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INFLUENCE OF SOIL-STRUCTURE INTERACTION MODELING ON THE RESPONSE OF SEISMICALLY ISOLATED BRIDGES

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ABSTRACT: The impact of the Soil-Structure Interaction (SSI) on seismic isolated bridges is investigated. Two stick models of two seismically isolated bridges and equivalent models of the functions of soil-pile group system are considered. The frequency-dependent impedance models consist of frequency-independent springs and dashpots as well as "gyromasses", which are elements proposed recently in the literature. Each "gyromass" plays the role of an ordinary mass, with the advantage of not altering the dynamics of the system since does not add additional inertial forces into the system. Appropriate assemblies of springs, dashpots, and "gyromasses" can match in the frequency domain even the most frequency-sensitive impedance functions providing an advantage over simple Voigt models (spring-dashpot in parallel) used frequently for SSI analyses. The models are suitable for use in time domain and are utilized in this study for the nonlinear time history analyses of the bridges, subjected to 2 sets, near fault (NF) and the far field (FF), of 20 motions. The paper considers factors which influence the SSI effects on a seismically isolated bridge, number and geometry of piles, spacing of piles, soil characteristics, flexibility of superstructure, and examines how the SSI modeling, Gyromodels vs Voigt models, influence the response parameters of such structures. The results are presented in terms of displacements and forces of the structural system and some general conclusions are drawn over the appropriate modeling systems of each case of study

KEYWORDS: Seismic Isolation; Soil Structure Interaction; Gyromass; Bridges; Seismic response.

1 INTRODUCTION

Seismic isolation research concerns with soil-foundation-structure-interaction issues since the late 1980's during the early years of development of this design and construction technology. Constantinou and Kneifati [1] were the first to present a study on the effects of SSI in seismic isolated buildings. In seismically isolated bridge structures SSI effects were first investigated approximately a decade later by Vlassis and Spyrakos [2]. The last 25 years the published literature in the subject of SSI of seismically isolated structures has evolved from considering linear models for the superstructure and frequency independent linear spring and dashpot models for the soil-foundation dynamic stiffnesses, to nonlinear hysteretic models for the superstructure and linear frequency dependent springs and dashpots for the soil-foundation behavior.

The last 15 years the literature (Vlassis and Spyrakos [2], Spyrakos and Vlassis [3], Tangaonkar and Jangid [4], Ucak and Tsopelas [5], Olmos and Roesset [6], Olmos et al. [7], Soneji and Jangid [8], Stehmeyer III and Rizos [9], Dezi et al. [10], Krishnamoorthy and Anita [11]) concentrating on seismically isolated bridges had centered around evaluating the effects of SSI over the non SSI case (fixed base). The only study attempting to evaluate the effect in the response of seismically isolated bridges due to different modeling approaches of the SSI is by Ucak and Tsopelas [5]. They presented results on the differences between modeling the SSI with frequency independent springs and dashpots (Voigt model with constant parameters) and models accounting for the frequency variations of the soil-foundation impedances through the addition of artificial masses. They concluded that for short stiff bridges (e.g. Bridge I) Voigt models adequately account for SSI for both FF and NF sets of seismic records. In addition, they found that 10% underestimation of the isolation system displacements is possible if frequency dependence of the soil-foundation system is omitted.

The current study builds on the work by Ucak and Tsopelas [5] with main objective being to evaluate how the SSI modeling affects the response parameters of seismically isolated bridges. Two bridge structures are considered: the first (Bridge I) is representative of a typical highway overcrossing with a stiff short pier, while the second one (Bridge II) could be part of a long multi-span bridge with flexible tall piers. The isolation system is assumed as bilinear hysteretic in nature, while the pier is linear elastic with5% inherent damping. The foundation system is a pile group rigidly connected to the rectangular pile cap. The total system is subjected to two sets of seismic motions: one set of far-field (FF) excitations and one set of near-fault (NF) seismic excitations. Two mechanical analog models are utilized to account for the soil-foundation impedances: the first is the simple Voigt model which consists of a linear spring and a linear dashpot in parallel with constant values of its parameters corresponding to the soil-foundation impedances for zero frequency (static values). This model does not account for the variations of stiffness and damping with frequency of oscillations. The second one called Gyromodel, consisting of systems of springs, dashpots and "gyromasses" (a new mechanical analog to mass originally introduced by Saitoh [12]) which is capable of modeling the variations of stiffness and damping of the soil-foundation system with frequency. This model can correctly account for those effects without introducing artificial masses in the dynamics of the system which could have altered the inertial components of the soil-foundation-structure system as was the case with the earlier proposed models (see [12] for more details). The values of the element parameters are calibrated from the semi-analytically obtained frequency sensitive dynamic soil-foundation impedances using software PILES by Kaynia and Kausel [13]. The results from the non-linear time history analyses are compared in terms of pier shear forces and isolation system displacement. As the presented structural configurations are typically used all over the world, the results presented in this work are expected to provide important insight on the effect of SSI modeling approach (Voigt vs Gyromodel) on the response of seismically isolated bridges to the engineers throughout the world.

2 SOIL-STRUCTURE INTERACTION

Many analysis methods for SSI have been proposed and advanced through the years. They can be categorized in two large groups: a) the direct solving methods and b) the substructure solving methods. Although direct analysis (modeling the exact problem with the help of finite elements and subjecting its boundaries to specific excitations) is the most accurate, it is also computationally expensive and the interpretation of results is rather difficult. In the substructure method the system is considered as being composed of separate parts/substructures. The link between the substructures is established if compatibility conditions are imposed. Such an approach allows for easier identification of the interaction's important parameters and helps quantifying how the interaction influences each part separately. To this direction, the seismic SSI problem can be divided to two major components. The first, soil response analysis, is the response of the soil as waves travel through the soil deposit. The second is coupled foundation-superstructure response, which is actually assumed to be a superposition (due to the linear elastic nature of the system) of the response of the pile foundation itself to the excitation in the absence of the superstructure's inertial forces (kinematic response) and the effect on the response of the superstructure due to the additional flexibility introduced into the structure by the presence of the foundation (inertial response).

Even though soil response analysis is one of the most important aspects of earthquake engineering, as it will determine the ground motion that will be experienced at the top of the soil without the presence of a structure or the so-called free field response. In the present study, no soil amplification analysis was performed, rather, the considered seismic motions (accelerograms) were used directly to excite the springs-dashpot-gyromass (Saitoh, 2007) assemblies, which were used to model the soil and foundation subsystem.

Extensive research has been conducted concerning the kinematic response of piled foundations [14].The results of these studies indicate that a scattered wave field could be generated which might result in differences in the free-field response and the motion that the pile cap will experience. Such differences (kinematic response) depend on the soil parameters, the relative stiffness between the soil strata and the material of the piles, the characteristics of the input motion (amplitude, frequency, duration) and the pile boundary conditions. According to the results so far, for pile groups resting on homogeneous soil profile, subjected to low frequency excitations, free-field motions can be used directly as an input motion. Since a homogeneous soil profile is adapted in this study, and considering that seismically isolated bridges are expected to undergo low frequency vibrations, it is a valid assumption without loss of accuracy to ignore the kinematic soil-pile interaction. The inertial response on the other hand, is utilized, by replacing the pile foundation with appropriate assemblies of springs, dashpots and gyromasses.

3 SOIL-FOUNDATION-BRIDGE SYSTEM MODELING

3.1 Bridge Systems

Two bridge structures are considered in this study; they are the same used in Ucak and Tsopelas [5]. The first (Bridge I) is representative of a typical highway overcrossing with a stiff short pier, while the second one (Bridge II) could be part of a long multi-span bridge with flexible tall piers. Figure 1a depicts the geometric characteristics of each bridge model. Stick models are used for both bridges, each consisting of a linear elastic pier at the top of which, a bilinear hysteretic isolation system carries the weight of the deck. At the bottom, the pier is monolithically connected to the pile-group cap. The deck, the pier and the foundation masses are assumed concentrated at the location of their center of mass. Figure 1b depicts the behavior of each component of the bridge models considered in the study.

Three pile group systems, a 5x5 a 3x3 and a 2x2, were considered for each bridge model in order to examine the effect of the number of piles on the overall seismic response of the structure. For the 5x5 pile group for the Bridge I model, the number and geometrical characteristics of the piles were chosen to be identical to the pile group supporting the pier of the Meloland road overcrossing bridges [15]. The 3x3 and the 2x2 pile groups have the same material properties and equivalent pile diameters so as to represent the same foundation area and result in approximately the same static impedances (ω=0) with the 5x5 pile groups. Apparently the different pile groups lead to different dynamic impedance functions (see Figures 3 and 4). The 3 equivalent pile group systems for the second bridge have larger diameters due to the larger superstructure supported. Figure 2 presents the geometry of the pile groups considered.

