial\hat{\alpha}|_{\hat{\alpha}=0}$ with $g = D, L, M$, and $\mathbf{F}_s(\alpha_o) \cong \mathbf{F}_s(0) + \mathbf{F}_{s/\hat{\alpha}}|_{\hat{\alpha}=0}\,\alpha_o$.

A frequency domain representation of Eq. (30) can be easily derived as:

$$\begin{aligned}\overline{\mathbf{F}}^o(\alpha_o, K) = &\, \mathbf{F}_s(\alpha_o) + \mathbb{C}_b^o(\alpha_o)\overline{\mathbf{V}}(K) \\ &- \frac{U}{B}\mathbb{C}_a^o(\alpha_o)\overline{\mathbf{q}'}(K) - \mathbb{K}_a^o(\alpha_o)\overline{\mathbf{q}}(K)\end{aligned} \tag{35}$$

where, as it is customary in bridge analysis, the reduced frequency $K = B\omega/U = 2k$ and the dimensionless time $S = Ut/B = s/2$, based on the cross-section width B, are introduced. As a notation rule, from now on, $f' = \dot{f}B/U$ will denote the first derivative of f with respect to S.

In agreement with the previously-recalled quasi-steady assumptions, Eqs. (30) and (35) should be completed by linearly superimposing contributions accounting for wake-induced loads, generally depending on the Strouhal number of $\mathcal{S}$. Signature effects are out of the aim of the present paper. Nevertheless, although in the analysis of bridge buffeting/flutter problems signature effects are generally not taken into account (e.g., [16, 17]), a wide review of well-posed models for describing wake-induced contributions can be found in [16, 40, 43].

It is worth pointing out that, in the framework of the present linear description, aeroelastic and buffeting loading contributions are not coupled each other and, as a result, self-excited forces have to be considered independent from any turbulent effects, and thereby referred to a laminar flow condition.

Time-domain quasi-steady formulation described by Eqs. (30) to (34) consistently generalizes many quasi-steady approaches recently proposed (e.g., [17, 40–42, 44–47]), which can be straight recovered by assuming specific reference points P_f and P_m, and/or by disregarding some aerodynamic or geometric contributions.

In the limit of the quasi-steady approach, the choice of the reference points P_f and P_m, that is of the parameters β_{rj} (with $r = f, m$ and $j = x, y$), is crucial because they affect the contribution of $\dot{\alpha}$ to the effective angle of attack. Such parameters, as it is widely recognized in the specialized literature, should be properly determined through dynamic experimental tests, and they could generally depend on the steady angle of attack, i.e. $\beta_{rj} = \beta_{rj}(\alpha_o)$. Different meanings to the role of reference points, as well as different choices for P_f and P_m, can be found through the literature. For instance, Diana et al. [48] considered only two parameters, namely β_{fx} and β_{mx}, assuming that the reference points belong to the axis $\hat{x}_o$ and that the height of $\mathcal{S}$ is small, so that $\beta_{fy} = \beta_{my} = 0$. Salvatori and Borri [49] proposed a generalization of the previous approach, considering three parameters similar to β_{rj}, one for each component of $\mathbf{F}_a^o$. Nevertheless, this model is not consistently and rationally derived, but it is only postulated as an extension of other quasi-steady formulations [16, 40]. Other authors (e.g., [19, 42, 50, 51]), similarly to the case of the thin airfoil, used only one reference point $P_f = P_m$ belonging to the axis $\hat{x}_o$ (i.e., $\beta = \beta_{fx} = \beta_{mx}$ and $\beta_{fy} = \beta_{my} = 0$). Many authors (e.g., [44, 52–54]) considered, as a unique reference point, the centroid of $\mathcal{S}$ (i.e., $\beta = 0$); Borri and Costa [55] employed the leading edge of the bridge cross-section (i.e., $\beta = -1/2$), and Chen and Kareem [56] performed some calculations considering the three-quarter chord point like in thin airfoil (i.e., $\beta = 1/4$). It is worth remarking that these assumptions have not to be considered in an absolute sense but, depending on the bluffness degree of $\mathcal{S}$, they should be experimentally verified case by case.

As a matter of fact, a consistent and general adaptation of the quasi-steady approach to a general dynamics is simply not possible, and quasi-steady formulation remains strictly valid only as a limit behavior, achievable when a constant value in time of the angle of attack can be supposed, that is when it is theoretically possible that flow reaches a steady state. This condition may occur only when time-dependent functions $\dot{p}(t)$, $\dot{h}(t)$ and $\alpha(t)$ are constant.

5 UNSTEADY MODELS FOR LONG-SPAN BRIDGES

Quasi-steady and quasi-static assumptions are not generally satisfied when typical wind-bridge interaction processes are addressed. As a matter of fact, bridge deck sections are usually characterized by a very elongated shape along the mean wind direction and, as a consequence, the characteristic chord-size of the section can not be considered as small if compared with the length scales of the turbulence components [11, 37]. Therefore, turbulence field around $\mathcal{S}$ can not be considered as perfectly correlated, and effects of chord-wise correlation should be taken into account. Moreover, since deck sections are not usually streamlined and they are characterized by low values of the Strouhal number, the harmonic contents involved in signature and undisturbed turbulence overlap, and the corresponding load contributions, can not be considered as independent each other and simply superposable. Furthermore, in the case of real bridge deck sections, flow memory effects can not be generally neglected, leading to a significant phase shift between self-excited (respectively, buffeting) forces and structural motions (respectively, wind gusts). Accordingly, wind-induced loads can not be generally considered as depending only on the instantaneous relative velocity between flow and structure, and thereby wind loads arising upon deck sections of long-span bridges usually require suitable unsteady descriptions. These latter are obtained mainly by extending the theoretical approach developed for the thin airfoil to the case of bluff sections via semi-empirical techniques. Nevertheless, the limit behavior described by the quasi-steady model should be recovered as a proof of physical consistence.

5.1 Time-domain description

Preserving the assumption that buffeting and self-excited contributions can be linearly superimposed (see Eqs. (30)), a pure time-domain formulation is established moving from the basic rationale that the history of both sectional motion and wind gusts can be considered as a series of infinitesimal step-wise increments. Therefore, the non-stationary evolution in time of wind loads due to such step-wise increments is described by indicial functions (Wagner-like indicial functions for self-excited forces, and Küssner-like for buffeting contributions), strictly depending on the steady part of the angle of attack α_o. If a step-wise change in the angle of attack induces, after a transient stage, a new steady state around $\mathcal{S}$, such an approach should be able to recover, as for the thin airfoil, the steady wind loads as an asymptotic behavior.

