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STRUCTURAL ANALYSIS OF CABLE-STAYED STRUCTURES IN THE CONSTRUCTION SEQUENCE OF BRIDGES BUILT BY CANTILEVERING

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ABSTRACT: Cable structures are often used as final or partial intermediate structures in construction stages of bridges like arch bridges or cable-stayed ones. These bridges are built often by cantilevering, i.e. with subsequent cantilever partial structures, which are cable-stayed too; hence, every construction stage is a cable structure to be analyzed. The methodology of structural analysis of these construction stages as well as the modeling of the structure for the determination of initial cable force are fundamental steps for establishing the actual state of stress and deformation.

In the paper, a general methodology of analysis for construction sequences of cable-stayed structures is presented, which can be used both for the design of cable-stayed bridges and arch bridges. The proposed methodology is based on the simple analysis of multiple partial elastic schemes, which follow the actual construction sequence. The aim is that of obtaining a convenient final geometry through the control of deformations from the first stage to the last one, coincident with the service life configuration. Geometry and internal forces are contemporary checked, as well as cable forces are determined without the need of too many stressing adjustments. Results of analyses, performed for different case-studies, are reported, summarized and commented, in order to show the reliability and the wide range of applicability of the proposed methodology of analysis.

KEYWORDS: Cable-stayed structures, construction sequence, forward analysis, partial elastic scheme.

1 INTRODUCTION

In cable supported structures, the cable is the fundamental element. It can be used in the final layout like in cable-stayed bridges or in partial intermediate schemes of construction stages like in arch bridges. The construction methodology most used for these bridges is the cantilever one, i.e. the sequence of cantilever segments, which are cable-stayed from towers, until the final scheme is achieved. In arch bridges the cable-stayed cantilever is present only during construction, when the arch segments are assembled, in order to avoid centerings and till the arch key is closed; after that, the provisional cables, already used for supporting the arch elements, are dismantled (fig. 1a). This technique was used for the first time in the construction of the St. Louis steel arch bridge over the Mississippi River, designed by J. Eads and completed in 1874. Later, in 1952, it was extended to the construction of concrete arches for the bridges of the Caracas-La Guaira motorway in Venezuela, to which E. Freyssinet contributed [1]. Only in particular cases, arch bridges can be built by using stays as permanent structural elements (bowstring bridges). In cable-stayed bridges instead cables remain always as definitive elements and they have the role of elastic supports of the deck, in service life too. Cable-stayed bridges are frequently built by cantilevering that consists in a sequence of partial cable structures (fig. 1b).

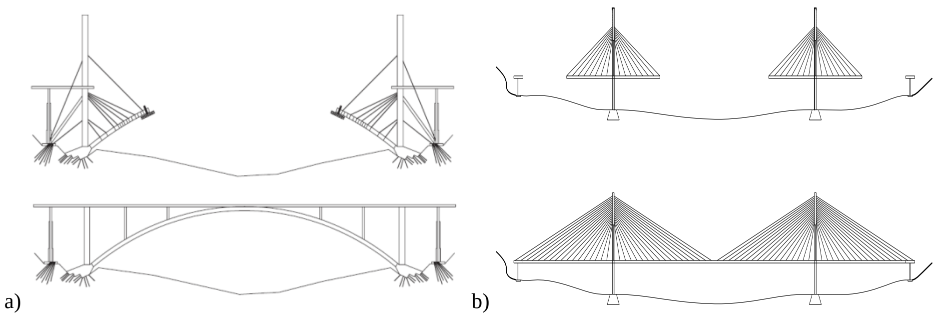


Figure 1. Cantilever construction of cable structures: a) arch bridge, b) cable-stayed bridge

Cantilever construction is characterized by a sequence in which geometric configuration, restraints and consequently stress and strain patterns vary many times till the final arrangement is achieved; this method is used today both for concrete and steel structures. For arch bridges, this method implies a different behavior of arch segments between construction stages and the final structure: in intermediate phases, the structure is mainly subjected to bending moments, as a curved beam on elastic supports, whereas after key closure, the arch gains its mainly axial behavior. This implies that, even if the arch shape was designed as the anti-funicular curve of permanent dead loads, bending moments appear in cantilever stages, and residual values remain in the arch after key closure and stays removal. These values of the bending moment will be added to those caused by deck construction, superimposed dead loads and moving loads.

Two requirements have to be fulfilled in the arch bridge: to achieve the exact geometry of the arch at the end of cantilever construction and to minimize the value of residual bending moments in the completed arch. These targets can be obtained by performing a convenient stay stressing sequence and by finding the

correct value of initial cable forces, stage by stage [2].
For cable-stayed bridges, the cantilever construction, unlike arch bridges, is characterized by a sequence of structures statically similar to the final one. The principal difference between the partial structures during construction and the final one is that a cantilever segment is attached to the last stay at each construction phase, modifying the geometry and load condition of every stage. This cantilever segment, both in prestressed concrete girders and in steel cross sections, may imply a stress state heavier than that occurring in service life. In the design of these bridges, the assessment of initial cable forces and the procedure of stay stress adjustments during erection is very important, but constitutes a hard task to achieve, in order to respect the requested geometric profiles of deck and towers at the end of construction [3]. It is not simple to state a convenient methodology of initial stay force determination for the following reasons: at the end of erection, the girder longitudinal profile must satisfy aesthetic and functional requirements, possibly presenting a convenient pre-camber; the towers must keep the vertical profile, in order to avoid second order effects and to satisfy architectural demands and even though geometrical requirements are satisfied in the so-called dead load configuration, after erection end, the system of stresses has to be checked to avoid high stress levels in the deck and tower members.

Another item to be considered in a cable structure with concrete or steel-concrete elements is related to time-dependent phenomena, because creep and shrinkage of concrete have a significant influence both on the final deformed configuration and on the internal force distribution. For arch bridges, the stress state can become intolerable during construction, with the danger of concrete cracking and excessive displacements. It has a greater consequence for slender arches with a limited rise [4] because creep strains can produce the change of the initial shape, leading to significant second-order effects in the arch. The effects of creep in arch bridges built by the suspended cantilever method were discussed in Granata *et al*. [5] while arch bridges built by the lattice cantilever method were discussed in Granata *et al*. [6]. For cable-stayed bridges, static scheme, during construction, evolves towards the final configuration of continuous beam on elastic supports. In these stages, the main effects of creep and shrinkage on internal forces are the stress relaxation due to elastic supports and the stress redistribution due to the addition of new restraints, especially after the closure of midspan. The development of deformations and internal forces during erection phases modifies stay and deck stresses as well as deck and pylon final profiles and, for example, an increase of deck vertical displacements and axial shortening occurs. For this reason, it is desirable to limit deformation and internal forces in the deck and pylon. This result can be achieved by adjusting in one or more phases pre-stress forces of stays during construction. The effects of creep in the staged erection of concrete cable-stayed bridges were investigated by Arici *et al*. [7] and by Schlaich [8] in composite

cable-stayed bridges.
Another problem to be considered in the medium and long span cable structures, especially in cable-stayed bridges, is the non-linear cable sag effect. It depends on the span range covered by bridges and on the related length of stays, for which the geometric non linearity has to be considered in many cases as well as the second order effects on the deck and the pylon [9, 10]. In composite and steel bridges these effects are emphasized.

In this paper a general methodology for establishing the initial cable force in the stressing sequence of cable-stayed structures is presented, which can be used both for the design of arch bridges and cable-stayed ones. There are also reported, summarized and commented the results of analyses performed for different case-studies.

2 STAY STRESSING SEQUENCES IN CABLE STRUCTURES

In cable structures, such as partial intermediate structures of suspended cantilever segments of arch bridges or cantilever segments of cable-stayed bridges, the deformed configuration and the internal force distribution depends significantly on the stay stressing sequence and especially on the value of initial cable forces. Different stay stressing sequences give different results on the kinematic and static point of view. Moreover, if the cable forces are not properly assessed, the actual evolution of the stress and strain state of cable structures may differ significantly from the design prevision and from the desired state, leading to not acceptable or not convenient results in terms of serviceability.