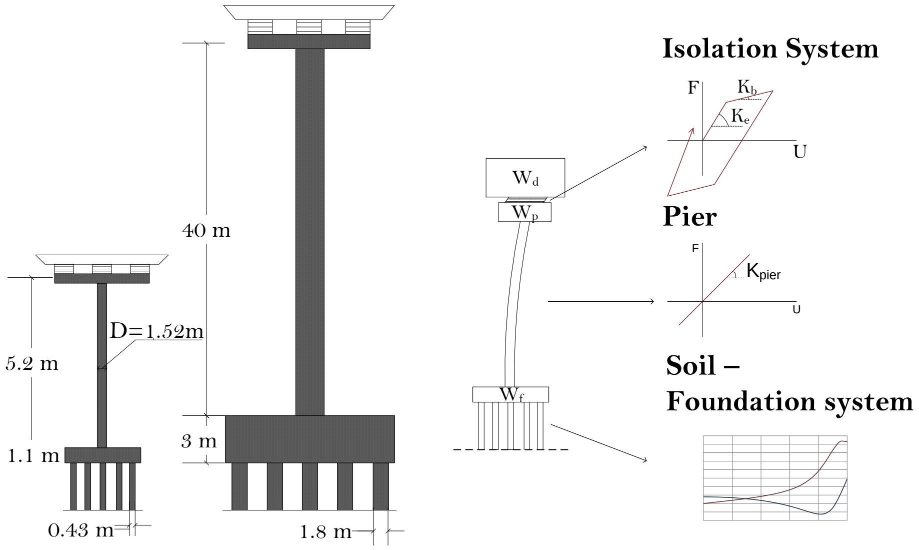


Figure 1. (a) Geometrical representation of Bridge I and Bridge II; (b) Mechanical modeling of bridge and foundation system

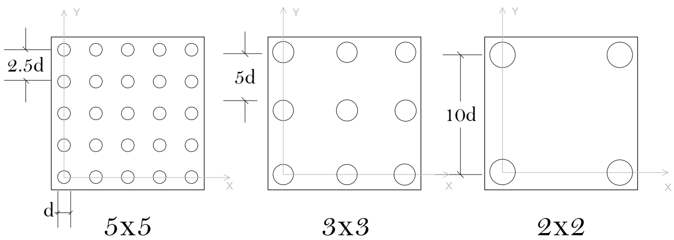


Figure 2. Geometry of 5x5 and equivalent 3x3 and 2x2 pile groups (for both bridges)

The surrounding soil is taken as homogeneous, while the foundation flexibility after interacting with soil is taken into account through lateral and rocking systems consisting of springs, dashpots and "gyromasses" [12]. There were no rotational inertial effects of the isolation system considered, at neither of the Bridges. Table 1 contains the geometric and mechanical properties of the two bridge systems.

Table 1. Properties of the bridge models considered

Bridge model	Bridge I	Bridge II
Deck mass, m_d (Mg)	265	1440
Isolation system period, T_b (sec)	2	4.5
Isolation strength ratio (F_y / W_d)	0.12	0.04
Pier mass, m_p (Mg)	38.5	620
Pier weight/ Deck weight	0.15	0.43
Pier height, h (m)	5.2	40
Pier elastic stiffness, k_p (kN/m)	1.24E5	1.09E5
Pier damping ratio, ξ	5%	5%
Foundation mass, m_f (Mg)	84	4248
Foundation moment of inertia, I_f (Mg m^2)	173	126200
Pile cap height, H_f (m)	1.1	3

3.2 Seismic Isolation System

The seismic isolation system is considered to behave as a bilinear hysteretic spring with smooth elastic to post yielding transition. Such a behavior could be representative of typical lead rubber bearings, as well as of sliding bearings with metallic yielding devices or restoring force capability. Its nonlinear hysteretic behavior was modeled using a model proposed by Ozdemir [16]. The variables controling the system are the yield strength (F_y), the elastic stiffness (K_e) and the post yielding stiffness (K_b). The values used in this study are defined also in Table 1.

3.3 Dynamic impedances of pile groups

The harmonic response of pile groups is substantially affected by the dynamic interaction between the soil and the piles and between the individual piles. The dynamic impedance of soil-foundation systems is evaluated as function of the frequency of oscillations under harmonic excitations. Following the early numerical studies by Wolf [17], Von Arx [18] and Nogami [19], several researchers have developed a variety of computational (rigorous and simplified) methods for computing the dynamic impedances of pile groups, lateral($\mathcal{K}_{xx}$), rocking ($\mathcal{K}_{rr}$) and the cross term ($\mathcal{K}_{xr}$). Considering a single pile for each sinusoidal excitation with a frequency ω, dynamic impedance is defined as the ratio between the magnitudes of excitation and of the resulting displacement or rotation at the pile head:

$$\mathcal{K}_{\mathrm{xx}} = \frac{P_o e^{i\omega t}}{u_o e^{i(\omega t+\phi)}} \tag{1}$$

where: $P_o e^{i\omega t}$ is the horizontal dynamic force,
$u_o e^{i(\omega t+\varphi)}$ is the resulting horizontal displacement.

It is preferable to express the dynamic impedances as:

$$\mathcal{K}_{xx} = K_{xx} + i\omega C_{xx} \tag{2}$$

where: K_{xx} is the "spring" coefficient,
C_{xx} is the "dashpot" coefficient or damping constant,
ω is the frequency of the harmonic input (under free interpretation the seismic event) (rad/sec) and
i equals to $(-1)^{1/2}$.

Similarly, the dynamic impedances related to rocking and the coupled mode of vibration are expressed as:

$$\mathcal{K}_{rr} = K_{rr} + i\omega C_{rr} \tag{3}$$

$$\mathcal{K}_{rx} = K_{rx} + i\omega C_{rx} \tag{4}$$

The dynamic stiffnesses of a pile group, in any vibration mode, can be computed using the dynamic stiffnesses of a single pile in conjunction with the use of superposition principle, originally developed for static loads by Poulos [20], and then for dynamic loads by Kaynia and Kausel [13], Sanchez-Salinero [21] and Roesset [22]. It can be used with confidence at least for groups with less than 50 piles. Dynamic interaction factors for various vibration modes are available in the form of non-dimensional graphs [23] and in some cases, closed form expressions derived from a beam on winkler foundation model in conjunction with simplified wave-propagation theory [24], [25]).

In this study, for the estimation of the dynamic impedances of pile groups, the boundary element software PILES [13] was utilized. The behavior of the piles and the ground is considered linear elastic, while the pile cap is considered rigid and not in contact with the ground. The ground is assumed to be horizontally layered and resting either on rigid bedrock or viscoelastic half space. The piles are characterized by their radius, mass per unit length, bending and axial rigidities and Poisson's ratio. Table 2 lists the values of the parameters of the soil model considered in PILES. Table 3 summarizes the geometric and mechanical properties of the pile groups analyzed here in; 2 pile groups (one for each bridge), 3 sets of pile configurations per group, 5x5, 3x3, 2x2, (same soil profile, with $E_p/E_s = 300$, $\rho_s/\rho_p = 0.7$).

Table 2. Properties of the soil profile to be studied

Number of layers (input PILES)	70 (max)	
Total Thickness (m)	21.5	
Soil Profile	Homogeneous Half-space	
Shear Wave Velocity, V_s (m/sec)	110	**$G = \rho_s V_s^2 = 22$ MPa**
Mass Density, ρ_s (kg/m^3)	1800	
Damping Ratio, ξ	0.10	**$E_s = 2(1+v)G = 0.062$ GPa**
Poisson's Ratio, v	0.40	

Table 3. Properties of the 3 equivalent pile groups of study

	Bridge I			**Bridge II**		
Pile Group Label	**5x5**	**3x3**	**2x2**	**5x5**	**3x3**	**2x2**
# piles, N	25	9	4	25	9	4
Pile Diameter, d (m)	0.43	0.7	1	1.8	3	4.5
Pile Length, L (m) **L/d**	21.5 50	21.5 31	21.5 21	21.5 12	21.5 7	21.5 5
Pile to Pile Distance, S (m) **S/d**	1.08 2.5	3.5 5	10 10	4.5 2.5	9 3	18 4
Mass Density, ρ_p (kg/m^3)	2500	2500	2500	2500	2500	2500
Modulus of Elasticity, E_p (GPa)	18.5	18.5	18.5	18.5	18.5	18.5

Figure 3 and 4 present the components of the dynamic impedance K_{xx} and K_{rr} of all the pile group cases and for the two bridges as functions of the dimensionless frequency of excitation $a_o = \omega d/V_s$, where ω is the frequency of the harmonic excitation (or of the seismic event). K_{xx} and ωC_{xx} are respectively the real and the imaginary parts of the horizontal dynamic stiffness of the pile group, K_{xx} .

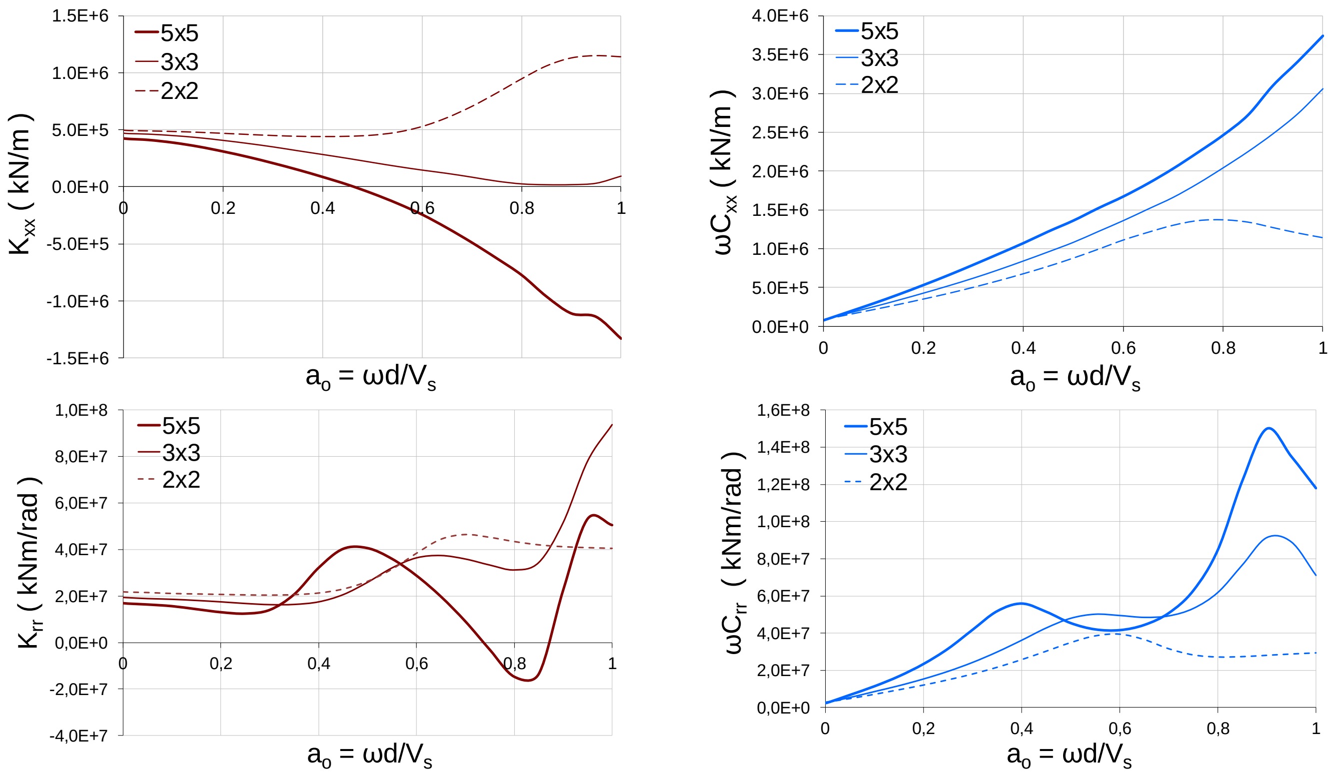


Figure 3. Components of the K_{xx} and K_{rr} impedances of 5x5, 3x3 and 2x2 pile groups for Bridge I