Under the assumption of linear superposition of flow memory effects, the unsteady wind loads acting upon $\mathcal{S}$ and induced by a general time history of small motions and small turbulent gusts can be represented by Duhamel's convolution integrals as in Eqs. (9) and (20). Nevertheless, due to unsteady effects arising from the bluffness character of bridge sections, the position of both the rear neutral point and the aerodynamic center can not be identified in a closed-form theoretical way and as independent from the type of bridge motion and wind gust, as it instead occurs for the thin airfoil. Accordingly, in order to overcome such a drawback and in the framework of the postulated linear superposition, several indicial responses are usually introduced, each of them associated to a specific motion/gust direction and to a specific force component.

In agreement with some available indicial approaches [9, 19, 41, 49], buffeting and self-excited unsteady loads can be respectively expressed as

$$\mathbf{F}_b(\alpha_o, S) = \int_{-\infty}^{S} \mathbb{C}_b(\alpha_o, S-\tau)\, \mathbf{V}'(\tau)\, d\tau \tag{36}$$

$$\mathbf{F}_a(\alpha_o, S) = -\int_{-\infty}^{S} \mathbb{C}_a(\alpha_o, S-\tau)\mathbf{q}''(\tau)\, d\tau - \int_{-\infty}^{S} \mathbb{K}_a(\alpha_o, S-\tau)\mathbf{q}'(\tau)\, d\tau \tag{37}$$

with

$$\mathbb{C}_b = \frac{\mathcal{P}_o B}{U} \begin{bmatrix} 2C_D^o\,\psi_{Du} & -(C_{D/\hat{\alpha}} + C_L^o)\,\psi_{Dv} \\ 2C_L^o\,\psi_{Lu} & -(C_{L/\hat{\alpha}} - C_D^o)\,\psi_{Lv} \\ 2BC_M^o\,\psi_{Mu} & -BC_{M/\hat{\alpha}}\,\psi_{Mv} \end{bmatrix} \tag{38}$$

$$\mathbb{C}_a = \mathcal{P}_o \begin{bmatrix} 2C_D^o\,\phi_{D\dot{p}} & -(C_{D/\hat{\alpha}} + C_L^o)\,\phi_{D\dot{h}} & 0 \\ 2C_L^o\,\phi_{L\dot{p}} & -(C_{L/\hat{\alpha}} - C_D^o)\,\phi_{L\dot{h}} & 0 \\ 2BC_M^o\,\phi_{M\dot{p}} & -BC_{M/\hat{\alpha}}\,\phi_{M\dot{h}} & 0 \end{bmatrix} \tag{39}$$

$$\mathbb{K}_a = -\mathcal{P}_o \begin{bmatrix} 0 & 0 & C_{D/\hat{\alpha}}\,\phi_{D\alpha} \\ 0 & 0 & C_{L/\hat{\alpha}}\,\phi_{L\alpha} \\ 0 & 0 & BC_{M/\hat{\alpha}}\,\phi_{M\alpha} \end{bmatrix} \tag{40}$$

$\psi_{rj}(\alpha_o, S)$ and $\phi_{rn}(\alpha_o, S)$ (with $r = D, L, M$, $j = u, v$, and $n = \dot{p}, \dot{h}, \alpha$) being the Küssner-like and Wagner-like indicial functions, respectively.

It is worth pointing out that such a description of unsteady buffeting and aeroelastic wind loads allows to consistently recover, as an asymptotic behavior, the quasi-steady description recalled in Section 4, when dimensionless indicial responses tend to 1, in agreement with the normalization indications provided by Scanlan [15]. In detail, by assuming step-wise changes $g(S) = \mathcal{H}(S)\tilde{g}$, with $g = u, v, \dot{p}, \dot{h}, \alpha$, and $\tilde{g}$ a constant value, it results (with $r = D, L, M$, $j = u, v$ and $n = \dot{p}, \dot{h}, \alpha$)

$$\begin{aligned} &\lim_{S\to+\infty} \mathbf{F}_b(S) = \mathbf{F}_b^o, \qquad \lim_{S\to+\infty} \mathbf{F}_a(S) = \mathbf{F}_a^o \\ \text{when} \quad &\lim_{S\to+\infty} \psi_{rj}(S) = \lim_{S\to+\infty} \phi_{rn}(S) = 1 \end{aligned} \tag{41}$$

If time-dependent wind loads acting upon $\mathcal{S}$ can be effectively described via the quasi-steady approach, the following relationships hold, involving Wagner-like indicial functions associated to the torsional degree-of-freedom (α) and coordinates of the reference points P_r (with $r = f, m$, see Fig. 3 and Eq. (34)):

$$\begin{aligned} \phi_{D\alpha}(S) &= [\mathbf{c}]_1 \delta(S) \frac{U}{\mathcal{P}_o C_{D/\hat{\alpha}} B} + 1 \\ \phi_{L\alpha}(S) &= [\mathbf{c}]_2 \delta(S) \frac{U}{\mathcal{P}_o C_{L/\hat{\alpha}} B} + 1 \\ \phi_{M\alpha}(S) &= [\mathbf{c}]_3 \delta(S) \frac{U}{\mathcal{P}_o C_{M/\hat{\alpha}} B^2} + 1 \end{aligned} \tag{42}$$

Moreover, present time-domain formulation allows to recover the closed-form description for the thin airofoil. In detail, by enforcing $C_D^o = C_{D/\hat{\alpha}} = p(S) = u(S) = 0$ and $C_{L/\hat{\alpha}} = -4C_{M/\hat{\alpha}} = -2\pi$, wind loads acting upon the thin airfoil are straight described via the present formulation by expressing the indicial responses in terms of Wagner and Küssner indicial functions, that is:

$$\begin{aligned} &\psi_{Lv}(S) = \psi_{Mv}(S) = \psi(S) \\ &\phi_{L\dot{h}}(S) = \phi(S) + \delta(S)/4 \\ &\phi_{M\dot{h}}(S) = \phi(S) \\ &\phi_{L\alpha}(S) = \phi(S) + \phi'(S)/4 + \delta(S)[\phi(0) + 1]/4 \\ &\phi_{M\alpha}(S) = \phi_{L\alpha}(S) - \delta(S)/2 - \delta'(S)/32 \end{aligned} \tag{43}$$

Previous relationships generalize those proposed by Chen and Kareem [19], which can be simply recovered by disregarding non-circulatory inertial terms related to h'' and α''.