The literature on stay stressing sequence and initial cable force determination is wide, because this topic has been considered by many authors and by different points of view. The simplest methodology to find cable forces in the cable structures is the substitution of structure profile with a series of simply supported beams in which supports are represented by stays. Another choice is the so called zero-displacement method, presented by Wang *et al*. [11] for cable-stayed bridges; in this method, stay forces are found by a procedure that makes the displacement of each anchorage point null because of applied loads in the final static scheme of the completed bridge. Another methodology is the force-equilibrium method, in which the system of cable forces is found by imposing a given distribution of bending moments in the cable structure, as done by Chen *et al*. [12] for cable-stayed bridges. Another important category of methodologies used to find the initial cable forces is the optimization method. They are based on target functions that represent the ideal state to be achieved by modifying the value of stay pre-stress and the sequence adopted. Three different kinds of target function are the most common:

1. Methodologies in which the target function is expressed in terms of displacements of established control points, generally the stay anchorages to

deck and pylons [11]. This type of target function is a good choice to find the initial cable forces in the arch bridges and cable stayed-bridges built by cantilevering;

2. Methodologies in which the target function is expressed in terms of bending moment values: in this case, the aim is to obtain a uniform distribution of bending moments in the structure avoiding peaks in any section [13]. It could be convenient for cable-stayed bridges, instead its application to arch bridges is possible but it could be problematic because of difficulties in finding a valid target bending moment function. In fact, even if this function is found and satisfied by the stay stressing sequence at the end of construction, there is no guarantee of the arch shape achieved. Alternatively Au *et al*. [14] introduced target function in terms of allowable stresses at the top and bottom fibers of arch sections;
3. Methodologies in which the target is the minimization of an energetic function. It is generally expressed in terms of the total strain energy of the structure [15, 16]. Results of this methodology are not of immediate practical use, especially in the case of arch bridges, because they are not directly referred to a convenient distribution of bending moment that satisfies the requested geometry [12].

It is important to underline that some methodologies solve the problem only in the final stage, not considering the transient forces in temporary stages. Some authors recently have considered the influence of construction stages, non-linear effect of cable sag and time-dependent phenomena.

For arch bridges built by suspended cantilever method, some procedures can be found in literature to optimize the value of prestressing stay forces in staged construction analyses. Li *et al*. [17], for the construction stages of the Baishagou bridge in China, performed an optimization method in which the target function was based on the squares of bending moment values. The same problem was faced by Au *et al*. [14] by imposing a dead load configuration with assigned limits to maximum compressive and tensile stresses in concrete sections for an arch bridge with 180 m of span [18]. Janjic *et al*. [13] performed the unit load method for the analysis and construction of the Pitz Valley bridge in Austria. In this case, the target function is an ideal distribution of bending moments in the arch until key closure. This method derives from the one developed for cable-stayed bridges. However, for cable-stayed bridges, especially with composite and steel deck, the optimization of bending moment diagram in the construction stages and in the final dead load configuration is the principal objective. Instead, for the arch bridge the principal objective is to build the arch with the desired theoretical profile, which has to be the anti-funicular curve of the permanent dead load.

The evaluation of the initial cable forces to be assigned to stays is the first step for the staged construction analysis. The most common procedures in the literature to perform a staged construction analysis are the backward and forward methodologies [19]. The former consists of ideally dismantling the bridge by following a backward procedure [20, 21]. This widely used methodology allows designers to find the initial cable forces, but it cannot take into account the influence of time-dependent phenomena such as creep and shrinkage in concrete. A forward procedure can consider these, because it follows the actual sequence of construction. Nevertheless, a direct forward analysis needs a preliminary determination of initial cable forces, so every backward analysis has to be followed by a forward one to find the actual state of stress and strain in the structure at each stage and in the final dead load configuration. Conversely, it is not possible to obtain good results from a direct forward analysis without a careful preliminary evaluation of initial cable forces.

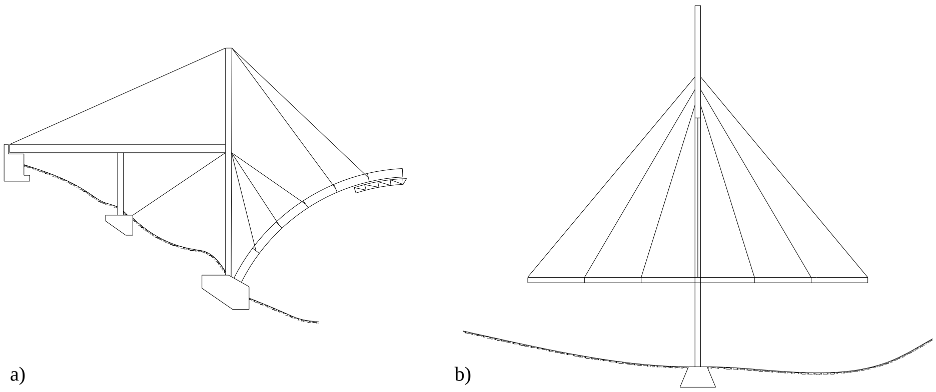
If time-dependent phenomena are neglected, backward and forward analyses should give the same results; unfortunately, this is not true. In fact, in the backward methodology, during the operation of bridge dismantling, the structure is not in a zero-stress state because it has already been loaded. Instead, in the actual sequence, every segment and every stay to be added are in a zero-stress state before assembly. To minimize this effect, a system of imposed forces and imposed strains should be applied to recover the zero-stress state of new segments to be assembled in the actual construction sequence [22, 23]. Moreover, a forward analysis can be useful for taking into account other related and significant topics:

- When a new segment is built by the cantilever method, by assembling it with the previous one already completed and by stressing the related stay, it is necessary to choose the right position of the new segment by following the tangent direction with respect to the tip of the previous segment, which is already deformed. This avoids the birth of discontinuities between geometrical axes of segments;
- When a stay is attached to a new segment, the actual length of the stay differs from the theoretical one, because of the displacements of the anchorage point in the deformed configuration. As a consequence, the stay force is different from the expected one, foreseen in the undeformed state. Therefore, a force variation must be induced to bring the anchorage point to the correct position (lack-of-fit force);
- The stay is not a rectilinear cable between the anchorage points, so a sag effect must be considered, which implies geometric nonlinear behavior [21]. This effect can be neglected for short stays (until~300 m), but it can be important for longer stays [24]. In concrete arch bridges, it can generally be neglected because it occurs occasionally in the case of very long arch spans, while it must be considered in medium and long span cable-stayed bridges.

3 PARTIAL ELASTIC SCHEME METHOD

The Partial Elastic Scheme Method (PES Method) is a simple procedure proposed for cable structures and theoretically applied to concrete and composite cable-stayed bridges built by cantilevering [7, 25] and to arch bridges built by suspended cantilevers [5] and lattice cantilevers [6].

Initial cable forces determination is done by partial elastic schemes of the structure (fig. 2), one for each construction phase, i.e. for each segment assembled by cantilevering and for each stay attached and tensioned, and it is analyzed by implementing the zero-displacement method or the force method at each stage on which permanent loads and pretension forces are applied. Hence, each partial elastic scheme is an independent structure with its geometry and its loads applied, useful for determining initial cable forces. Geometric non-linearity due to cable sag can be taken into account by performing an iterative procedure in which the modified Dischinger elastic modulus is introduced through the Ernst hypothesis [21].



a) b)

Figure 2. Partial elastic scheme: a) arch bridge, b) cable-stayed bridge

In the displacement method, the design constraints are expressed in terms of displacement requirements while in the force method they are settled in terms of static requirements. The first case occurs when the geometric configuration has to be reached during and after the construction sequence while the second case occurs when the target is to define a convenient bending moment diagram or a convenient distribution of stay forces into deck or arch during and at the end of construction. In both cases, it is important to choose the smallest number of constraints for a good mathematical conditioning of the problem. These conditions are imposed in the control points, which are generally established at the anchorages of the stays, in the joints with tower and deck, for convenience.