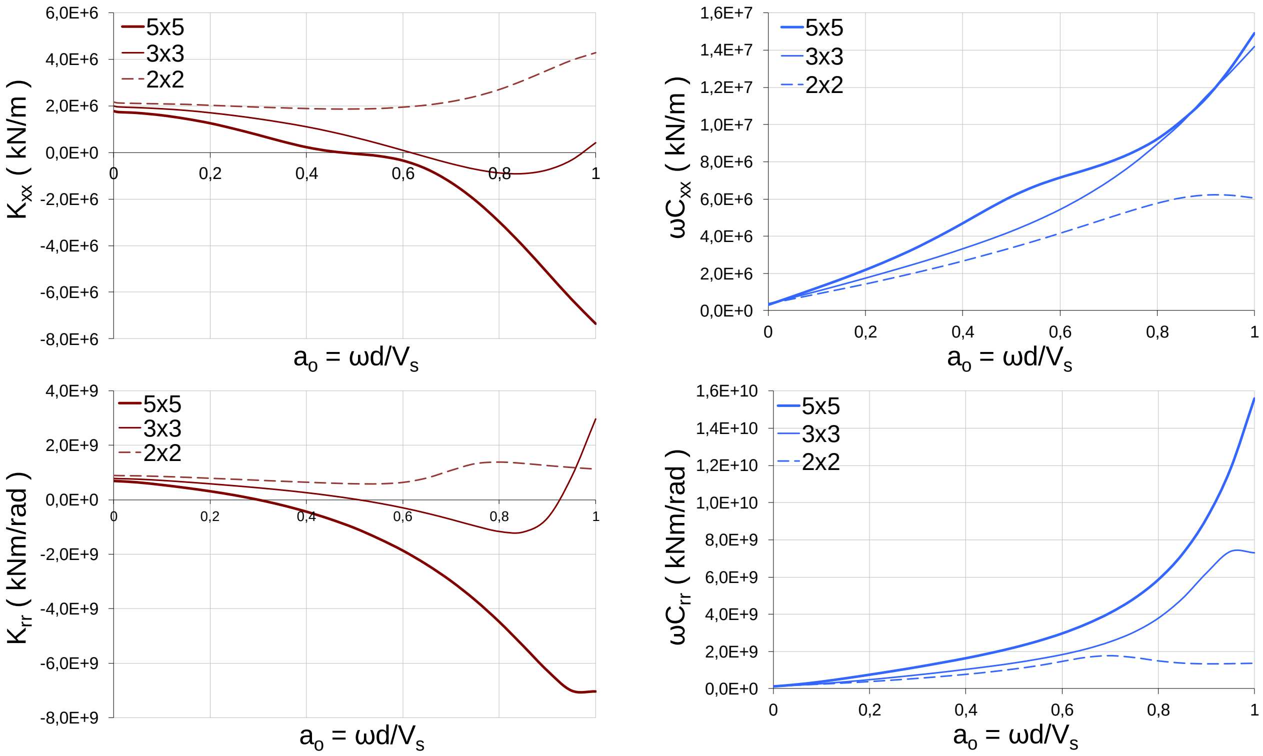


Figure 4. Components of the K_{xx} and K_{rr} impedances of 5x5, 3x3 and 2x2 pile groups for Bridge II

Figures 5 and 6 present the dimensionless horizontal and rocking impedances (dynamic interaction factors) for all pile groups kai both bridges considered in the study. The horizontal dynamic stiffness is normalized by the horizontal static stiffness of the corresponding pile group $\mathrm{N} \cdot K_{x,single}(\mathrm{a_o} = 0)$ and the rocking dynamic stiffness is normalized by the rocking static stiffness of the corresponding pile group $\mathrm{N} \cdot \sum (x_i^{\,2} \cdot K_{x,single}(\mathrm{a_o} = 0))$.

In this study the coupling effect of horizontal-rocking oscillations are not considered (K_{rx} impedances are neglected).

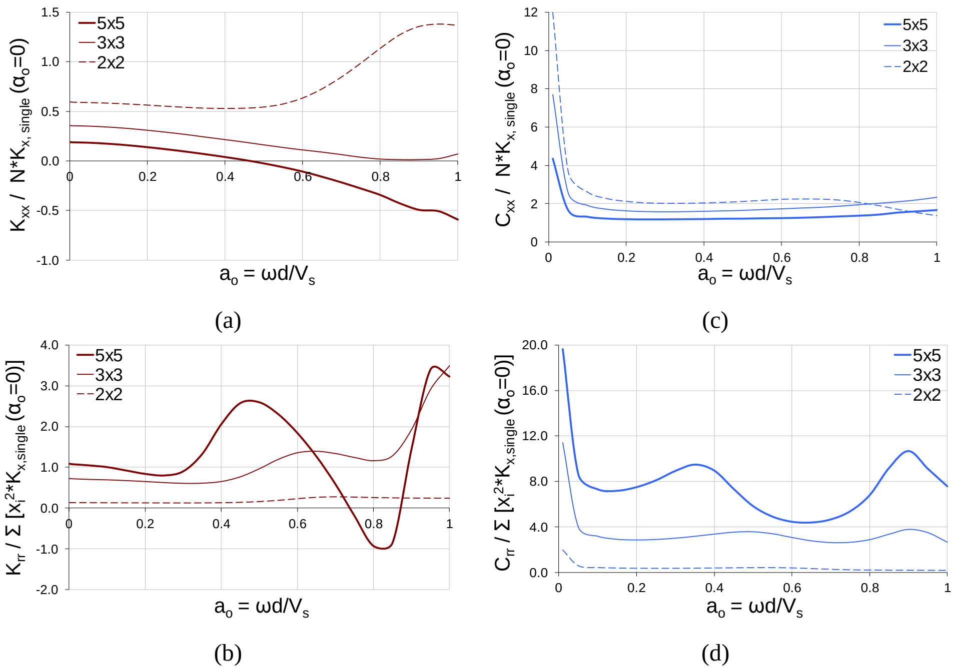


Figure 5. (a) and (b) Dimensionless stiffness constants of the horizontal and rocking dynamic impedances; (c) and (d) Dimensionless damping constants of the horizontal rocking dynamic impedances for Bridge I

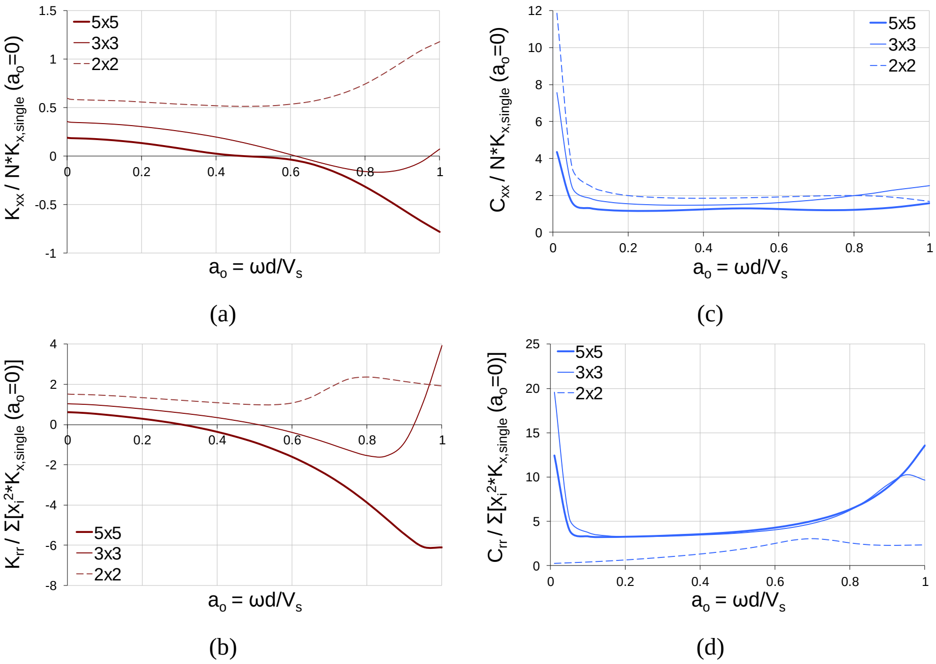


Figure 6. (a) and (b): Dimensionless stiffness constants of the horizontal and rocking dynamic impedances; (c) and (d): Dimensionless damping constants of the horizontal rocking dynamic impedances for Bridge II

3.4 Spring-Dashpot Model (Voigt)

The nonlinear behavior of the isolation system prohibits the use of frequency domain analysis techniques to treat the SSI problem. In order to overcome this incompatibility between the frequency dependence of the soil-foundation springs and the requirement for nonlinear time history analysis, it is customary to introduce an approximation omitting the frequency dependency of the soil-foundation system considering that the springs and dashpots have constant, frequency–independent values corresponding to the impedance values for a_o=0. This assumption, in the present study, might not be unreasonable considering the fact that the isolation period, which is fairly large, is expected to dominate the overall seismic response of the bridge. These low excitation frequencies correspond to a range of a_o between 0 and 0.1 thus effectively the “static” values of the spring and damping coefficients.