Unsteady wind loads can be also expressed, into the time domain, through convolution integrals involving aerodynamic impulse response functions $I_{rj}(\alpha_o, S)$ (with $r = D, L, M$ and $j = u, v, p, h, \alpha$) and the fluctuating wind velocity (buffeting contributions) or displacement components (self-excited forces) [15, 51, 56]:

$$\mathbf{F}_b(\alpha_o, S) = \frac{\mathcal{P}_o}{U} \int_{-\infty}^{S} \mathbb{I}_b(\alpha_o, S-\tau)\, \mathbf{V}(\tau)\, d\tau \tag{44}$$

$$\mathbf{F}_a(\alpha_o, S) = \frac{\mathcal{P}_o}{B} \int_{-\infty}^{S} \mathbb{I}_a(\alpha_o, S-\tau)\, \mathbf{q}(\tau)\, d\tau \tag{45}$$

where $\mathbb{I}_b$ and $\mathbb{I}_a$ are the impulse-response matrices:

$$\mathbb{I}_b(\alpha_o, S) = \begin{bmatrix} I_{Du} & I_{Dv} \\ I_{Lu} & I_{Lv} \\ I_{Mu} & I_{Mv} \end{bmatrix}, \qquad \mathbb{I}_a(\alpha_o, S) = \begin{bmatrix} I_{Dp} & I_{Dh} & I_{D\alpha} \\ I_{Lp} & I_{Lh} & I_{L\alpha} \\ I_{Mp} & I_{Mh} & I_{M\alpha} \end{bmatrix} \tag{46}$$

By comparing Eqs. (44) and (45) with Eqs. (36) and (37), and by omitting the dependency on α_o for the sake of compactness, it results

$$\mathbb{I}_b(S) = \frac{U}{\mathcal{P}_o} \left[\mathbb{C}_b(0)\delta(S) + \mathbb{C}'_b(S)\right] \tag{47}$$

$$\begin{aligned} \mathbb{I}_a(S) = &- \frac{B}{\mathcal{P}_o} \left[\mathbb{C}_a(0)\delta'(S) + \mathbb{C}'_a(0)\delta(S) + \mathbb{C}''_a(S) \right. \\ &\left. + \mathbb{K}_a(0)\delta(S) + \mathbb{K}'_a(S)\right] \end{aligned} \tag{48}$$

Starting from the Fung's formulation [6] and in agreement with the remarks given in [7], the indicial approach has been introduced for the first time in bridge analysis by Scanlan and co-workers [8], by considering only lift and moment contributions. They described self-excited wind loads induced by heaving and pitching motions of the girder by considering different indicial responses relevant to both the effective angle of attack at the section mid-chord point (namely related to α and h') and the effective rate of change of the angle of attack (α'). Nevertheless, following this formulation, no difference should appear in indicial response when step-wise changes in h' or in α are considered. Although such an approach can be successfully applied in the case of almost streamlined profiles (such as real wings or some modern bridge deck sections), it usually fails for bluff bodies; thereby many authors have proposed to assume that the indicial responses associated to the different downwash contributions have to be different [15, 57, 58]. Accordingly, disregarding any drag and horizontal effect, available time-domain formulations are mostly based on two indicial functions for each generalized force. They are combined by convolution with downwash terms described by the displacement functions $\alpha(s)$ and $h(s)$ [59, 60], or by $\alpha(s)$ and $h'(s)$ [9, 13, 15, 19, 26, 61]. Therefore, downwash contributions related to the pitch rate $\alpha'(s)$ are usually not explicitly included, despite of the thin airfoil theory, since indicial functions associated to α' are often considered as redundant (e.g., [8, 15, 40]). Many indicial representations in agreement with previous considerations can be found in the specialized literature, including both self-excited drag force and the along-wind deck's displacement (e.g., [9, 19, 59]), as well as describing buffeting loads (e.g., [13, 40]. Nevertheless, many authors considered different normalization conditions with respect to Eqs. (36) and (40) and, since the scalar weights of the indicial functions do not match the quasi-steady formulation, in these cases the consistency asymptotic condition (41) is not satisfied.

Moreover, referring to the most popular time-domain descriptions of aeroelastic wind loads on long-span bridges, inertial effects are usually neglected or they are at most intended to be implicitly incorporated within the indicial response. In fact, the adopted ϕ-like indicial functions are generally obtained by rational approximation techniques from the experimentally-determined flutter derivatives, thereby preventing to formally distinguish between circulatory and non-circulatory terms. As a consequence, when inertial effects are not negligible, indirectly-extracted indicial functions for bluff sections have no direct correspondence with the Wagner's one (that instead describes circulatory effects only), because they include inertial contributions related to the identification procedure of the flutter derivatives. Such a consistency violation can be significant in the description of the first transient stage of the aerodynamic response to step-wise motions (i.e., for very small values of S) and tends to disappear for high values of S (namely, when the response tends to be steady), inertial effects tending there to vanish.

In order to overcome such a drawback, a new general time-domain description of self-excited wind loads on elongated bridge sections with a mildly bluff character has been recently proposed by de Miranda et al. [27], and generalized in [62] by including lateral (i.e., along-wind) effects. In detail, moving from the thin-airfoil rationale, pseudo-circulatory and pseudo-non-circulatory contributions are superimposed. The first ones are assumed to be representative of memory-dependent downwash-related effects, whereas the latter are regarded as independent from the section motion history. Pseudo-non-circulatory contributions are described as linearly depending on components of $\mathbf{q}'$ and $\mathbf{q}''$, whereas pseudo-circulatory terms are associated to generalized downwash functions depending on components of $\mathbf{q}$ and $\mathbf{q}'$. Such a description has been proven to be fully feasible. In fact, the identification (via suitable numerical or experimental techniques) of the aerodynamic response to few quasi-step motions (namely, one for each degree of freedom) and to an angular harmonic motion, combined with consistency conditions arising from the asymptotic behavior in quasi-stationary regimes, allows to give an effective estimate of the complete set of parameters needed to apply the referred loading model [27].