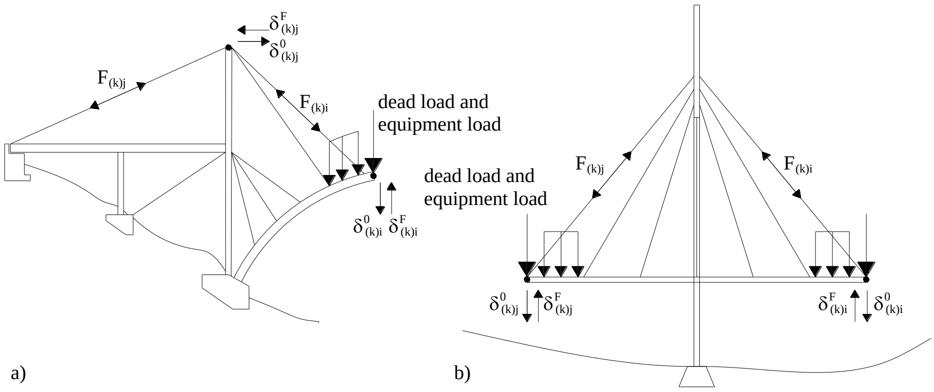


Figure 3. Partial elastic scheme with kinematic constraints: a) arch bridge, b) cable-stayed bridge

Let us consider a cable structure that is the partial scheme of an arch or a cable-stayed bridge, built by cantilevering, in N_s stages. The k-th partial scheme, related to the k-th construction stage, is shown in fig. 3. Let $\mathbf{F}_{(k)}$ be the array of pretension forces $F_{(k)i}$ of n stays to be pre-tensioned in that stage. Let $\boldsymbol{\delta}_{(k)}$ be the array of displacements of m control points, conveniently established in the joints of cables and structure related to the n stays, and due to loads applied in the reference elastic scheme. Let $\boldsymbol{\delta}^0{}_{(k)}$ be the array of displacements due to dead loads in the control points. The influence matrix $\mathbf{D}_{(k)}$ ($m \times n$) can be assembled, which gives displacement $\boldsymbol{\delta}_{(k)ij}$ of control point i due to the unitary value of cable force $F_{(k)j}$. According to the definition of the partial elastic scheme, the following equation can be written:

$$\boldsymbol{\delta}_{(k)} = \mathbf{D}_{(k)}\mathbf{F}_{(k)} + \boldsymbol{\delta}^0_{(k)} \tag{1}$$

By applying the displacement method to the partial elastic scheme, values of cable forces can be found imposing a target array to the control points displacements $\boldsymbol{\delta}^*{}_{(k)}$ which corresponds to a given shape or camber to the deck, assuring also the verticality of pylons in the cable-stayed bridges and the desired theoretical profile when the method is applied to arch bridges. Hence eq. (1) can be re-written in the form:

$$\boldsymbol{\delta}^*_{(k)} = \mathbf{D}_{(k)}\mathbf{F}_{(k)} + \boldsymbol{\delta}^0_{(k)} \tag{2}$$

where $\boldsymbol{\delta}_{(k)} = \boldsymbol{\delta}^*{}_{(k)}$ and $\mathbf{F}_{(k)}$ represents the array of imposed strains given to stays in order to achieve the target displacements.

If $\mathbf{D}_{(k)}$ is a square matrix, i.e. number of control points is equal to the number of stays, the direct solution of eq. (2) is possible and it represents an exact solution of imposed strain (pre-stress) values $\mathbf{F}_{(k)}$ to be given to the stays. In some cases $\mathbf{D}_{(k)}$ is a rectangular matrix with more constraints (control points) than variables (pre-stress forces), hence the exact solution cannot be found because it is not possible to invert the matrix. In this case, an approach for minimizing the error of solution results can be adopted and a good approximation can be found through the following relation:

$$\mathbf{K}^{\mathbf{D}}_{(k)}\mathbf{F}_{(k)} + \mathbf{D}^{T}_{(k)}\left(\boldsymbol{\delta}^{0}_{(k)} - \boldsymbol{\delta}^{*}_{(k)}\right) = \mathbf{0} \tag{3}$$

where $\mathbf{K}^{\mathbf{D}}_{(k)} = \mathbf{D}^{T}_{(k)}\mathbf{D}_{(k)}$ is a square and symmetric matrix [26].

Solution of eq. (2) or (3) supplies the array $\mathbf{F}_{(k)}$ of stay prestressing forces in the *k*-th reference elastic scheme. In this way it is evident that each elastic scheme, in which the displacement algorithm is applied, is composed of a partial structure referred to a construction stage and it is analyzed by applying permanent loads and prestressing forces related only to that stage. If stays are attached to the structure for the first time in that scheme, $F_{(k)i}$ represents the initial cable force, otherwise it represents an adjustment of a previously attached cable.

The displacement method is often applied in the form of zero-displacement algorithm in which the target array of control point displacements $\boldsymbol{\delta}^{*}_{(k)}$ is a zero-vector. In fact, in the arch bridge, in which the strength capacity is due to the curved geometry, the main aim is to build the structure according to the desired theoretical profile. In the cable-stayed bridges, instead, it is important to give initial cable forces for achieving the configuration of continuous beam on rigid supports at each construction stage and especially in the final dead load configuration, because in this way changes are not expected from the redistribution of stresses due to creep according to the application of the 1st theorem of linear viscoelasticity [27, 28].

In fig. 3 only the new stays attached are stressed in the reference scheme, but it is possible to adjust other stays previously attached, if it is judged convenient by the designer; in this way the dimensions of influence matrix $\mathbf{D}_{(k)}$ increase but the computational burden does not increase significantly. The choice of pre-tensioning one or more stays at each stage is an important design parameter in order to implement a convenient sequence of stay tensioning during construction and it is strictly related to the forward staged construction analysis to be performed in the model. Sequences with one or two stays pre-tensioned at each stage have been considered by authors for cable-stayed bridges [7], in which a two-phase stressing procedure can be useful in order to reduce effects of creep in cable-stayed bridges with concrete deck or to reduce geometry problems due to the coupling of steel and concrete in composite decks. However, progressive shortening of concrete deck and pylons in time due to axial forces and to creep and shrinkage effects remain and cannot be avoided [8]. For this reason deck and pylons must be built longer to compensate the axial shortening. A double stressing sequence is useful to achieve a better precision in the geometric shape and a more convenient bending moment diagram for composite cable-stayed bridges, during construction and in the final dead load configuration because in some cases a single stay stressing procedure does not permit to achieve the exact desired geometric profile [25]. On the other hand, a multiple stressing of each stay in different phases implies technological problems, because it is necessary to shift the stressing equipment stay by stay. Moreover for cable-stayed bridges too many stress adjustments are not convenient, because they imply that cable free length is marked many times, gripping strands in areas where marks exist from previous wedge seating, with the consequence of a local strength reduction. Alternatively a single stressing sequence has been considered by Granata *et al*. [5] for arch bridges built by suspended cantilevers and by Granata *et al*. [6] for arch bridges built by lattice cantilevers in which the PES method has been implemented. For short span arch bridges, a single stressing procedure is sufficient to achieve the desired results, i.e. the desired theoretical profile. Some authors, instead, prefer to re-stress many stays at each stage [29, 30], but this is not necessary when a convenient value of initial cable force is found, because more stressing phases of the last stays attached can lead to the relaxation of the first ones or to increasing deformations of the arch segments already assembled with the consequence of increasing bending moments in the construction stages of the arch.

In a dual way with respect to the previous approach, it is possible to apply the PES method, introducing a procedure based on the control of mechanical properties, applying it from the static point of view (force method). Let $\mathbf{c}_{(k)}$ be the array of internal forces $\mathbf{c}_{(k)i}$ of m control points due to the loads applied in the reference elastic scheme and $\mathbf{c}^0_{(k)}$ is the array of internal forces in the control points, due to dead loads. The influence matrix $\mathbf{C}_{(k)}$ $(m \times n)$ can be assembled, which gives internal forces $\mathbf{c}_{(k)ij}$ of control point i due to the unitary value of cable force $F_{(k)j}$.

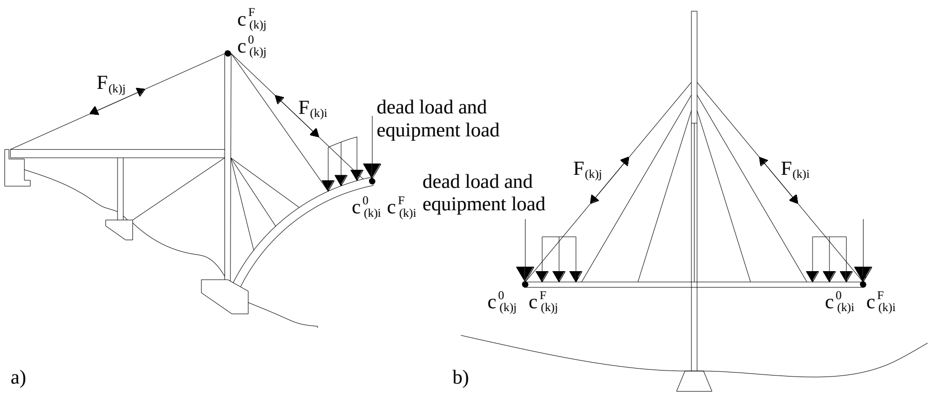


Figure 4. Partial elastic scheme with static constraints: a) arch bridge, b) cable-stayed bridge