The simple Voigt model (see Figure 7a), consisting of a spring and a dashpot, connected to the foundation mass, is the easy modeling alternative for the impedance functions, as discussed above. The constant values of the springs and dashpots in Voigt model make the analysis easier but are not representing the real behavior of the dynamic impedances (Figures 3 to 6). An alternative modeling approach introducing a spring-damper-artificial mass secondary subsystem attached to the foundation mass with appropriately calibrated parameters, as is the basis of the procedure used by De Barros and Luco [26] and Wolf and Somani [27], can be used to account for the frequency dependent behavior of the soil-foundation system. However, such models alter the dynamics of the soil-foundation-structure system because the additional mass of the secondary system contributes to the inertial forces and moments of the system thus altering the actual input (right hand side of the equations of motion) to the superstructure.

3.5 The Gyromodel

In order to include the SSI effects in the time history analysis, Saitoh [12] introduced a system consisting of basic mechanical elements (springs and dashpots) together with elements he called “gyromasses”. This system is capable of representing frequency dependent impedance functions while eliminating the shortcomings of the models introduced by De Barros and Luco [26] and others. The gyromass is a mechanical element which has the same dimension as the mass. When it is used as an independent unit it generates a reaction force proportional to the relative acceleration of the nodes between which the gyromass is placed. The mechanical analogy of the gyromass is given by Saitoh [12].

The system utilized in this study to model the SSI effects, named for the rest of the study "Gyromodel", consists of three subassemblies of springs, dashpots and gyromasses. The first subassembly, called in Saitoh [12] base system, contains a spring unit, a dashpot unit, and a gyromass unit connected in parallel. The second and third subassemblies, called in Saitoh [12] core systems, have a spring in series with a gyromass and dashpot in parallel, as depicted in Figure 7.

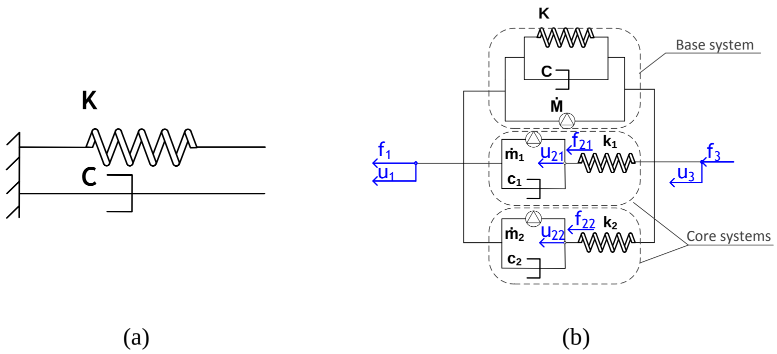


Figure 7. (a)Voigt model (b)Gyromodel system with the spring-dashpot-gyromass sub assemblies

Equation (5), as presented in Saitoh [12], relates translational force and displacement between the two ends of a Gyromodel system (see Figure 7) in frequency domain (a_o). A similar equation with a similar set of variables is utilized for the rotational degree of freedom and relates the rotation of the system with the corresponding moment developed.

$$F_{GYROMODEL} = F(a_o) = K\left\{ \begin{array}{l} 1+\sum_{i=1}^{N}\frac{\beta_i\left[\mu_i a_o{}^2(\mu_i a_o{}^2-1)+\gamma_i a_o{}^2\right]}{\left(1-\mu_i a_o{}^2\right)^2+\gamma_i{}^2 a_o{}^2}+ \\ \qquad + i a_o\left[\sum_{i-1}^{N}\frac{\beta_i\gamma_i}{(1-\mu_i a_o{}^2)^2+\gamma_i{}^2 a_o{}^2}+\gamma_o\right] \end{array} \right\} u(a_o) \tag{5}$$

where: N is the total number of core systems,

$a_o = \dfrac{\omega d}{V_s}$ dimensionless frequency of excitation

$\mu_o = \dfrac{\dot{M} V_s{}^2}{d^2 K}$ dimensionless mass coefficient of the base system

$\gamma_o = \dfrac{C V_s}{dK}$ dimensionless damping coefficient of the base system

$\gamma_i = \dfrac{c_i V_s}{dk_i}$ dimensionless damping coefficient of each core system,

$\mu_i = \dfrac{\dot{m}_i V_s{}^2}{d^2 k_i}$ dimensionless mass coefficient in each core system and

$\beta_i = \dfrac{ki}{K}$ relative stiffness of each core and base system.

The aforementioned coefficients of the Gyromodels for translational and rotational DOF are evaluated/calibrated against the soil-foundation impedances through the frequency range considered, utilizing a non-linear least squares algorithm [29] in the complex domain [28]. The calibrated values of the Gyromodel's parameters for the three different pile groups, for both Bridges, are shown in Tables 4 and 5.

Figure 8 compares the semi analytically (PILES) obtained dynamic impedances and the ones obtained via Equation (5) for Bridge I. It is apparent that the Gyromodel system through appropriate calibration of its constants/variables can represent very accurately in the frequency domain the translational and rotational dynamic impedances of the soil foundation system considered in this study.

Table 4. Coefficients of Gyromodel for the bending impedances (horizontal degree of freedom)

Coefficient	**Bridge I**			**Bridge II**		
	5x5	**3x3**	**2x2**	**5x5**	**3x3**	**2x2**
$K_{xx} / N^*K_{x,single}$	0.19	0.36	0.60	0.19	0.36	0.60
γ_o	2	4.2	0.13	5	3	0.6
μ_o	5	1.1	1.6	2.5	1.2	1.1
β_1	1.2	0.9	0.03	0.28	1	0.08
γ_1	2	0.5	10	0.8	0.55	5
μ_1	1.9	0.82	2	3.4	0.8	0.1
β_2	6	0.14	2.5	2.7	1.22	2
γ_2	0.4	0.17	1.1	0.2	0.4	0.9
μ_2	0.5	0.02	1.2	0.65	0.8	1

Table 5. Coefficients of Gyromodel for the rocking impedances (rotational degree of freedom)

Coefficient	**Bridge I**			**Bridge II**		
	5x5	**3x3**	**2x2**	**5x5**	**3x3**	**2x2**
$K_{rr} / \Sigma (x_r^2 * K_{x,single})$	1.09	0.72	0.13	0.61	1.04	1.52
γ_{ro}	2.2	1.2	1.32	5.5	1.4	1.3
μ_{ro}	2.5	0.5	0.2	3.3	0.6	0.8
β_{r1}	1.45	0.85	0.48	8	2.6	0.04
γ_{r1}	0.23	0.27	1.8	0.01	0.33	2
μ_{r1}	1.18	1.14	4.7	0.01	1.03	1.1
β_{r2}	1.65	1.3	0.23	6.7	0.2	0.55
γ_{r2}	1.95	1.5	0.6	0.23	0.2	0.75
μ_{r2}	6.5	3.1	3.2	0.79	0.2	2.4

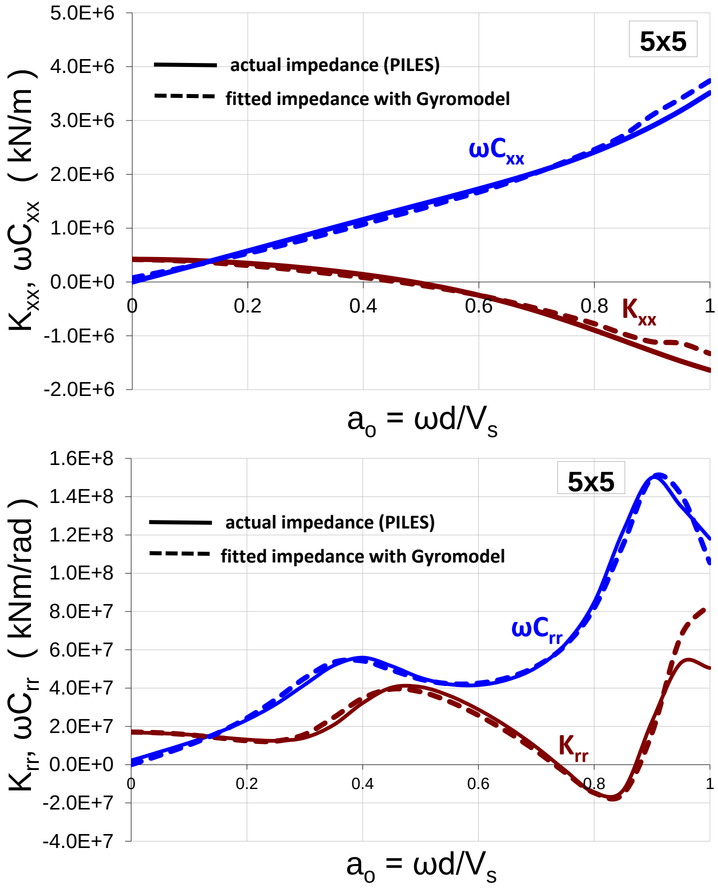


Figure 8. The actual (PILES) and calibrated/fitted (Gyromodel) translational Kxx and rotational Krr impedances, of the 5x5 pile group, for Bridge I

3.6 Equations of motion

The equations of motion in the time domain governing the response of the bridge models are presented below for both Voigt model and Gyromodel. The Voigt model consists of a linear spring and a linear viscous dashpot and is the simplest way to account for the soil-foundation structure interaction. The use of such a model by engineers is very common, however is not able to account for the frequency dependence of the soil-foundation behavior. Voigt models are utilized for both translational and rotational degrees of freedom of the soil-foundation. The coupled bending-rocking effects (K_{rx}) are neglected. Figure 9 presents the bridge system with the Voigt/Gyromodel in the un-deformed and deformed configurations.