5.2 Frequency-domain description

Buffeting wind loads are conventionally expressed in the frequency domain by correcting the quasi-steady formulation through frequency-dependent functions, accounting for the lack of pressure correlation around $\mathcal{S}$. In particular, Fourier transform of buffeting loads can be expressed in the form:

$$\begin{aligned}\overline{\mathbf{F}}_b(\alpha_o, K) &= \widetilde{\mathbb{C}}_b(\alpha_o, K)\overline{\mathbf{V}}(K) \\ &= \frac{\mathcal{P}_o B}{U}\begin{bmatrix} 2C_D^o\,\chi_{Du} & -(C_{D/\hat{\alpha}} + C_L^o)\,\chi_{Dv} \\ 2C_L^o\,\chi_{Lu} & -(C_{L/\hat{\alpha}} - C_D^o)\,\chi_{Lv} \\ 2BC_M^o\,\chi_{Mu} & -BC_{M/\hat{\alpha}}\,\chi_{Mv}\end{bmatrix}\overline{\mathbf{V}}(K) \end{aligned} \tag{49}$$

where complex functions $\chi_{rj}(\alpha_o, K)$ (with $r = D, L, M$ and $j = u, v$) have the meaning of aerodynamic transfer operators between fluctuating wind velocities and buffeting sectional forces. These functions generally depend on the steady part of the angle of attack and on the reduced frequency K based on the fluctuation frequency of the wind gusts. The square moduli of such functions are called aerodynamic admittance functions, and they describe the frequency-dependent variation of the force level with respect to the steady state. In the limit of the quasi-steady response, the following relationships hold

$$\lim_{K\to 0^+}\widetilde{\mathbb{C}}_b(\alpha_o, K) = \mathbb{C}_b^o(\alpha_o), \qquad \lim_{K\to 0^+}\chi_{rj}(\alpha_o, K) = 1 \tag{50}$$

Fourier transforms of Eqs. (36) and (44) lead to the following relationship among time- and frequency-dependent functions describing buffeting unsteady loads:

$$\widetilde{\mathbb{C}}_b(K) = \frac{\mathcal{P}_o}{U}\overline{\mathbb{I}}_b(K) = \mathbb{C}_b(0) + \overline{\mathbb{C}_b'}(K) \tag{51}$$

or equivalently (with $r = D, L, M$ and $j = u, v$)

$$\chi_{rj}(K) = \psi_{rj}(0) + \overline{\psi_{rj}'}(K) = \mathrm{i}K\int_0^\infty \psi_{rj}(S)\mathrm{e}^{-\mathrm{i}KS}dS \tag{52}$$

$$\psi_{rj}(S) = \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{\chi_{rj}(K)}{K}\mathrm{e}^{\mathrm{i}KS}dK \tag{53}$$

The comparison among Eqs. (22)-(23) and Eqs. (52)-(53) suggests that, in analogy with the thin-airfoil theory and as highlighted in [107], aerodynamic admittance functions $\chi_{rj}(K)$ in the frequency domain correspond to Sears-like complex functions

$$\Theta_{rj}(K) = \chi_{rj}(K) = \mathcal{F}_S^{(rj)} + \mathrm{i}\mathcal{G}_S^{(rj)}(K) \tag{54}$$

dual (in the sense of the Fourier transform) to indicial Küssner-like functions $\psi_{rj}(S)$ in the time domain.

The first attempt to incorporate in a quasi-steady formulation for bridge deck sections the rationale introduced by Sears [5] for the thin airfoil, based on the introduction of a frequency filter mapping turbulence components in the buffeting wind loads, has been performed by Davenport [11]. By assuming wind gusts as a stationary random process and under the strip assumption [38], Davenport described (not accounting for signature turbulence effects) drag force, lift force and torsional moment acting upon the deck cross-section by introducing five aerodynamic admittance functions (one for the drag, and two for lift and moment), very similar to Sears' relationship. Such a formulation has been successively generalized by Scanlan [15, 17] by introducing two admittance functions for each loading component, as in Eq. (49).

As far as self-excited forces are concerned, they are conventionally expressed in the frequency domain as:

$$\overline{\mathbf{F}}_a(\alpha_o, K) = -\widetilde{\mathbb{C}}_a(\alpha_o, K)\overline{\mathbf{q}'}(K) - \widetilde{\mathbb{K}}_a(\alpha_o, K)\overline{\mathbf{q}}(K) \tag{55}$$

with

$$\widetilde{\mathbb{C}}_a(\alpha_o, K) = -\mathcal{P}_o \begin{bmatrix} KP_1^* & KP_5^* & KP_2^* \\ KH_5^* & KH_1^* & KH_2^* \\ BKA_5^* & BKA_1^* & BKA_2^* \end{bmatrix} \tag{56}$$

$$\widetilde{\mathbb{K}}_a(\alpha_o, K) = -\mathcal{P}_o \begin{bmatrix} K^2P_4^* & K^2P_6^* & K^2P_3^* \\ K^2H_6^* & K^2H_4^* & K^2H_3^* \\ BK^2A_6^* & BK^2A_4^* & BK^2A_3^* \end{bmatrix} \tag{57}$$

where $A_j^*(\alpha_o, K)$, $H_j^*(\alpha_o, K)$ and $P_j^*(\alpha_o, K)$ (with $j = 1, ..., 6$) are the so called aeroelastic or flutter derivatives. These latter are dimensionless real functions depending on the section shape, on the steady part of the angle of attack, and on the reduced frequency K of oscillation. From Eq. (55) it clearly appears that flutter derivatives can be regarded as frequency filters mapping the motion of the deck section $\mathcal{S}$ into the self-excited forces. In the limit of the quasi-steady response, the following relationships hold

$$\lim_{K\to 0^+} \widetilde{\mathbb{C}}_a(\alpha_o, K) = \mathbb{C}_a^o(\alpha_o), \qquad \lim_{K\to 0^+} \widetilde{\mathbb{K}}_a(\alpha_o, K) = \mathbb{K}_a^o(\alpha_o) \tag{58}$$

It is worth pointing out that, assuming flutter derivatives as known frequency-dependent functions, the limit condition $(58)_1$ gives a straight indication on the consistent position of reference points P_r (with $r = f, m$) introduced in the quasi-steady approach (see Fig. 3 and Eq. (34)). Moreover, it is simple to prove that the present frequency-domain formulation allows to recover the closed-form description for the thin airfoil, by expressing flutter derivatives not associated to the along-wind motion in terms of the complex Theodorsen circulatory function [16, 27, 63].

Starting from the Theodorsen formulation and following an heuristic approach, the concept of flutter derivatives was firstly introduced by Scanlan and Tomko [12], by defining the self-excited lift and moment loads as functions of six (namely, A_j^*, H_j^*, with $j = 1, .., 3$) flutter derivatives, related to cross-wind and torsional deck motions. With increasing spans, the influence of self-excited contributions associated with the motion in the along-wind direction has become not longer negligible [14, 64, 65] and in many recent studies the complete set of the 18 flutter derivatives has been generally considered (e.g., [21, 22, 66–70]).

In the specialized literature, other representations of the aeroelastic forces can be found, less popular than the Scanlan's one. For instance, mention can be made to the ONERA representation (often referred to as Küssner representation) [71] and to the description proposed by Zasso [72]. Nevertheless, such alternative representations can be simply converted to the classical Scanlan's one.