According to the *k*-th partial elastic scheme (fig. 4), the following equation can be written:

$$\mathbf{c}_{(k)} = \mathbf{C}_{(k)}\mathbf{F}_{(k)} + \mathbf{c}^{0}_{(k)} \tag{4}$$

If $\mathbf{C}_{(k)}$ is a square matrix, i.e. the number of control points is equal to the number of stays, the direct solution of eq. (4), with $\mathbf{c}^{*}_{(k)} = \mathbf{c}_{(k)}$, is immediate and it represents the exact solution of the imposed stress values $\mathbf{F}_{(k)}$ to be given to the stays, corresponding to the prestressing forces. In many cases $\mathbf{C}_{(k)}$ is a rectangular matrix because the values of internal forces to be controlled are more than the axial forces of stays and in these cases the exact solution cannot be found. An approximated solution can be found also in this case with a procedure of error minimization, through the following relation:

$$\mathbf{K}^{\mathbf{C}}_{(k)}\mathbf{F}_{(k)} + \mathbf{C}^{\mathrm{T}}_{(k)}\left(\mathbf{c}^{0}_{(k)} - \mathbf{c}^{*}_{(k)}\right) = \mathbf{0} \tag{5}$$

where $\mathbf{K}^{\mathbf{C}}_{(k)} = \mathbf{C}^{\mathrm{T}}_{(k)}\mathbf{C}_{(k)}$ is a square and symmetric matrix and $\mathbf{c}^{*}_{(k)} = \mathbf{c}_{(k)}$ is the target array of internal forces in the control points.

Solution of eq. (4) or (5) gives the array $\mathbf{F}_{(k)}$ of prestressing forces in the *k*-th reference elastic scheme. This second method (force method), which is alternative to the previous one based on displacements, is used in composite and steel cable-stayed bridges when the main aim is to obtain a convenient bending diagram for the dead load configuration, associated to the required deck shape, in order to minimize stresses of steel elements and upper concrete slab, avoiding over dimensioning of structural elements and concrete cracking.

In the Partial Elastic Scheme Method the array $\mathbf{F}_{(k)}$ of prestressing forces given to stays is found for every stage. These forces have to be applied to each stay in a forward procedure that should follow the actual construction sequence for evaluating the actual state of stress. This procedure is alternative to the classical backward analysis, which is not good for cable-stayed bridges and especially not effective for arch bridges; in fact by performing the dismantling of the arch bridge in the backward approach, a negative value of the stay force can be found in many cases and this value cannot be used as initial cable force. Negative values of stay forces are acceptable only if they represent a partial relaxation of stays in intermediate phases, but they must remain in tension in the entire sequence. This fact is not common for cable-stayed bridges, but it can occur often in the construction sequence of an arch bridge until the relaxation of many cables in the last construction phases [29].

An important difference between cable-stayed bridges and arch bridges in application of PES method is that, in the arch bridge, the final adjustment needs only to recover construction errors or unacceptable deformed geometry of the arch before the key closure, while, in cable-stayed bridges, the last adjustment is always necessary to recover the effects of superimposed dead loads caused by finishing works of the bridge and/or construction errors. In fig. 5 the flow chart of Partial Elastic Scheme Method is shown.

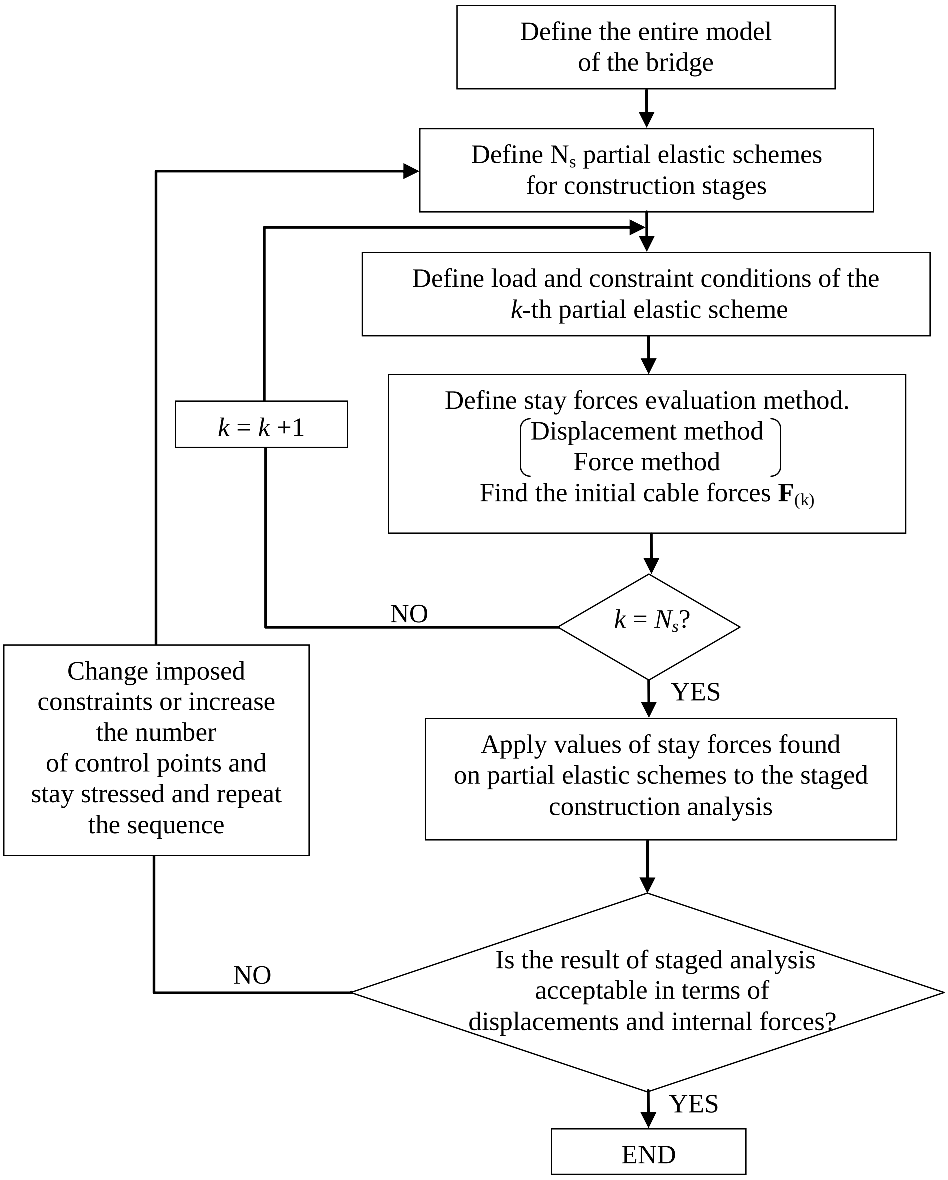


Figure 5. Flow chart of PES method

4 APPLICATIONS

In order to illustrate the proposed procedure, a few applications were performed implementing the Partial Elastic Scheme Method on cable structures and especially on arch and cable-stayed bridges built by cantilevering. Results of analyses performed for different case-studies are reported, summarized and commented, in order to show the reliability and the wide range of applicability of the proposed methodology of analysis.

The Partial Elastic Scheme Method with implementation of zero-displacement algorithm was applied in a model of concrete arch bridge built by suspended cantilever method in Granata *et al*. [5] and in a model of concrete cable-stayed bridge in Arici *et al*. [7].

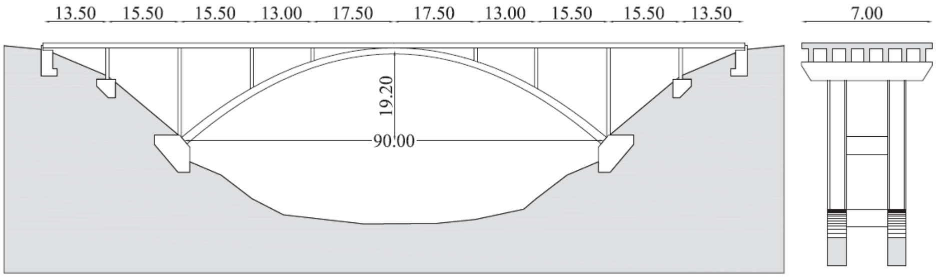


Figure 6. Geometry of arch bridge

The case-study of arch bridge has a span of 90 m and it is composed of two twin concrete arches with a parabolic profile and a rise of 19.20 m, which makes a rise/span ratio of f/l = 0.213. The bridge geometry is shown in fig. 6. The twin arches lie on parallel planes at a mutual distance of 3.20 m, each one having a rectangular cross section with a constant base of 1.00 m and a variable height of 1.80 m at arch footings and 1.30 m at the key segment. The upper deck, 7.00 m wide, is composed of 10 spans of different lengths; only six spans are on the arch, and the other four are supported by piers and abutments.