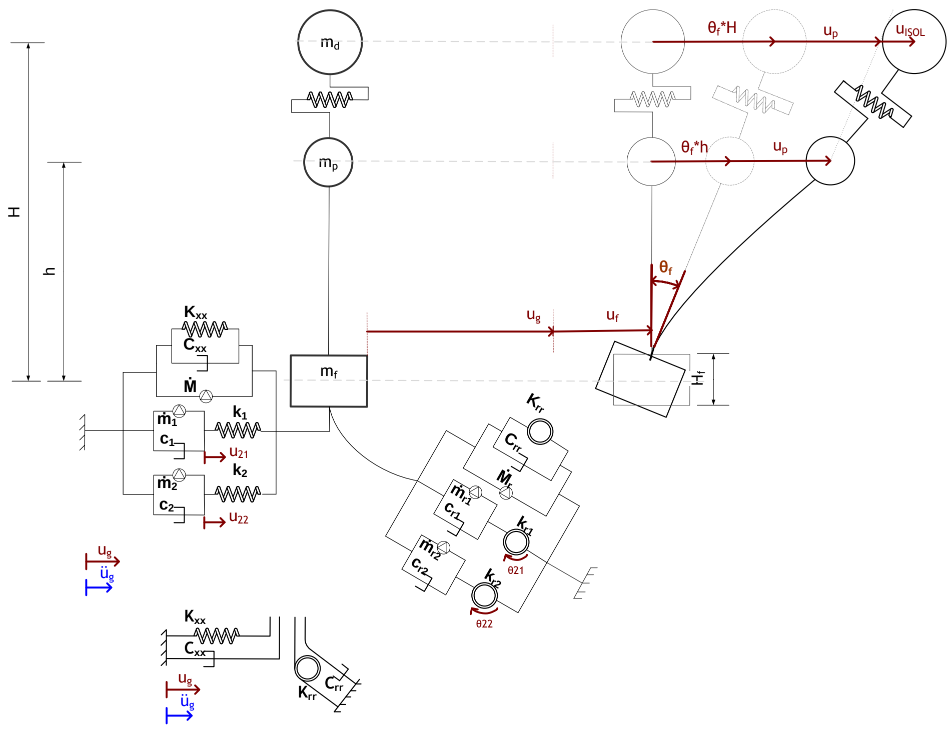


Figure 9. Un-deformed and deformed configuration of the bridge system with Gyromodels and Voigt models

Using dynamic equilibrium the equations of motion of the system are obtained:

$$\sum F_{x,deck}=0 \Rightarrow m_d\ddot{u}_{d,tot} + m_d\ddot{u}_g + f_{ISOL}=0 \quad (6)$$

$$\sum F_{x,pier}=0 \Rightarrow m_p\ddot{u}_{p,tot} + m_p\ddot{u}_g + f_p - f_{ISOL}=0 \quad (7)$$

with $f_p = k_p u_p + c_p\dot{u}_p$

$$\sum F_{x,\,foundation}=0 \Rightarrow m_f\ddot{u}_f + m_f\ddot{u}_g + f_{GYROMODEL}\ or\ f_{VOIGT} - f_P=0 \quad (8)$$

with $f_{GYROMODEL} = \left\lfloor f_{base} + f_{core1} + f_{core2} \right\rfloor$

$f_{VOIGT} = K_{xx}u_f + C_{xx}\dot{u}_f$

and $f_{base} = \dot{M}\ddot{u}_f + K_{xx}u_f + C_{xx}\dot{u}_f$

$f_{core1} = k_1(u_f - u_{21}) = \dot{m}_1\ddot{u}_{21} + c_1\dot{u}_{21}$

$f_{core2} = k_2(u_f - u_{22}) = \dot{m}_2\ddot{u}_{22} + c_2\dot{u}_{22}$

$$\sum M_{foundation} = 0 \Rightarrow$$

$$I_f \ddot{\theta}_f + (M_{GYROMODEL} \text{ or } M_{VOIGT}) - (f_{GYROMODEL} \text{ or } f_{VOIGT})\frac{H_f}{2} - f_P \frac{H_f}{2} = 0 \tag{9}$$

with $M_{GYROMODEL} = \lfloor M_{base} + M_{core1} + M_{core2} \rfloor$

$M_{VOIGT} = K_{rr}\theta_f + C_{rr}\dot{\theta}_f$

and $M_{base} = \dot{M}_r \ddot{\theta}_f + K_{rr}\theta_f + C_{rr}\dot{u}_f$

$M_{core1} = k_{r1}(\theta_f - \theta_{21}) = \dot{m}_{r1}\ddot{\theta}_{21} + c_{r1}\dot{\theta}_{21}$

$M_{core2} = k_{r2}(\theta_f - \theta_{22}) = \dot{m}_{r2}\ddot{\theta}_{22} + c_{r2}\dot{\theta}_{22}$

where: u_g is the ground displacement,
u_f is the foundation displacement,
u_{ISOL} is the isolation system displacement,
θ_f is the rotation of the foundation,
m_d is the mass of the deck,
m_p is the mass of the pier,
m_f is the mass of the foundation,
k_{isol} stiffness of the isolation system (the behavior of isolation is bilinear hysteretic and is modeled by Ozdemir's model),
k_p is the linear elastic stiffness of pier,
c_p is the damping coefficient of the pier,
I_f is the mass moment of inertia of the pile group cap,
H_f is the height of pile cap and
K_{xx}, C_{xx}, K_{rr}, C_{rr} are the spring and damping constants of the Voigt models calculated previously from the foundation-soil impedances for for a_o=0

The system of equations is transformed to state-space form, after reduction of order, and solved utilizing a predictor corrector scheme, based on 4th order Runge-Kutta algorithms, suitable for solving 1st order nonlinear ordinary differential equations.. The bridge system has 4 DOFs, however in the case of the presence of the Gyromodels the total number of DOFs increases by four more to 8.

3.7 Seismic excitations

Two sets of ground motion time histories are considered here in and are introduced in both bridge models, for the three equivalent pile groups (5x5, 3x3, 2x2) and the two SSI modeling approaches of each pile group (Voigt and Gyromodel). The first set, referred as NF (near fault), consists of 20 ground motions, assembled by Somerville et al. [30]. The motions are recordings by National Earthquake Hazards Reduction Program (NEHRP [31]), from earthquakes with multiple fault mechanisms, magnitude range of 6.7-7.4 and epicentral distances between 0 and 10 km. These recordings correspond to medium to soft soil (site class D conditions). Table 6 presents the NF set of ground motions.

Table 6. List of the NF set of ground motions

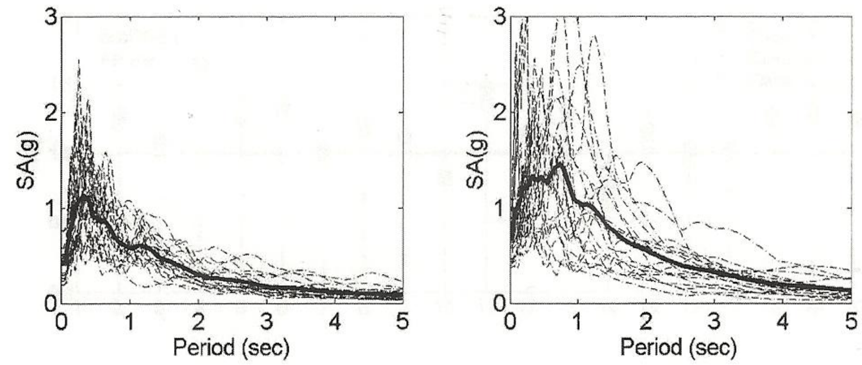
Record ID	Seismic event	Station	Component	Scale factor
1,2	1978 TABAS		N, P	1.00
3,4	1989 Loma Prieta	Los Gatos	N, P	1.00
5,6	1989 Loma Prieta	Lex Dam	N, P	1.00
7,8	1992 Cape Medocino	Petrolia	N, P	1.00
9,10	1992 Erzincan		N, P	1.00
11,12	1992 Landers	Lucerne	N, P	1.00
13,14	1994 Northridge	Rinaldi	N, P	1.00
15,16	1994 Northridge	Olive View	N, P	1.00
17,18	1995 Kobe	Kobe	N, P	1.00
19,20	1995 Kobe	Takatori	N, P	1.00

The second set, referred as FF (far field) is identical to the set used in Whittaker et al. [32] and Constantinou and Quarshie [33]. It consists of ten pairs of scaled acceleration time histories from six actual earthquakes with magnitudes larger than 6.5 and epicentral distances between 10 and 20 km. The recordings correspond to soft rock or stiff soil. The scale factors were chosen so as the average spectrum of all the response spectra from the 20 motions to match a target design spectrum, as presented in AASHTO, for soil type II, A=0.4. The procedure is described analytically in Tsopelas et al. [34].

Table 7 presents the FF set of ground motions. The two sets of motions of Tables 6 and 7 are shown graphically in Figure 10. There is also their average spectrum, indicated with bold line.

Table 7. List of the FF set of seismic motions

Record ID	Seismic event	Station	Component	Scale factor
1,2	1992 Landers	Joshua (DMG)	90, 0	1.48
3,4		Yermo (CDMG)	270, 360	1.28
5,6	1989 Loma Prieta	Gilroy 2 (CDMG)	0, 90	1.46
7,8		Hollister (CDMG)	0, 90	1.07
9,10	1994 Noorthridge	Century (CDMG)	90, 360	2.27
11,12		Moorpark (CDMG)	180, 90	2.61
13,14	1949 W. Washington	325 (USGS)	N86E, N04W	2.74
15,16	1954 Eureka	022 (USGS)	N79E, N11W	1.74
17,18	1971 San Fernando	241 (USGS)	N00W, S90W	1.96
19,20		458 (USGS)	S00W, S90W	2.22



(a) (b)

Figure 10. Response spectra (a) FF set of motions; (b) NF set of motions

4 ANALYSES RESULTS

4.1 Bridge I

Non-linear time history analyses of the two bridge models (Bridge I and Bridge II) with different foundations (2x2, 3x3, and 5x5 pile groups) utilizing two different soil-structure interactions models (Voigt model and Gyromodel) subjected to both Far Field (FF) and Near Fault (NF) sets of seismic excitations are performed to evaluate and compare the superstructure (deck) and substructure (pier) responses in order to quantify the effects of the soil-structure interaction modeling on these structures. The most critical response quantities for the design of a seismically isolated bridge structure are the isolation system displacement (isolation drift) and the shear force in the pier (pier drift is proportional to the pier shear due to the linear elastic behavior assumption of the pier). In the present study the results of the parametric analyses are presented in terms of ratios as isolation drift ratio (IDR) and pier shear ratio (PSR). Since the objective of this study is to evaluate the effect of soil-structure–interaction modeling, the analyses considering the Gyromodel are compared against the analyses utilizing the Voigt model. Thus IDR and PSR are defined as follows:

$$IDR = \frac{Isolation \quad Drift_{GYROMODEL}}{Isolation \quad Drift_{VOIGT}} \tag{10}$$

$$PSR = \frac{Pier \quad Shear_{GYROMODEL}}{Pier \quad Shear_{VOIGT}} \tag{11}$$

Figure 11 summarizes the results of the IDR and the PSR for Bridge I, when subjected to the FF set of seismic motions for all three pile groups considered (2x2, 3x3 and 5x5). The IDR is constant (±1% max deviation), around unity for all seismic excitations and for all three pile groups considered, indicating the insensitivity of the isolation system displacement on the SSI modeling in this particular case. It should be noted that the impedances for all three pile group do not show substantial variability between them and that the variation with frequency for a wide range of frequencies is constant [35]. For the PSR the variation around unity is now slightly larger, ranging between +2% and -4% between all the seismic excitations. The pier shear ratio is also more sensitive to the pile group. The 2x2 pile group results in higher response compared to the other two for every seismic excitation and with the max differences reaching almost 6% between the 2x2 and the 5x5 pile groups. The larger sensitivity of the PSR as compared to IDR is attributed to the higher frequency content of the pier shears relative to isolation system displacements.