Fourier transforms of Eqs. (37), (44) and (45) lead to the following relationships among time- and frequency-dependent functions describing aeroelastic unsteady loads:

$$\begin{aligned}\frac{\mathcal{P}_o}{B}\overline{\mathbb{I}}_a(\alpha_o, K) &= -\mathrm{i}K\widetilde{\mathbb{C}}_a(\alpha_o, K) - \widetilde{\mathbb{K}}_a(\alpha_o, K) \\ &= -\mathrm{i}K\left[\mathbb{C}_a(0) + \overline{\mathbb{C}'_a}(K)\right] - \left[\mathbb{K}_a(0) + \overline{\mathbb{K}'_a}(K)\right] \qquad (59)\end{aligned}$$

The comparison among Eqs. (12), (13) and (59) suggests that, in analogy with the thin airfoil and as also adopted in [9, 27], flutter derivatives and indicial Wagner-like functions can be related each other by introducing Theodorsen-like complex functions $\mathcal{C}_{rj}(K)$ (with $r = D, L, M$ and $j = \dot{p}, \dot{h}, \alpha$), such that

$$\begin{aligned}\mathcal{C}_{rj}(K) &= \mathcal{F}_T^{(rj)}(K) + \mathrm{i}\mathcal{G}_T^{(rj)}(K) \\ &= \mathrm{i}K\int_0^\infty \phi_{rj}(S)\,\mathrm{e}^{-\mathrm{i}KS}\,dS = \phi_{rj}(0) + \overline{\phi'_{rj}}(K) \qquad (60)\end{aligned}$$

$$\phi_{rj}(S) = \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{\mathcal{C}_{rj}(K)}{K}\mathrm{e}^{\mathrm{i}KS}\,dK = \frac{2}{\pi}\int_0^\infty \frac{\mathcal{F}_T^{(rj)}}{K}\sin(KS)\,dK \qquad (61)$$

Accordingly, since Eq. (60), frequency-dependent self-excited forces can be recast by considering the following relationship (see Eq. (59)):

$$
\begin{aligned}
&\frac{\mathcal{P}_o}{B}\bar{\mathbb{I}}_a(\alpha_o, K) = -\mathrm{i}K\widetilde{\mathbb{C}}_a(K) - \widetilde{\mathbb{K}}_a(K) \\
&= \mathcal{P}_o \begin{bmatrix} -\mathrm{i}K2C_D^o\mathcal{C}_{D\dot{p}} & \mathrm{i}K(C_{D/\hat{\alpha}} + C_L^o)\mathcal{C}_{D\dot{h}} & C_{D/\hat{\alpha}}\mathcal{C}_{D\alpha} \\ -\mathrm{i}K2C_L^o\mathcal{C}_{L\dot{p}} & \mathrm{i}K(C_{L/\hat{\alpha}} - C_D^o)\mathcal{C}_{L\dot{h}} & C_{L/\hat{\alpha}}\mathcal{C}_{L\alpha} \\ -\mathrm{i}K2BC_M^o\mathcal{C}_{M\dot{p}} & \mathrm{i}KBC_{M/\hat{\alpha}}\mathcal{C}_{M\dot{h}} & BC_{M/\hat{\alpha}}\mathcal{C}_{M\alpha} \end{bmatrix}
\end{aligned} \tag{62}
$$

Therefore, by combining Eqs. (56) and (57) with Eqs. (60) and (62) it is possible to establish a strong duality between time-domain and frequency-domain descriptions of motion-related wind loads on long-span bridges, by expressing flutter derivatives in terms of real and imaginary parts of the Theodorsen-like complex functions $\mathcal{C}_{rj}(K)$. Following such an approach and employing the novel time-domain description proposed by de Miranda et al. [27], and generalized in [62], pseudo-circulatory and pseudo-non-circulatory contributions can be clearly distinguished within the framework of the classical Scanlan's formulation. In this case, when the thin airfoil is addressed, Theodorsen-like circulatory functions consistently reduce to

$$
\begin{aligned}
&\mathcal{C}_{L\alpha}(K) = \mathcal{C}_{M\alpha}(K) = \mathcal{C}_{L\dot{h}}(K) = \mathcal{C}_{M\dot{h}}(K) = \mathcal{C}(K) \\
&\mathcal{C}_{r\dot{p}}(K) = \mathcal{C}_{D\dot{h}}(K) = \mathcal{C}_{D\alpha}(K) = 0
\end{aligned} \tag{63}
$$

and the non-trivial flutter derivatives in Eqs. (56) and (57) reduce to

$$H_1^* = -2\pi\frac{\mathcal{F}_T}{K} \tag{64}$$

$$H_2^* = -\pi\left[\frac{1}{2K} + \frac{\mathcal{F}_T}{K}\left(\frac{1}{2} - a\right) + 2\frac{\mathcal{G}_T}{K^2}\right] \tag{65}$$

$$H_3^* = -\pi\left[\frac{a}{4} + 2\frac{\mathcal{F}_T}{K^2} - \frac{\mathcal{G}_T}{K}\left(\frac{1}{2} - a\right)\right] \tag{66}$$

$$H_4^* = \pi\left[\frac{1}{2} + 2\frac{\mathcal{G}_T}{K}\right] \tag{67}$$

$$A_1^* = \pi\left(\frac{1}{2} + a\right)\frac{\mathcal{F}_T}{K} \tag{68}$$

$$A_2^* = \pi\left[-\frac{1}{4K}\left(\frac{1}{2} - a\right) + \frac{\mathcal{F}_T}{2K}\left(\frac{1}{4} - a^2\right) + \frac{\mathcal{G}_T}{K^2}\left(\frac{1}{2} + a\right)\right] \tag{69}$$

$$A_3^* = \pi\left[\frac{1}{8}\left(\frac{1}{8} + a^2\right) + \frac{\mathcal{F}_T}{K^2}\left(\frac{1}{2} + a\right) - \frac{\mathcal{G}_T}{2K}\left(\frac{1}{4} - a^2\right)\right] \tag{70}$$

$$A_4^* = -\pi\left[\frac{a}{4} + \frac{\mathcal{G}_T}{K}\left(\frac{1}{2} + a\right)\right] \tag{71}$$

recovering, for $a = 0$, the results proposed by Scanlan in [16].

5.3 Identification of aerodynamic force parameters

The assessment of unsteady aerodynamic forces in the time domain requires the identification of aerodynamic impulse or indicial response functions. In order to directly identify indicial responses of bridge decks, allowing also the indirect extraction of flutter derivatives, only few successful experimental techniques can be found in literature (e.g., [28, 29, 73]), mainly due to the drawbacks associated with the controllability of a suitable quasi-step description, as well as with the experimental replicability of an exact step function. As an alternative and/or support to the experimental methodologies, different computational approaches have been recently proposed, aiming to furnish direct estimates of aerodynamic indicial responses. Referring to two-dimensional grid-based methods, two different strategies are generally employed. The first considers a motionless solid region immersed in the fluid domain, simulating the step-response by suitable flow boundary conditions [57, 74–76]. The second directly simulates the motion of the solid domain within the flow, prescribing a smoothed-ramp motion of the section during a finite time in order to overcome the computational problems involved by the exact step-wise condition [27, 34, 77–79]. Nevertheless, all the adopted numerical strategies have been proven to be satisfactory only in the case of streamlined or at most mildly bluff sections. Accordingly, the direct evaluation of indicial aerodynamic response for bluff bridge sections remains a crucial and difficult task that has to be retained still an open and challenging issue.