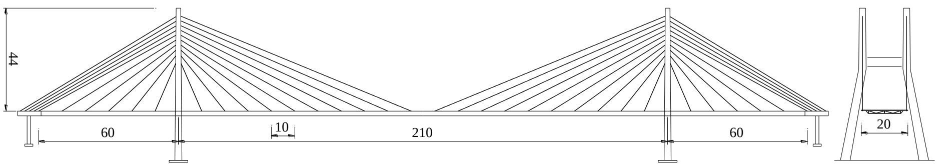


Figure 7. Geometry of cable-stayed bridge

The geometry of the case-study investigated for the cable-stayed bridge is shown in fig. 7. The main spain is 210 m long while side spans are 60 m long. The stay arrangement is mixed (harp and fan) with two symmetric planes of stays anchored at the edge of a multicellular box 20 m wide and 2.10 m high. Spacing of stay anchorages is 10 m in the deck and 2 m in the pylon. A parabolic profile is assumed for the deck with a maximum camber at the midspan of 1.60 m with respect to the bridge accesses.

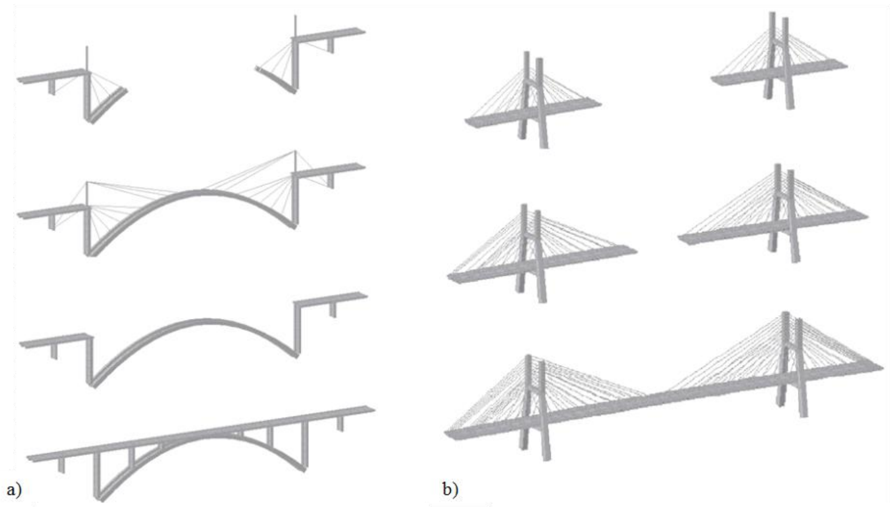


Figure 8. Examples of cantilever construction stages: a) arch bridge, b) cable-stayed bridge

The construction of the arches by suspended cantilever method follows a symmetric sequence for the two half structures after completion of lateral spans (fig. 8a). An arch is completed in 11 stages, with eight provisional stays and two backstays and the casting of nine segments. Stays have an equivalent steel diameter of ϕ_1 = 0.0298 m for the first three couples and ϕ_2 = 0.0353 m for the other five couples. Backstays have a diameter of ϕ_3 = 0.0462 m, anchored to a steel auxiliary pylon. Each segment is 6 m long and the key segment is 2 m long.

The cable-stayed bridge is built by symmetrical cantilevering till the back span has been completed. The other backstays are then anchored on a flexible pier, allowing longitudinal movements (fig. 8b). Equivalent steel diameter of stays is ϕ_s = 0.125 m, while earth anchored stays have the diameter ϕ_{bs} = 0.180 m.

Applying the Partial Elastic Scheme Method with the implementation of the zero-displacement algorithm, the main aim is to achieve the desired theoretical profile both for the arch and the cable-stayed bridge. Moreover the result of vertical tower with the achievements of the theoretical profile of the deck together with the final configuration next to that of a continuous beam on rigid support is searched for the cable-stayed bridge, in order to minimize the effects of stress redistribution due to creep. After the values of initial cable forces are found by the Partial Elastic Scheme Method, a linear elastic forward procedure was performed in both cases, in order to evaluate the actual values of forces and displacements in all construction stages and to take into account time-dependent phenomena. Geometric non-linear effects have been neglected for the limited length of the bridges analyzed.

With reference to the arch bridge, the deformed configuration in every construction stage approaches with very good approximation to the desired theoretical profile. This result is very important because the arch is a structure in which the role of the curved geometry on the strength mechanism is primary.

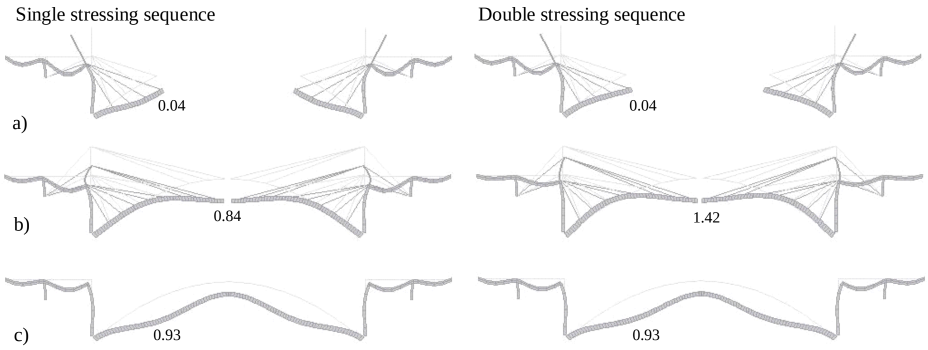


Figure 9. Deformed configuration [cm] of the arch in different stages with single and double stressing sequence

Fig. 9 shows the deformed configuration of an intermediate stage (fig. 9a), just before the key closure (fig. 9b) and after stays removal (fig. 9c), with single and double prestressing sequences. In short span arch bridge, single and double stressing sequences give practically the same results in terms of deformed configuration of the final stage, while a more marked deformed profile is registered in the intermediate stages when a double stressing procedure is implemented; this is consequence of the stay forces acting on the curved beam that is not still an arch, with a major flexibility and the consequent increase of bending moments in intermediate stages.

For the cable-stayed bridge a similar comparison between single and double stressing sequences is performed. In both cases, when the structure is completed, all stays are re-stressed and adjusted to achieve the final dead load configuration after the superimposed dead load is applied. For each elastic scheme, the results obtained by the double prestressing procedure seem to be more convenient, approaching the configuration of the beam on rigid supports.

The consequence is that the application of the zero-displacement algorithm in the cable-stayed bridge involves not only the optimization of the geometric profile of deck and tower but also the optimization of the stress state, especially in terms of bending moment diagram. It is evident that diagram approximates very well that of continuos beam on rigid supports if two-phase stressing is adopted (fig. 11b). If single stressing sequence is considered, the results obtained are good but not excellent. In fact, there are higher vertical displacements (fig. 10a) and especially the bending moment diagram (fig. 11a) differs significatly to the bending distribution of continuos beam on rigid supports. So, in a cable-stayed bridge, one or two-stressing phases, unlike the arch bridge, give different results and in the case of single stressing phase, the effects of time-dependent phenomena become more relevant and never negligible.