Figure 12 present the IDR and PSR for the NF seismic excitation set. Similar observations are made to the FF set of motions. For this set of seismic motions, also, the IDR is insensitive to the SSI modeling (Gyromodel vs Voigt model) and the pile group considered. The PSR although shows a slight sensitivity to both SSI modeling (2.5% max difference) and pile group considered (4% max difference between 2x2 and 5x5 pile groups), for all practical purposes it is also considered insensitive.

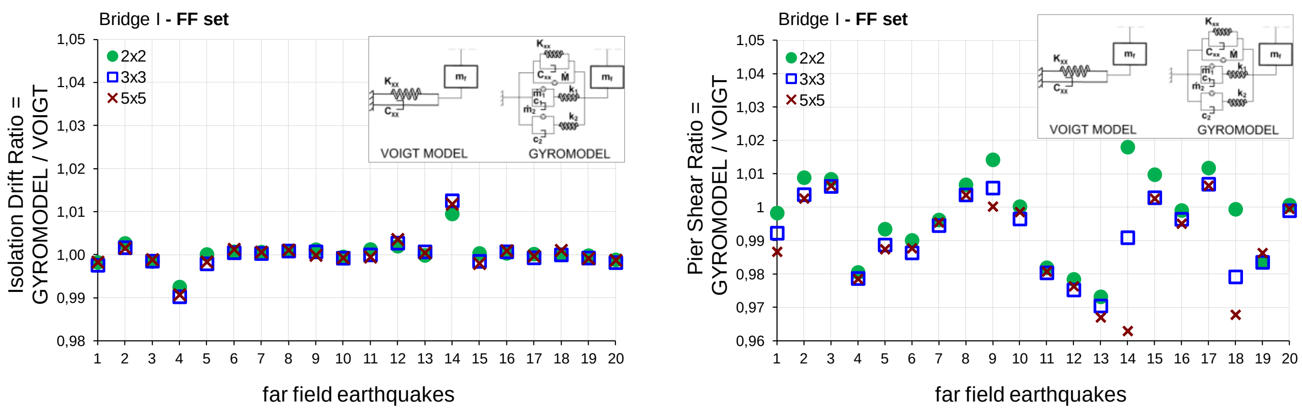


Figure 11. Isolation Drift Ratios and Pier Shear Ratios (Gyromodel/Voigt) for Bridge I and FF set of seismic motions.

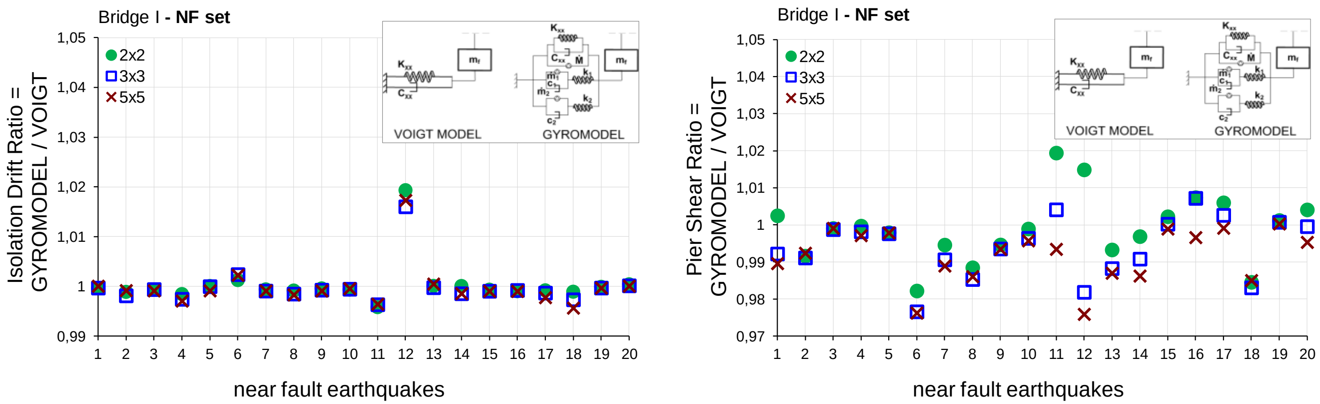


Figure 12. Isolation Drift Ratios and Pier Shear Ratios (Gyromodel/Voigt) for Bridge I and NF set of seismic motions

4.2 Bridge II

Figure 13 show the IDR and PSR for Bridge II and the FF set of motions. The differences in the isolation drift between the Voigt and the Gyromodel models, for all three pile groups considered, range between -10% and +10%. These values are much larger than the differences observed for Bridge I. Over all the isolation drift (IDR) seem to be insensitive to the 3 pile groups considered but it is not insensitive to the SSI modeling since there are seismic motions, 5 out of 20, where the observed differences between Voigt and Gyromodel are larger than 5% and even reach 10%. The PSR also show larger differences than Bridge I, 16% maximum. This indicates that using an SSI model such as the Gyromodel, which is able to capture the frequency dependency of the impedances, than the simple Voigt model, the shear forces in the pier are on the average 10% smaller over this FF motion set. PSR is sensitive to the pile groups with differences between them ranging from 2% (seismic motion # 3) to 10% (seismic motion #9).

Figure 14 show the IDR and PSR for the NF set of motions. The IDR response of the Bridge II under the NF motions appears similar to the corresponding response for the Bridge I, where the IDR shows insensitivity between the 3 pile groups and the seismic motions (IDR in the range +1% and -2% for 19 out of the 20 motions). Only NF16 motion (1994 Northridge, Olive View station, Fault Parallel component) shows difference between Voigt and Gyromodel reaching 8%. Overall IDR appears consistently lower (slightly though) than 1, indicating that the more accurate model for SSI results in slightly lower isolation system displacements. It is also interesting to observe that for the NF set, IDR for the 5x5 pile group is consistently lower than the other 2 pile groups which is a behavior opposite from the one observed for the FF set of motions.

The PSR shows similar behavior to the FF set of motions, -15% maximum and -8% on the average over all seismic motions. Again it indicates that using a more accurate SSI model (Gyromodel) than the simple Voigt the shear forces in the pier are on the average 8% smaller over this FF motion set. PSR is sensitive to the pile group with differences between them ranging from 0% (motion # 17) to 8% (motion #2).

Figure 15 presents Bridge II (5x5 pile group) response parameters for two seismic motions; the first is the NF16 seismic excitation (Northridge 1994 Olive View station, see Table 6) which results in the largest differences between Gyromodel and Voigt model, and the second is the FF11 seismic excitation (Northridge 1994, Moopark station, see Table 7).

Observing Figure 15a, b, and c for NF16 seismic motion there is approximately 10% difference in the isolation displacements between the Voigt and the Gyromodel, 15% difference for the pier drift and 17% difference for the pier shear and 13% for the foundation force and foundation's displacement.

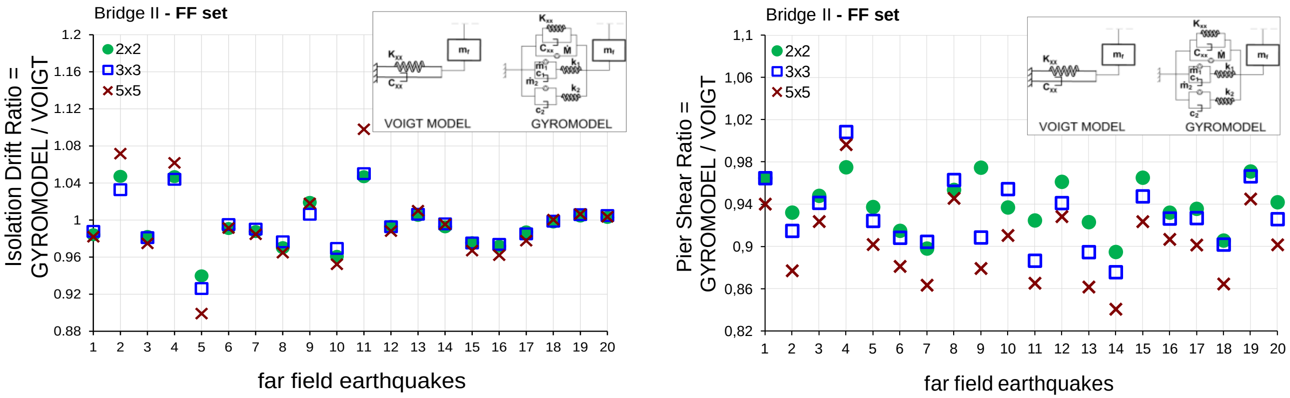


Figure 13. Isolation Drift Ratios and Pier Shear Ratios (Gyromodel/Voigt) for Bridge II and FF set of seismic motions