On the contrary, techniques for identifying frequency-domain force parameters, such as flutter derivatives and admittance functions, can be considered more effective, and a large data set based on many sectional geometries is available, especially as regards flutter derivatives.

In particular, although promising results have been obtained by means of computational fluid dynamics [34, 80–85], flutter derivatives are generally extracted by experimental wind tunnel tests [12, 64, 86–93], mainly based on forced or free vibration methods. Similarly, the identification of the buffeting actions induced by controlled wind gusts is employed to extract aerodynamic admittance functions. In detail, aiming to include the influence of the signature turbulence characteristics, as well as to separately identify aerodynamic admittance functions associated to different wind force components, many experimental and numerical investigations have been carried out in the last years [37, 94–102]. On the other hand, since the accurate evaluation of admittance functions consistently extracted within the framework of a given model formulation, may be difficult to achieve, the Sears's function is often employed for studying the buffeting response of long-span bridges [61, 103–105]. Nevertheless, the use of the Sears's function is strictly reasonable for bridges with a cross-section very streamlined, its use for bluffer deck sections being not justified [37] and generally inducing an underestimation of the bridge buffeting response [105].

Starting from the direct knowledge of frequency-dependent parameters, aerodynamic functions in time-domain descriptions could be estimated following an indirect way, by recurring to the strong duality (via the Fourier synthesis) between time-domain and frequency-domain descriptions. Nevertheless, both flutter derivatives and admittance functions are normally known only at discrete values of the reduced frequency K. Accordingly, the direct use of the aforementioned relationships to quantify impulse and indicial response functions by means of inverse Fourier transforms is not effective. Therefore, approximate continuous functions are required for describing frequency-dependent force parameters allowing to identify suitable time-domain responses. In order to overcome such a drawback and by recalling Eqs. (54) and (60), rational approximations as in Eqs. (15) and (16) are usually employed, for both self-excited contributions (e.g., [8, 9, 34, 35, 51, 56]) and buffeting ones (e.g., [19, 35, 106]). A suitable choice of parameters n, a_j, b_j is needed, and the identification of model parameters is usually performed through the least squares method, by minimizing error functions between the values of the unsteady coefficients calculated by the experimentally (or numerically) evaluated flutter derivatives and their expressions in terms of the indicial coefficients [8, 19].

5.4 Approximate relationships among force parameters

As already observed in the ideal case of the thin airfoil, fluctuating gust components and motion components of the section give rise to different physical phenomena and local pressure distributions. Nevertheless Eq. (18), relating Sears and Theodorsen functions, allows to state the possible existence of closed-form relationships between self-excited force parameters (namely, flutter derivatives and Wagner-like indicial functions) and buffeting ones (aerodynamic admittance functions and Küssner-like indicial functions). Therefore, at least within the limits of a linearized approach and for sections with a reduced bluffness degree, similar inter-relations can be expected in analogy with the thin airfoil. Accordingly, starting from the idea that the integral measures of the aerodynamic forces could exhibit no great differences when gust components u and v are respectively substituted by section velocities $-\dot{p}$ and $-\dot{h}$ (since the same relative flow conditions), many authors postulated phenomenological relationships between buffeting and self-excited forces (e.g., [32, 33, 42, 107]).

In time-domain approaches this occurrence leads to the use of the same indicial functions for both self-excited and buffeting loads (namely, $\phi_{j\dot{p}} = \psi_{ju}$ and $\phi_{j\dot{h}} = \psi_{jv}$ with $j = D, L, M$), as well as such an assumption allows to postulate inter-relations among flutter derivatives and aerodynamic admittance functions in the frequency-domain description [42]. In detail, by assuming that aeroelastic forces induced by harmonic motions p and h are equal to buffeting contributions associated to turbulent velocities $u(t) = -\dot{p}(t) = -\mathrm{i}\omega p(t)$ and $v(t) = -\dot{h}(t) = -\mathrm{i}\omega h(t)$, respectively, it is possible to prove that the following relationships hold:

$$\begin{aligned} 2C_D^o\chi_{Du} &= -K(P_1^* - \mathrm{i}P_4^*), & (C_{D/\hat{\alpha}} + C_L^o)\chi_{Dv} &= K(P_5^* - \mathrm{i}P_6^*) \\ 2C_L^o\chi_{Lu} &= -K(H_5^* - \mathrm{i}H_6^*), & (C_{L/\hat{\alpha}} - C_D^o)\chi_{Lv} &= K(H_1^* - \mathrm{i}H_4^*) \\ 2C_M^o\chi_{Mu} &= -K(A_5^* - \mathrm{i}A_6^*), & C_{M/\hat{\alpha}}\chi_{Mv} &= K(A_1^* - \mathrm{i}A_4^*) \end{aligned} \tag{72}$$

It is simple to highlight, by employing Eqs. (32), (33), (50) and (58), that relationships (72) are fully consistent in the framework of the quasi-steady assumption. In the case of lack of experimental measurements of the aerodynamic admittance functions and within the aforementioned limits, Eqs. (72) can be used for an analytical derivation of the complete set of functions χ_{rj} from the measured flutter derivatives. Starting from experimental observations, Scanlan [15] firstly introduced some relationships among aerodynamic admittance functions and flutter derivatives, later generalized in [17, 52]. Since different force conventions, Scanlan's inter-relations are in partial agreement with Eqs. (72). Similar relationships have been also discussed in [19], but they are not consistent with the quasi-steady limit behavior herein proposed.

Since flutter derivatives should model self-excited contributions not coupled with buffeting terms and referred to laminar incoming flow, the aerodynamic admittance functions obtained via Eqs. (72) take into account only a certain amount of signature effects, but they do not include the lack of correlation of turbulence components around the section. Scanlan [17, 32] suggested to weight the admittance functions evaluated via flutter derivatives by a function resulting from the integration of the coherence function associated to turbulence components along the contour of the deck section (chord-wise admittance function).