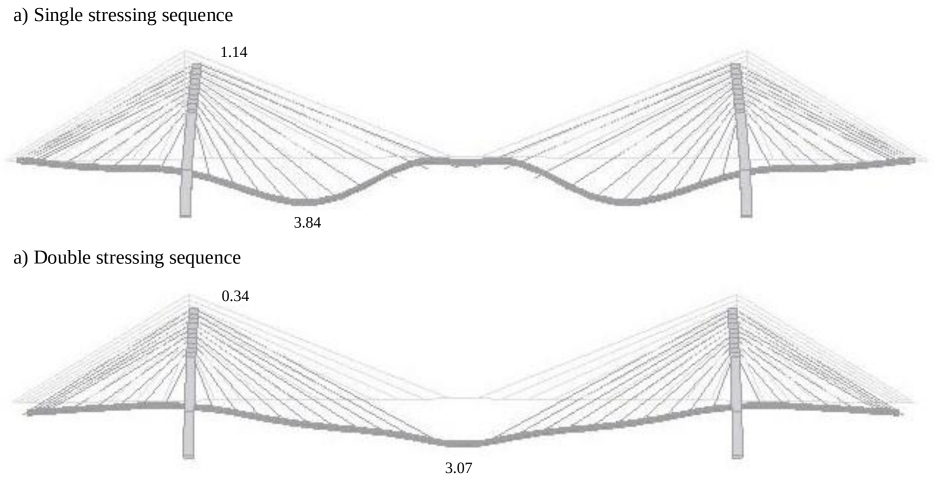


Figure 10. Comparison of deformed shapes [cm] with creep redistribution at time *t* = 10000 days: a) single stressing sequence, b) double stressing sequence

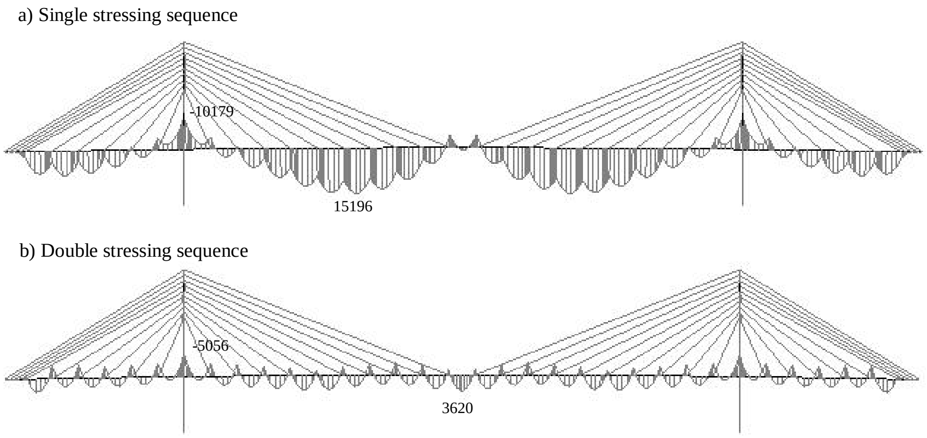


Figure 11. Comparison of dead load bending moments [kNm] with creep effects at time *t* = 10000 days: a) single stressing sequence, b) double stressing sequence

In a similar way, the application of zero-displacement algorithm in the arch bridge involves not only the optimization of the final geometric profile but also that of bending moment diagram. The stay system, during construction, introduces a positive bending moment at arch springs before closing the arch. In fig. 12 the bending moment diagram is shown before key closure (a) and after arch completion with stays removal (b) in the case of the single stressing sequence adopted (the result for double stressing sequence is similar). It is worth noting that the optimization of bending moment diagrams is much more important in cable-stayed bridges than in arch bridges, because in the latter ones, after the key closure, the axial behaviour is predominant (fig. 13).

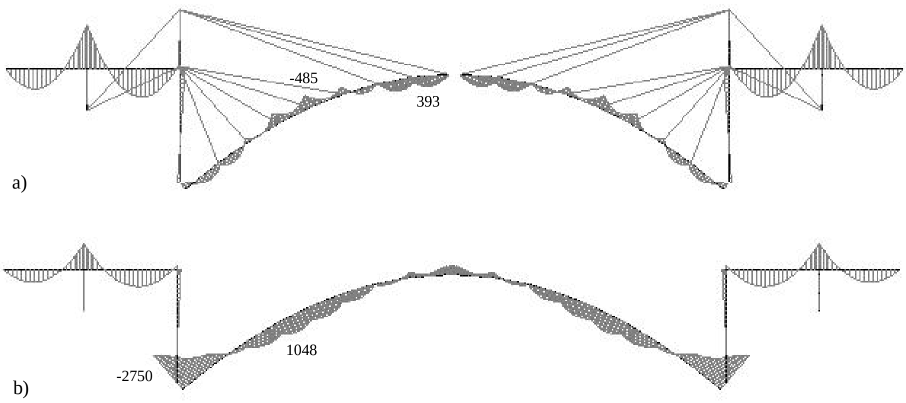


Figure 12. Bending moment diagram [kNm]: a) before key closure, b) after arch completion and stays removal

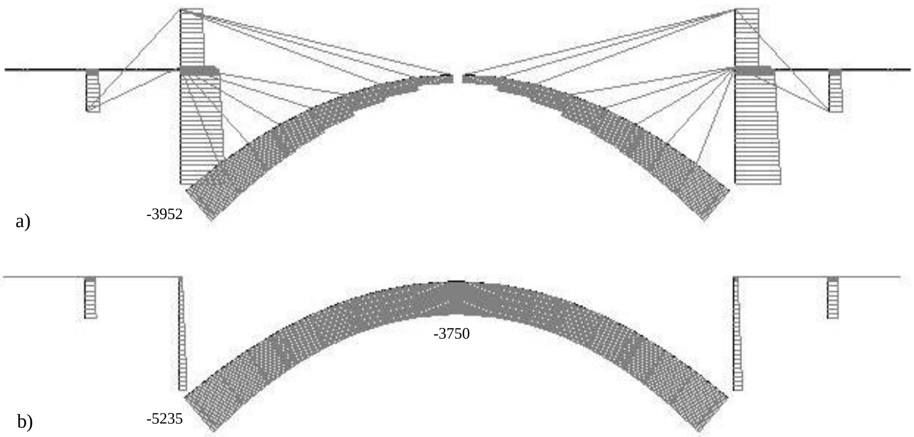


Figure 13. Axial force diagram [kN]: a) before key closure, b) after arch completion and stays removal

Figs. 14 and 15 show the distribution of stay stresses during the erection sequence, stage by stage, for cable structures with single and double stressing procedures. Fig. 14 refers to the arch bridge while fig. 15 refers to the cable-stayed one. Each line depicted refers to a single cable by following the development of the stress state during the whole sequence. By implementing the proposed procedure, the value of the initial cable force in each new stay attached increases gradually with respect to the previous one. This behavior is due to choice of imposing null-displacement in the tip of the last cantilever segment assembled.

In the arch bridge maximum values of stress are always present in the backstays (named A and B). With the proposed procedure, the first group of stays, already tensioned, maintains an almost constant value of axial force in the successive stages. When instead a procedure with adjustments is performed, the values of force adjustments are limited; consequently, the stress value of stays previously attached remains almost constant when new segments are built. This implies that it is not convenient to adjust the previous tensioned stays, because it does not provide any significant improvement in the stressing sequence. This is mainly true for short span bridges, whereas a double stressing sequence with stay adjustments can be useful for long span bridges [14]. If axial forces of the previous stays remain unchanged, no additional displacements of previous stay anchorages are induced by the actions induced on the last stay. Moreover, if additional displacements on the previous joints are negligible, bending moments in the arch do not vary significantly in the partial structure already built.

With the Partial Elastic Scheme Method, the first cables attached do not relax during the construction sequence. By using other procedures instead, the first stays attached can lose their tension and high values of positive bending moments can appear in the arch. This can be seen by comparing the proposed methodology with the actual construction of a long span bridge such as the Bloukrans in South Africa [2], in which the first stays are de-tensioned, and maximum bending moment in the arch increases significantly in intermediate stages.

In the same way, in the cable-stayed bridges, the stays maintain an almost constant value of axial force in the construction stages. However, one-stressing procedure implies higher values of prestressing at each stage, while a gradual prestressing can be achieved with the two-stressing procedure, maintaining cable forces to smaller values of axial force, especially in longer stays. Moreover, only two-phase stressing sequence gives a bending moment diagram similar to continuous beam on rigid support, so in cable-stayed bridges the two-phase stressing sequence is much more convenient.