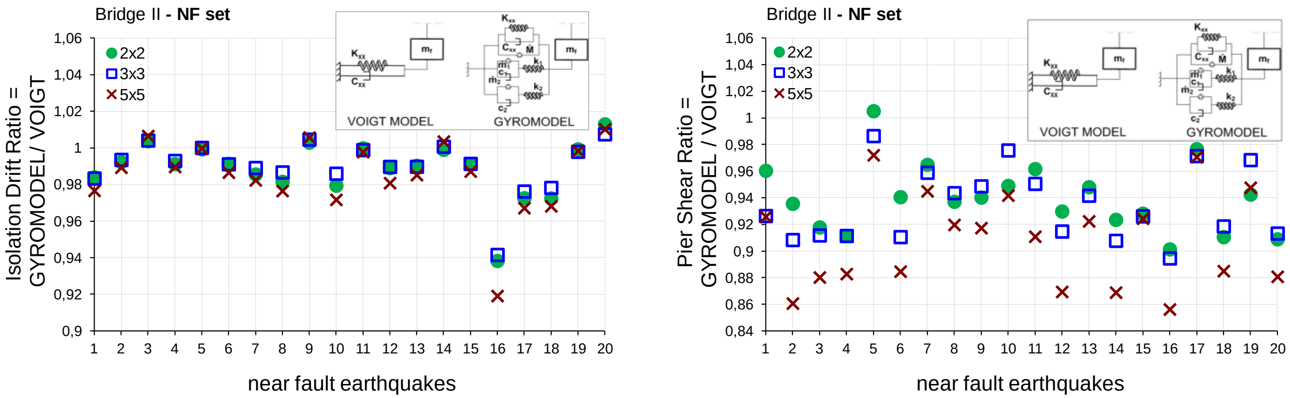


Figure 14. Isolation Drift Ratios and Pier Shear Ratios (Gyromodel/Voigt) for Bridge II and NF set of seismic motions

It is consistent that all Gyromodel responses are smaller than the ones obtained using the Voigt model. For the seismic motion, FF11 (Northridge 1994, Moopark station) there is approximately 15% difference in the isolation system displacements, 16% difference for the pier drift, 19% difference for the pier shear, and 10% for the foundation force and foundation's displacement. Again, the responses of Voigt model are larger than the Gyromodel except the response of the isolation system where the opposite holds as shown in Figure 15d, e, f.

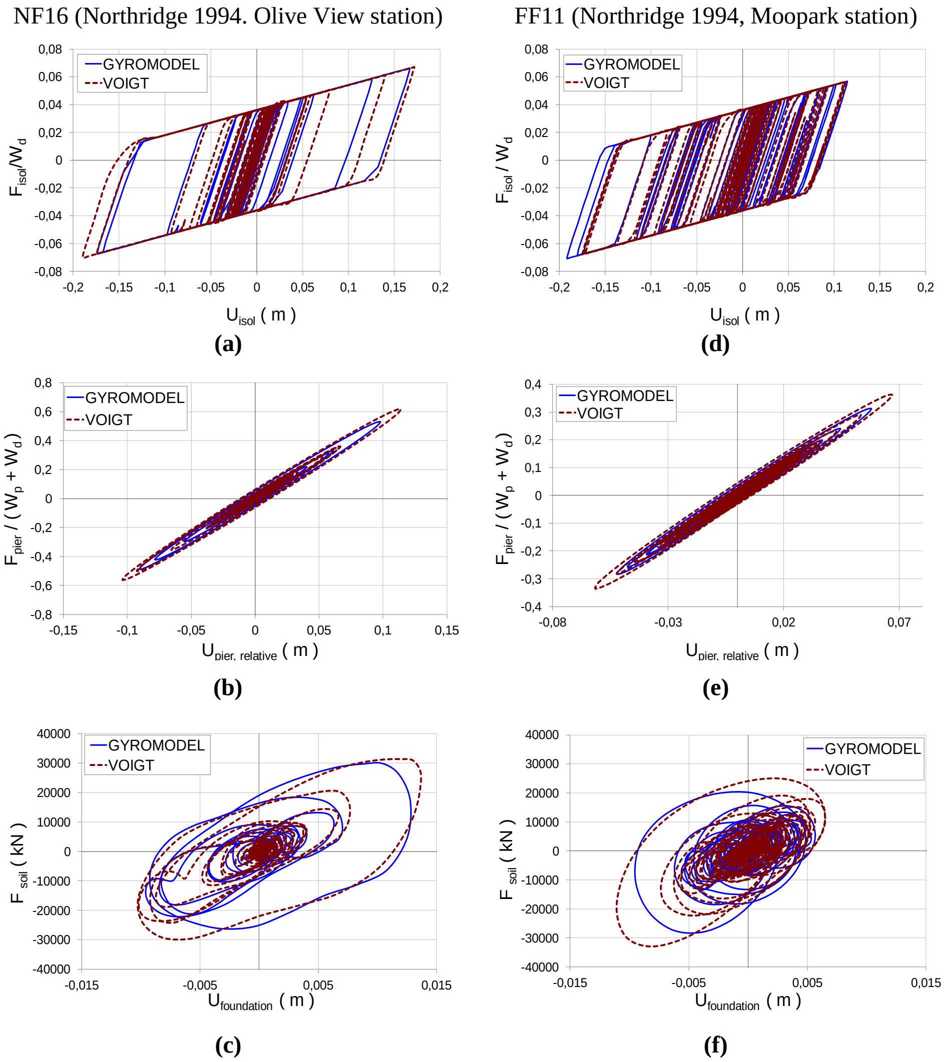


Figure 15. Bridge responses for Bridge II, 5x5 pile group. NF16 motion (Northridge 1994. Olive View station): (a) Isolation system hysteresis, (b) Pier shear vs pier drift, (c) Foundation shear vs foundation drift. FF11 event (Northridge 1994, Moopark station): (d) Isolation system hysteresis, (e) Pier shear vs pier drift, (f) Foundation shear vs foundation drift

5 THE EFFECT OF E_p/E_s

The amplitudes of the dynamic impedances of the soil-foundation system considered here in depend on the number of piles in the pile group, the stiffness of the piles, and the stiffness of the soil; therefore it is natural to investigate the influence of E_p/E_s on the two different modeling approaches of the SSI effects. Bridge II founded on a 2x2 pile group with d=1.8 m (same as previously considered 5x5 pile group) and S/d=10 is utilized in the analyses studying the effect of E_p/E_s. The system will be investigated for 2 soil cases, E_p/E_s=300 and E_p/E_s=1000, with the characteristics of both soil profiles being presented in Table 8. The first soil case is the same used in the previous analyses. The second was chosen so as to represent the ratio E_p/E_s=1000, with the pile constructed by concrete. The Far Field set of seismic motions is used. It should be noted that the case of E_p/E_s=1000, as shown in the table, represents extreme soil properties (V_s= 63 m/s). However, this shear wave velocity level is very close to the values of the soil in the Mexico City valley [4].

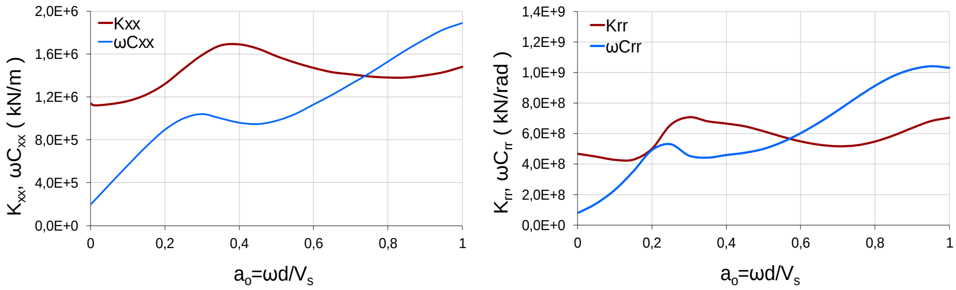
Table 8. Properties of soil profiles

	Soil Case #1	**Soil Case #2**
Total Thickness (m)	21.5	21.5
Soil Profile	Homogeneous Halfspace	Homogeneous Halfspace
Shear Wave Velocity, V_s (m/sec)	110	63
Mass Density, ρ_s (kg/m^3)	1800	1800
Damping Ratio, ξ	0.10	0.10
$G = \rho_s V_s^2$ (MPa)	22	7.1
$E_s = 2(1+v)G$ (MPa)	62	20
E_p/E_s	**300**	**1000**
Poisson's Ratio, ν	0.40	0.40

Figures 16 and 17 plot the soil-foundation impedances for the horizontal and rotational (rocking) degrees of freedom, when E_p/E_s=300 and 1000 respectively. Although for E_p/E_s=1000 the modulus of elasticity of the soil drops approximately 3 times compared to the case of E_p/E_s=300 their shape is similar, that is the peaks and valleys of the impedance curves happen almost at the same frequencies. The values (amplitudes) of dynamic stiffness change significantly and are proportional to the soil's stiffness (E_s).

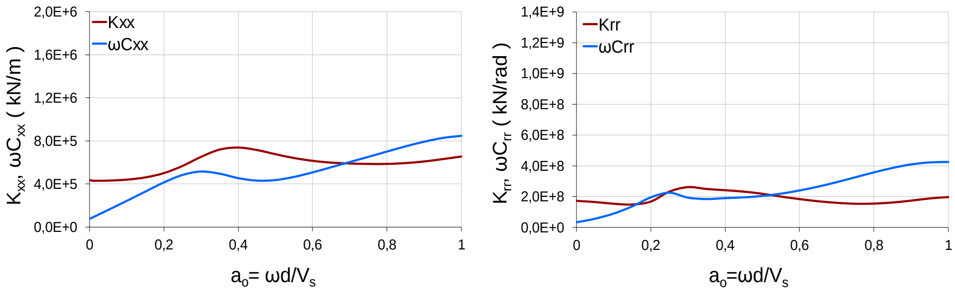
On the other hand the damping, which is represented by a measure of the slope of the blue curve in Figures 16 and 17, as it varies with frequency (a_o)

takes values depending on the frequency distribution of the excitation's main energy Therefore the soil-foundation when modeled with a gyromodel may respond with smaller damping than the Voigt's model when the excitations energy is concentrated in the frequency ranges coinciding with the valleys of the blue curves in the Figures 16 and 17.