Postulating that flow patterns induced by a bridge section undergoing harmonic oscillations in cross-wind and torsional directions are similar, approximate inter-relations among flutter derivatives can be also introduced. Therefore, by assuming $\dot{h}(t) = \mathrm{i}\omega h(t) = U\alpha(t)$ and by observing that self-excited force components associated to a torsional motion α are angularly shifted by α with respect to the ones induced by a vertical motion h, the following relationships hold in the framework of a linearized approximation:

$$\begin{aligned} KP_5^* &= K^2P_3^* - C_L^o, & P_6^* &= -KP_2^* \\ KH_1^* &= K^2H_3^* + C_D^o, & H_4^* &= -KH_2^* \\ A_1^* &= KA_3^*, & A_4^* &= -KA_2^* \end{aligned} \tag{73}$$

Also in this case it is simple to prove, by employing Eqs. (33) and (58), that relationships (73) are fully consistent in the framework of the quasi-steady assumption. Matsumoto [89], starting from experimental results on rectangular cylinders, proposed for H_j^* and A_j^* relationships similar to those reported in Eqs. (73), but not accounting for static coefficient C_D^o and then lacking for consistency with respect to the quasi-steady limit behavior. Scanlan et al. [63], postulating the analogy between the deck sections and the thin airfoil, introduced similar relationships for A_j^* and H_j^* that recover the herein reported results in the limit of small values of the reduced frequency.

6 SOME UNCONVENTIONAL EFFECTS

An accurate response analysis of wind-bridge interaction, mainly when long-span and highly flexible structures are addressed, should take into account the three-dimensional character of the flow acting on the structure, as well as aerodynamic nonlinear coupling effects.

All the previously-stated considerations are based on the main assumption that the incident mean wind is at a right angle to the longitudinal axis of the bridge deck. Nevertheless, this may not always be the case when the bridge is located in a complex and heterogeneous topography or in the case of extreme wind conditions. Accordingly, effects of yaw winds may need to be included in bridge design, as well as they should be also taken into serious consideration for long-span bridges during their construction stages [108, 109]. A classical approach aiming to include skew winds in bridge buffeting analysis is essentially based on the decomposition approach (the so-called cosine and sine rules). In other words, the mean yaw wind is decomposed into two components: one is normal and the other parallel to the bridge span. The contribution of the parallel mean wind component is then separately analyzed from that of the normal mean wind component [108, 110]. Nevertheless, such an approach reveals some difficulties in its application, mainly related to the consistent definition of the turbulence wind decomposition with respect to the bridge axis, as well as to the evaluation of the buffeting response due to parallel component and its composition with the orthogonal-based one. As wind tunnel tests reveal, the decomposition approach usually produces, especially for high turbulent winds, an underestimation of the bridge buffeting response under yaw wind [111], as well as it is generally inapplicable to the estimation of flutter critical wind speed [112]. In particular, yaw wind effects could reduce the values of critical wind speed inducing flutter onset. In order to overcome such drawbacks, alternative formulations [113, 114], as well as numerical approaches [117] and experimental procedures [112, 115, 116] allowing to treat the three-dimensional wind field without any decomposition, have been recently proposed.

The three-dimensional character of the flow around the bridge deck is also associated with the loss of span-wise [1, 118, 119] and chord-wise [120–122] correlation of the wind-induced forces. In other terms, due to the turbulent character of the flow and to its not uniform distribution on the bridge deck, aerodynamic forces are generally not perfectly correlated both along the bridge axis and section-chord direction. As it is customary in bridge analysis, span-wise and chord-wise correlation are usually treated as independent. The chord-wise correlation is directly taken into account by means of the aerodynamic admittance functions, as previously recalled, and in the linearized framework it affects only buffeting contributions and does not depend on the modal structural response. The span-wise correlation, instead, may affect both self-excited and buffeting contributions, and generally it can be strongly dependent on modal shape and amplitude characterizing the bridge response.

Although self-excited forces are commonly assumed to be fully correlated along the span-wise direction, Scanlan [123] showed that span-wise effects on flutter derivatives may arise from modal response of the bridge and from turbulent character of the approaching flow (due to both undisturbed and signature effects). Moreover, he noted that a loss of span-wise correlation for self-excited forces generally induced a stabilization in the single-mode torsional flutter. Nevertheless, although such a stabilizing effect on the single-mode torsional flutter, it is not obvious that same effect applies to multimode coupled flutter cases. In agreement with [124] and by assuming that flutter derivatives reduce in coherence through an exponential law, the uniform span-wise distribution of a given flutter derivative D_i^* to be employed within a fully correlated analysis framework results in:

$$D_i^*(K) = \sqrt{\int_0^1\int_0^1 D_i^*(\alpha_{oA})\, D_i^*(\alpha_{oB})\, \Phi(\zeta_A)\Phi(\zeta_B)\, \mathrm{e}^{-c|\zeta_A-\zeta_B|}\, d\zeta_A\, d\zeta_B} \quad (74)$$

where $\zeta_P = z_P/\ell$ is the dimensionless span coordinate (with $P = A, B$), $\alpha_{oP} = \alpha_o(\zeta_P)$, and $\Phi(\zeta)$ is the bridge deck modal shape in consideration, ℓ being the span length, z the along-the-span coordinate and c a coherence coefficient, experimentally valued. It is worth observing that, apart from coherence effects, flutter derivatives may be variable along the deck axis because of the variability of the steady angle of attack α_o with z [21]. Moreover, in order to account in the linearized framework for a certain amount of coupling between approaching turbulence and self-excited wind forces, Scanlan proposed to evaluate flutter derivatives considering experimental tests in turbulent flow regimes [16].

As far as the buffeting forces are concerned, a common choice is based on the assumption that they have the same span-wise correlation as the incoming wind fluctuations [19]. Nevertheless, this assumption is not completely verified by experimental tests, which have highlighted that, due to the flow dynamics in the separated flow region [125], buffeting contributions may have higher span-wise correlation than the incident wind fluctuations. In order to account for span-wise correlation of buffeting forces, and referring to the formulation proposed in [106], the linearized buffeting forces acting on a motionless element of the bridge deck with length ℓ and corresponding to arbitrary wind fluctuations can be expressed in the time domain by a double convolution integral:

$$\mathbf{F}_b(\alpha_o, S)\ell = \frac{\mathcal{P}_o \ell}{U} \int_{-\infty}^{S} \int_{-\infty}^{\tau_2} \mathbb{I}_b^{\mathbb{J}}(\alpha_o, S-\tau_2, \tau_2-\tau_1)\mathbf{V}(\tau_1)\, d\tau_1\, d\tau_2 \tag{75}$$