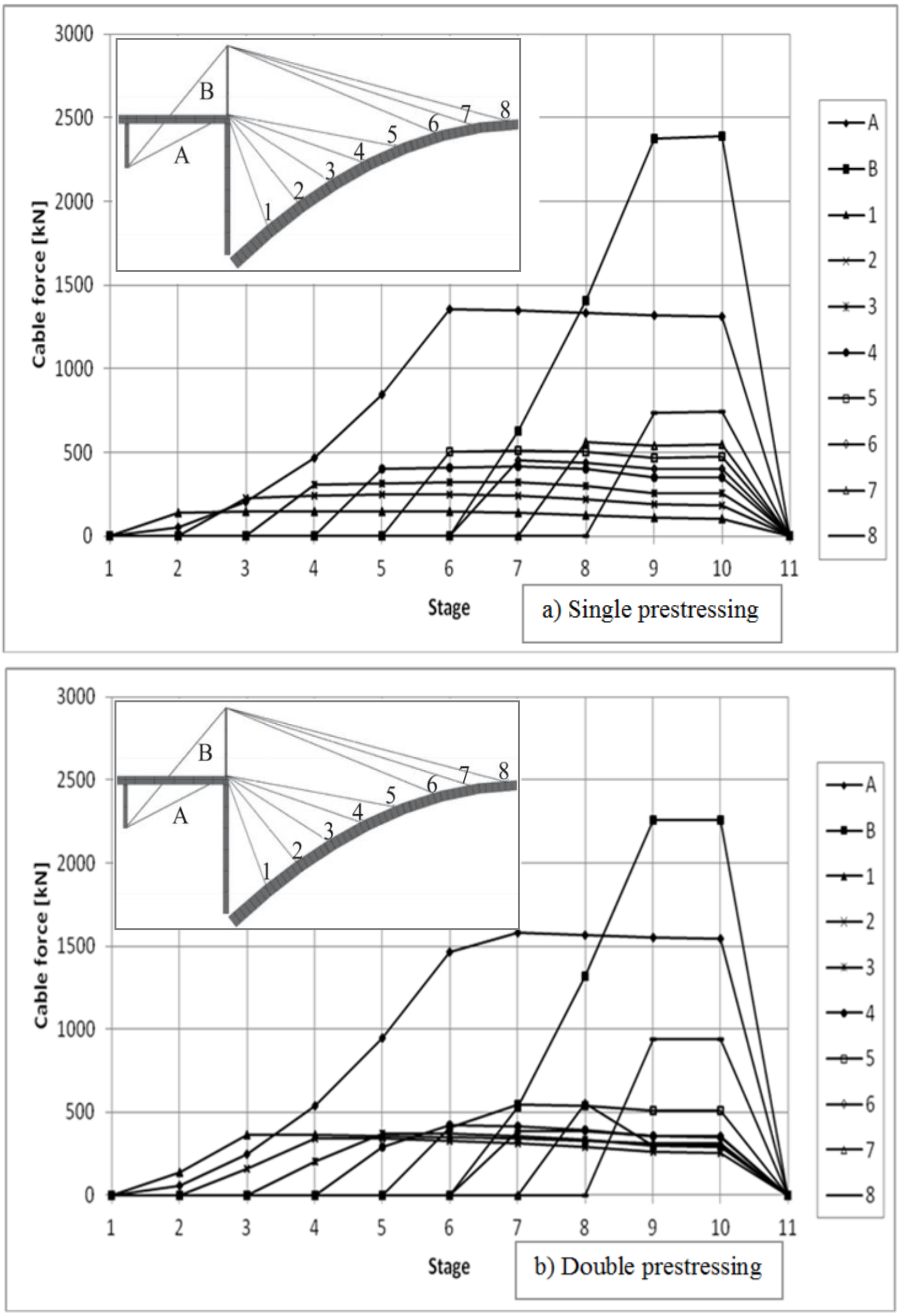


Figure 14. Sequence of stay stressing and cable force behavior in the arch bridge: a) single prestressing, b) double prestressing

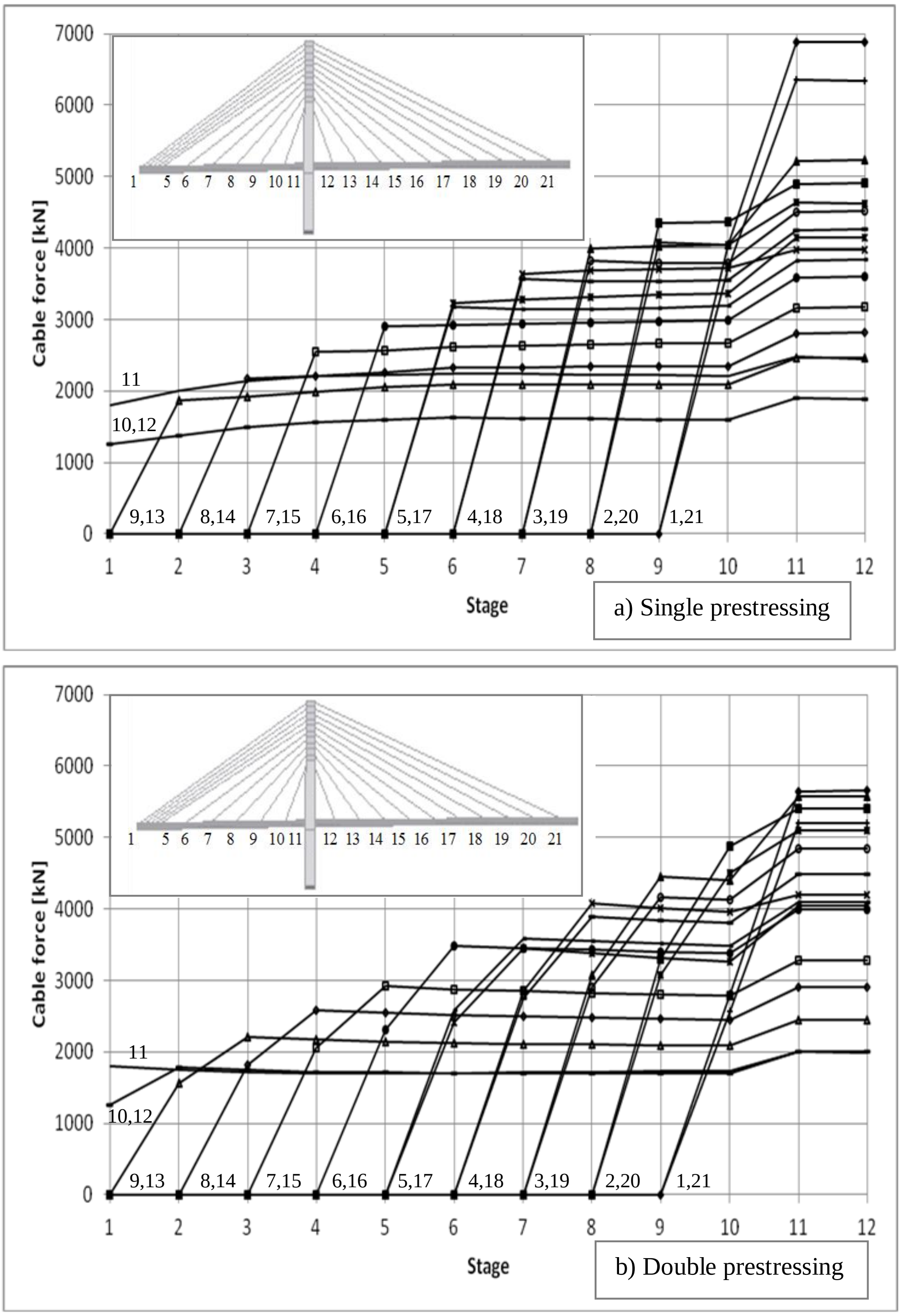


Figure 15. Sequence of stay stressing and cable force behavior in the cable-stayed bridge: a) single prestressing, b) double prestressing

The Partial Elastic Scheme Method with implementation of zero-displacement algorithm has been applied also to an arch bridge with span of 198 m, taken from a real case-study [31], composed of a stiff arch with a bi-cellular boxed cross-section and an upper deck with a single-cell cross-section [6]. In this case, good results are obtained, as in the previous ones, both in term of deformed shape and bending moment diagram during the whole suspended cantilever construction and after key closure and stays removal. Value of the axial force during construction does not vary significantly, confirming that this result is closely related to the proposed method.

The proposed method was also applied to the same cable-stayed bridges shown in fig. 7, changing the deck from prestressed concrete box to composite one [25]. If the cross section is steel-concrete composite, the different elements of section are assembled in different times and this aspect must be considered in the forward staged construction analysis. About the flexural effects of creep and shrinkage, the behavior of composite beams has to be considered, because they are non-homogeneous elements with respect to creep. In fact these elements show an internal redistribution of stresses between concrete and steel members and the change of the centroid position of the so-called creep-transformed section [32], which is the homogenized section modified by the creep coefficient. Another consideration has to be done about the anchorage point of the stay to the deck; when it is set at the upper fibre of the deck section or in an eccentric position with respect to the cross section centroid, a concentrated value of bending moment is generated at each anchor point, which can be significant.