(a) (b)

Figure 16. (a) *Kxx*(ω) for Bridge II , with the 2x2 pile group; (b) *Krr*(ω) for Bridge II, with the 2x2 pile group, (d=1.8m, S/d=10, E_p/E_s=300)

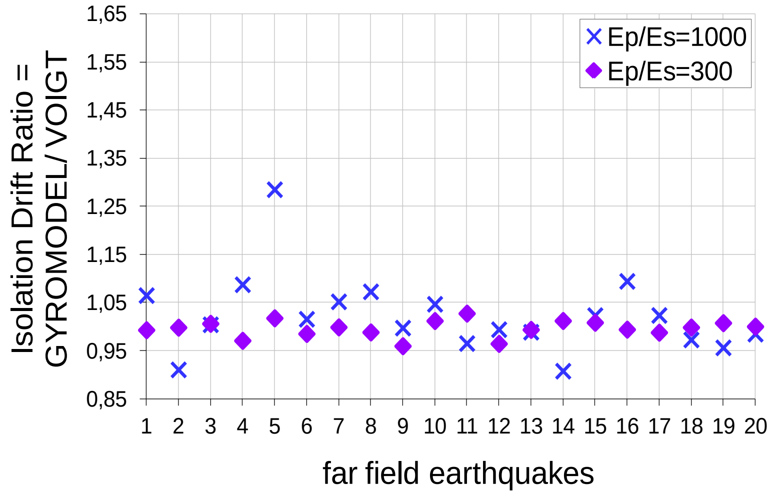


(a) (b)

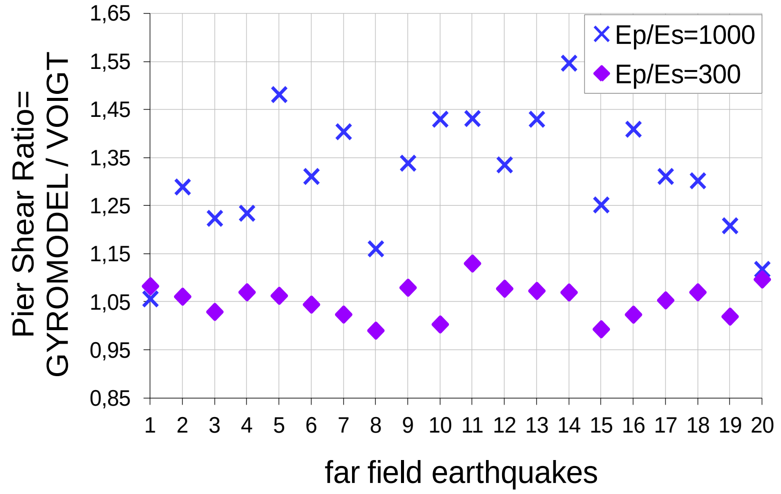
Figure 17. (a) *Kxx*(ω) for Bridge II , with the 2x2 pile group; (b) *Krr*(ω) for Bridge II, with the 2x2 pile group, (d=1.8m, S/d=10, E_p/E_s=1000)

Figure 18 presents the Isolation Drift Ratios and Pier Shear Ratios for Bridge II on a 2x2 pile group with E_p/E_s= 300 and 1000 modeled with a Gyromodel and a Voigt model, when excited with the far field (FF) set of motions. There are minor differences (5% to 10%) between the two modeling approaches for the Isolation Drift Ratio with the exception of one motion (FF05) where Gyromodel analysis gives 28% larger values. Considering the Pier Shear Ratio for the case of E_p/E_s=300 the differences between Gyromodel and Voigt are of the order of 5%. However, for the case of E_p/E_s=1000 the discrepancies are much larger ranging between 15% and 55% higher shear forces when the frequency variation of the stiffness and damping are taken into account with the Gyromodel.

Time history results for isolation system displacements, pier shear forces vs pier drifts, and foundation forces vs foundation drifts are presented in Figure 19 for the FF05 seismic event. The larger responses obtained when frequency dependence is explicitly modeled in the system are easily seen. These differences can be attributed on the fact that damping in Voigt model is overestimated compared to the damping as it varies with frequency with the Gyromodel, see Figure 17a.



(a)



(b)

Figure 18. (a)IDR for Bridge II and FF set; (b)PSR for Bridge II and FF set (2x2 pile group, d=1.8m, S/d=10)

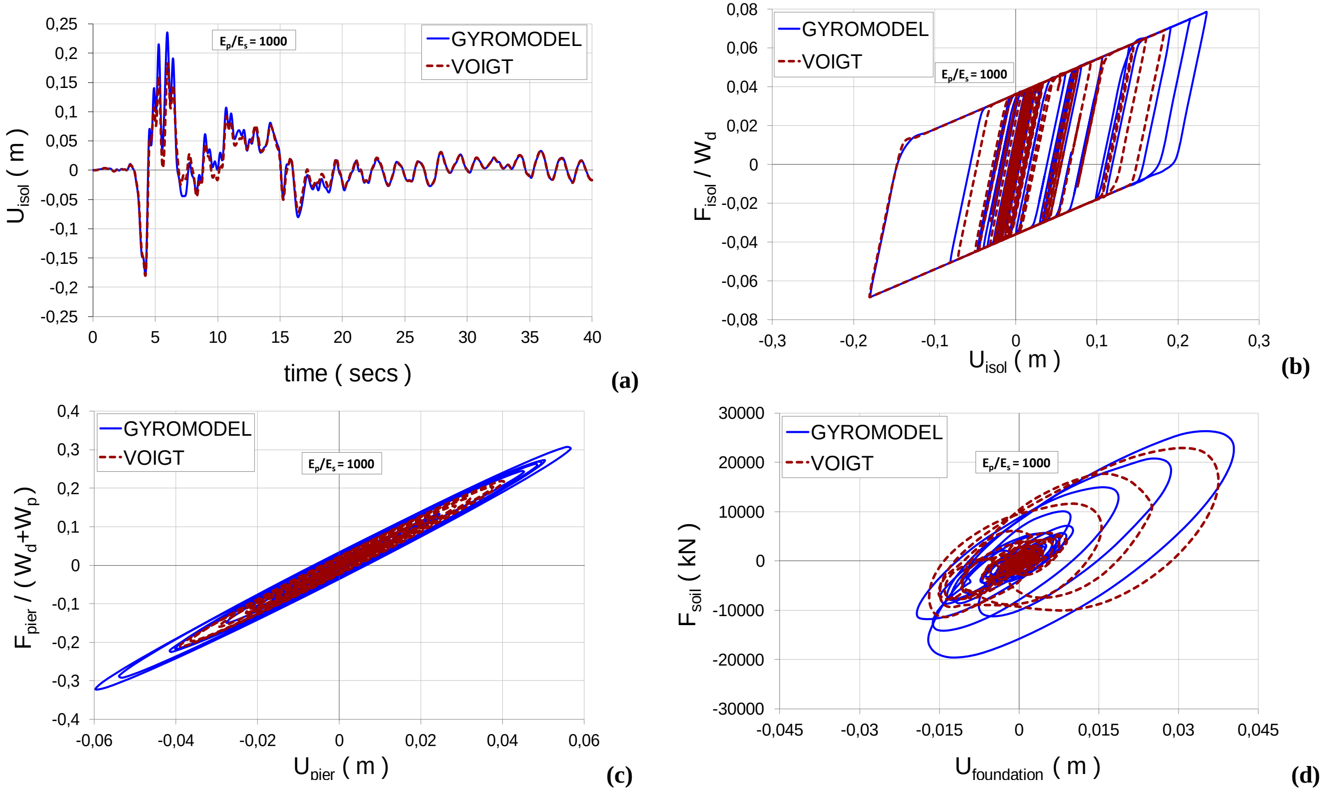


Figure 19. (a) Isolation system displacement, (b) Isolation system hysteretic loop, (c) Pier shear vs pier drift, (a) Foundation shear vs foundation drift for Bridge II, 2x2 pile group (d=1.8m, S/d=10) E_p/E_s=1000, and FF05 seismic motion

6 CONCLUSIONS

The main objective of this study was to evaluate, qualitatively if possible, the effect of the SSI modeling in seismically isolated bridges. Hence, the most important conclusions are:

- This study, utilizing the Gyromodel to account for the frequency dependence of impedances without the introduction of artificial mass in the system, for short stiff bridges (similar to Bridge I) and for the isolation system and the pier response, shows insensitivity of the SSI model used for all three pile groups considered, for both FF and NF seismic motions. For these bridge structural systems there is no loss of accuracy if SSI is incorporated via simple spring dashpot models with frequency independent parametrs. These results are in agreement with the corresponding conclusions in Ucak and Tsopelas [5].
- In tall and flexible bridges (Bridge II) the SSI modeling with frequency independent Voigt models resulted in ±10% differences compared to the Gyromodel for the isolation system displacements. For the pier response, shear force and drift, the observed differences are larger reaching 15%. Again the current results confirm the corresponding conclusions in Ucak and Tsopelas [5]. It is recommended that bridge designers should take an additional step beyond Voigt assemblies to account in their modeling for the frequency dependence of the soil-foundation springs.
- If simple Voigt models are used to account for SSI in a bridge system with a flexible pile group resting on a very soft soil it appears that both isolation system displacements and pier shear forces can be underestimated by as much as 50%. These differences observed for the FF set of motions (no analyses were performed with the NF set of motions). Even though this case, of such a small value of shear wave velocity (V_s=63 m/sec), is very uncommon the results are of some interest when someone considers the nonlinear nature of the soil response in case of strong seismic shaking.
- The results of the last case of analysis Bridge II, 2x2 pile group, E_p/E_s= 1000, FF set of motions revealed the importance of the relative stiffness between the soil and the foundation concerning the choice of SSI models. When flexible soil-foundation systems are considered for analysis and design, it is recommended that frequency dependence of soil-foundation springs must be accurately accounted in modeling utilizing models such as the “Gyromodels” which do not introduce artificial masses into the dynamics of the system. Otherwise large underestimation of the response variables might be possible.

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