with

$$\mathbb{I}_b^{\mathbb{J}}(\alpha_o, S_1, S_2) = \begin{bmatrix} J_{Du}(S_1)\, I_{Du}(S_2) & J_{Dv}(S_1)\, I_{Dv}(S_2) \\ J_{Lu}(S_1)\, I_{Lu}(S_2) & J_{Lv}(S_1)\, I_{Lv}(S_2) \\ J_{Mu}(S_1)\, I_{Mu}(S_2) & J_{Mv}(S_1)\, I_{Mv}(S_2) \end{bmatrix} \tag{76}$$

where the vector $\mathbf{V}(S)$ collects the wind fluctuations at the center of the deck element, I_{jr} are the aerodynamic impulse functions defined as in Eqs. (46), and J_{jr} are impulse functions representing the spatial correlation characteristics (with $j = D, L, M$ and $r = u, v$). Accordingly, the frequency description of buffeting loads can be put in the form:

$$\begin{aligned} \overline{\mathbf{F}}_b(\alpha_o, K)\ell &= \ell \widetilde{\mathbb{C}}_b^{\mathbb{J}}(\alpha_o, K)\overline{\mathbf{V}}(K) \\ &= \frac{\mathcal{P}_o \ell\, B}{U} \begin{bmatrix} 2C_D^o\, \chi_{Du}\overline{J}_{Du} & -(C_{D/\hat{\alpha}} + C_L^o)\, \chi_{Dv}\overline{J}_{Dv} \\ 2C_L^o\, \chi_{Lu}\overline{J}_{Lu} & -(C_{L/\hat{\alpha}} - C_D^o)\, \chi_{Lv}\overline{J}_{Lv} \\ 2BC_M^o\, \chi_{Mu}\overline{J}_{Mu} & -BC_{M/\hat{\alpha}}\, \chi_{Mv}\overline{J}_{Mv} \end{bmatrix} \overline{\mathbf{V}}(K) \end{aligned} \tag{77}$$

where $\overline{J}_{jr}(K)$ (with $j = D, L, M$ and $r = u, v$) are the Fourier transforms of time-dependent functions $J_{jr}(S)$, and are usually referred to as joint acceptance functions, given by [19]

$$|\overline{J}_{jr}(K)|^2 = \int_0^1\!\!\int_0^1 \mathrm{coh}_{jr}(\zeta_A, \zeta_B, K)\, d\zeta_A\, d\zeta_B \tag{78}$$

coh_{jr} being the frequency-dependent span-wise coherence function associated to the aerodynamic force j and to the wind fluctuation r [16]. Following [11, 95], position (78) can be generalized in order to account for a certain amount of coupling between buffeting contributions and modal bridge response, by defining the joint acceptance functions as

$$|\overline{J}_{jr}(K)|^2 = \int_0^1\!\!\int_0^1 \mathrm{coh}_{jr}(\zeta_A, \zeta_B, K)\, \Phi(\zeta_A)\, \Phi(\zeta_B)\, d\zeta_A\, d\zeta_B \tag{79}$$

As for other time-domain parameters, techniques for direct determination of impulse functions J_{jr} can be considered to be as not well established yet, whereas experimental approaches for evaluating frequency-dependent span-wise coherence have been extensively developed. Therefore, these impulse functions in time domain can be quantified through their dual relationships in frequency domain, usually treated via rational approximations.

Remarkable unconventional effects, that should be taken into account in order to provide an accurate prevision of the bridge dynamical response to wind actions, but herein only cited for the sake of compactness, are related to the aerodynamic influence of bridge-section details (such as median dividers, edge safety barriers or parapets) [126–128], towers and cables [129, 130], as well as to the presence of trains and/or vehicles moving along the bridge deck [131, 132].

Finally, it is worth pointing out that often, in very flexible long-span bridges, traditional linear aerodynamic force models can be ineffective. As previously discussed, linear models assume that the variation of the effective angle of attack is small enough so that: aerodynamic forces can be linearized at the statically deformed bridge configuration; the variation of the aerodynamic parameters is negligible. Nevertheless, for typical bridge decks, such aerodynamic parameters are generally high sensitive to the effective angle of incidence. As a consequence, even for small levels of turbulence, the effective angle of incidence due to the structural motion and wind fluctuations may largely vary, so that aerodynamic nonlinearities can not be neglected [133–136]. In order to account for these effects, the main idea is based on the extension of the quasi-steady theory, not considering a linear approximation but including higher order terms [41]. Nevertheless, in order to accomplish simple and effective analysis, several approaches can be found in specialized literature, based on the aerodynamic-response decomposition associated to the frequency level [19, 48, 56]. Accordingly, the effective angle of attack can be decomposed in low-frequency (large scale, including static component) and high-frequency (small scale) components, corresponding to the lower and higher frequencies than a cut-off one (e.g., the lowest natural bridge frequency): $\hat{\alpha}(t) = \hat{\alpha}^{low}(t) + \hat{\alpha}^{high}(t)$. Therefore, each nonlinear aerodynamic force is separated in turn into low- and high-frequency components. The high-frequency components are linearized around $\hat{\alpha}^{low}(t)$ and further separated into self-excited and buffeting contributions, in agreement with the traditional linearized approach; whereas the low-frequency contributions are modeled as a nonlinear function of $\hat{\alpha}^{low}(t)$:

$$\mathbf{F}(t) = \mathbf{F}(\hat{\alpha}) \cong \mathbf{F}(\hat{\alpha}^{low}) + \left.\frac{d\mathbf{F}}{d\hat{\alpha}}\right|_{\hat{\alpha}=\hat{\alpha}^{low}} \hat{\alpha}^{high} = \mathbf{F}^{low} + \mathbf{F}_b^{high} + \mathbf{F}_a^{high} \quad (80)$$

Accordingly, high-frequency contributions can be described in classical way (in frequency-domain by flutter derivatives and aerodynamic admittance functions, in time-domain by indicial or impulse response functions), whereas the low-frequency aerodynamic forces, since the small frequency values, can be suitably expressed by quasi-steady theory by means of a nonlinear format as in Eq. (29).

7 CONCLUDING REMARKS

Recent advances in the modeling of aerodynamic forces on bridge decks in both frequency domain and time domain have been reviewed, generalizing some common diffused inconsistencies and removing some not-well-posed assumptions. Accordingly, a unified approach has been presented, proving theoretical consistence with both the classical thin airfoil theory and the quasi-steady formulation (the latter regarded as an asymptotic behavior). Analytical arguments based on the Fourier synthesis have been employed to establish the strong duality between time-domain and frequency-domain descriptions. Exact and approximate relationships, useful for practical applications, among aerodynamic parameters involved in the description of static, self-excited and buffeting force components, have been comprehensively discussed and compared with the actual state of the art. Emphasis has been also given to a number of open topics and advanced strategies in bridge aerodynamics, aiming to include some unconventional effects, often neglected in wind-bridge interaction analyses.

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