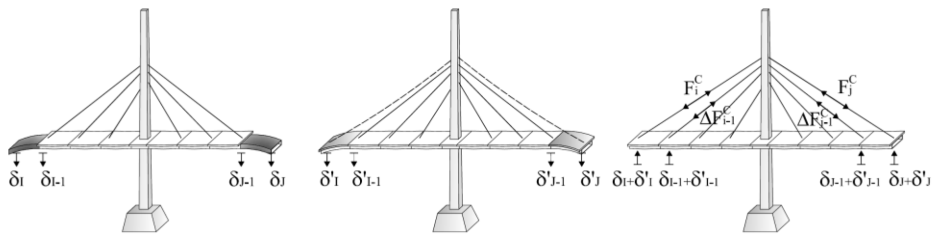


Figure 16. Double stressing sequence in composite cable-stayed bridges

Initial cable forces determination is done by partial elastic schemes of the structure, one for each construction phase, i.e. for each segment assembled by cantilevering and for each stay attached and tensioned. When steel elements and slab casting are built in two following times, different partial elastic schemes of the two subsequent assembling phases have to be considered for each stage. Three different approaches are considered in Granata *et al*. [25]. In the approach that provides the best results, steel elements of the cantilever segment are assembled and the new stay is attached without modifying its length. The slab is cast and then the stay is stressed to the desired value by recovering both displacements δ_I and δ'_I. At the same time the previous stay, already stressed in the previous stage, is adjusted in order to compensate the displacements δ_{I-1} and δ'_{I-1} of the related anchorage point, due to the actual construction stage (fig. 16). With this approach, the final bending moment diagram approximates the moment distribution of an equivalent continuous beam on rigid supports and this behavior is achieved not only in the final stage (fig. 17) but also in every intermediate stage during construction. It is a great advantage because bending moments have similar values in all different steps of the sequence. However, it has to be considered that in composite bridges the bending diagram of a cantilever phase, in which only the steel members are assembled, has often the same significance of that related to the phase in which the stay is tensioned, but with a different sign: negative moments in the cantilever phase and positive ones in tensioning phases. The proposed stay stressing sequence shows a balancing of bending moments, giving similar values for positive and negative diagrams. Through the staged construction analysis, it can be seen that values of maximum negative and positive moments result of the same order of magnitude, stage by stage. Approach described gives also good results in terms of deformed configuration (fig. 18).

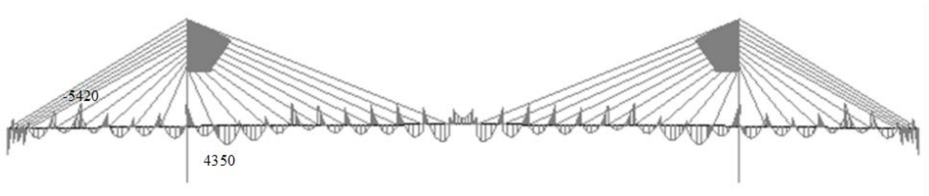


Figure 17. Final bending moment diagram [kNm] in the composite cable-stayed bridge

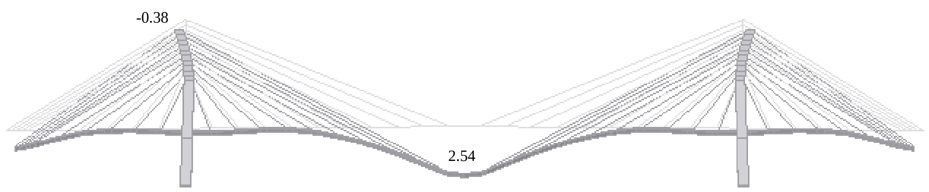


Figure 18. Final deformed shape [cm] in the composite cable-stayed bridge

Figure 19 shows the sequence of stay stressing during the bridge construction, stage by stage. It can be seen that double stressing sequence, also in a composite bridge, gives good results in terms of stress stay variation during construction, maintaining the value of cable forces almost constant after the adjustment. It is a good consequence of initial cable forces found by the Partial Elastic Scheme Method, experienced also in the previous applications. Moreover in the early phases, when the stay is stressed and adjusted, the value of forces increases sweetly without sudden variations, contrarily to what occurs with a one-step stressing sequence. The final adjustment given to all stays compensates not only the additional loads but also the errors accumulated in the actual erection sequence.

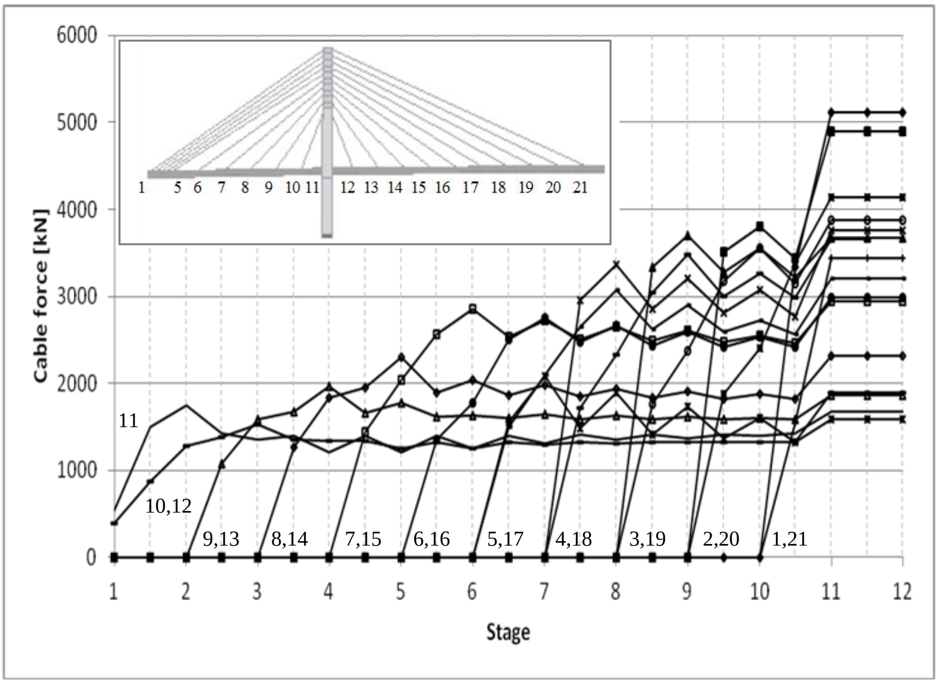


Figure 19. Sequence of stay stressing and cable force behavior in the composite cable-stayed bridge

Reference to the evaluation of errors on the actual stressing sequence can be found in [26].

5 CONCLUSIONS

A method for evaluating the initial cable force and for analyzing the stressing sequence of cable structures has been presented. The proposed method, called Partial Elastic Scheme (PES) Method, is based on the analysis of multiple simple elastic schemes of partial structures representing the construction stages of these structures. The evaluation of initial cable forces is a fundamental step for implementing a forward staged construction analysis in finite element procedures, because the cable structures, such as cable-stayed bridges and arch bridges built by suspended cantilevers, need the definition of initial prestressing for each stay to be assembled in the construction process. The deformed configuration and the achievement of the design geometry for deck and piers or for the arch shape depend strictly on the initial cable forces. Hence, a reliable method for establishing a-priori stay prestressing forces is necessary as input data for the following step of the design process consisting of the construction analysis.

The proposed method supplies a practical tool to engineers for finding initial cable forces with the double aim of achieving an acceptable geometry in the dead load configuration and also a convenient state of stress for stays and for the suspended structure, especially in terms of internal forces (e.g. bending moments); this aim is fulfilled at each construction stage, till the final one.

The PES method gives very good results both for cable-stayed and arch bridges of medium spans in the linear range (with length of stays less than 300 m); its main advantages are:

- Partial elastic schemes are simple static ones to be analyzed and they are independent one from the each other;
- The displacement or force methods can be applied in the same way, with no changes in the formulation, i.e. the final configuration can be found alternatively with optimization in terms of kinematical or static constraints;
- The proposed method can be applied to cable-stayed bridges, in which the final configuration is cable-stayed too, as well as to arch bridges built by suspended cantilevers, in which the final configuration is the closure of the arch key with the stays removal;
- A forward staged construction analysis can be performed, since the values of initial cable forces (stay prestressing) is given a-priori as result of the independent partial static schemes and in this way analyses including time-dependent phenomena such as shrinkage and creep of concrete can be carried out immediately;
- The method can be applied indifferently to concrete and steel-concrete cable-stayed structures and the procedures of stay stressing as well as the necessary adjustments can be evaluated case-by-case without modifying the approach;
- Results on numerical examples of case-studies confirm that the zero-displacement method, associated to the partial elastic schemes with one or two re-stressing of stays, gives always the best results in terms of geometric configuration and state of stress, approaching the convenient configuration of beams on rigid restraints.

A comparison between different cable-stayed structures is given to show the wide range of applicability of the proposed method.

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